



## Volume 32, Issue 3

### Capacity choice in a mixed duopoly with a foreign competitor

Jorge Fernández-Ruiz  
*El Colegio de México*

#### Abstract

This paper analyzes a mixed duopoly in which a public firm and a (possibly partially) foreign-owned firm choose their capacity scales before competing in quantities. We show that the private firm chooses over-capacity, as in previous literature, except if it is completely foreign-owned. In this polar case, the private firm chooses the cost-minimizing capacity scale. We also show that the change in the nationality of the private firm does not essentially alter the public firm's choice, since this firm chooses under-capacity if products are substitutes and over-capacity if they are complements, just as it does when it faces a domestic competitor.

---

I gratefully acknowledge the suggestions of an anonymous referee, which greatly improved this paper. In particular, the suggestion to consider the possibility of partial (instead of total) foreign ownership of the private firm

**Citation:** Jorge Fernández-Ruiz, (2012) "Capacity choice in a mixed duopoly with a foreign competitor", *Economics Bulletin*, Vol. 32 No. 3 pp. 2653-2661.

**Contact:** Jorge Fernández-Ruiz - [jfernan@colmex.mx](mailto:jfernan@colmex.mx).

**Submitted:** June 08, 2012. **Published:** September 25, 2012.

## 1. Introduction.

In this paper we examine firms' capacity choices in a mixed duopoly where a public firm with social welfare objectives competes with a foreign firm with profit objectives in a context where each firm chooses its capacity scale before making its output choice.

The idea that firms may gain a strategic advantage by choosing a capacity scale in excess of their needs has been analyzed in the context of pure private oligopolies by Dixit (1980), Brander and Spencer (1983) and Horiba and Tsutsui (2000), among others.

Several authors (Wen and Sasaki (2001), Nishimori and Ogawa (2004), Lu and Poddar (2005), Ogawa (2006) and Bárcena-Ruiz and Garzón (2007)) have extended the above analysis to mixed oligopolies, where public and private firms coexist. These authors show that the result that firms choose excess capacity does not always hold in mixed oligopolies. In the papers closest to ours, Nishimori and Ogawa (2004) and Ogawa (2006) examine a mixed duopoly in which firms make their capacity choices before competing in quantities. Nishimori and Ogawa (2004) consider a homogeneous product market and show that the private firm chooses over-capacity but the public firm chooses under-capacity. Ogawa (2006) extends Nishimori and Ogawa's model to a framework with product differentiation and shows that the result about the private firm is preserved, since this firm continues to choose over-capacity. But, the public firm chooses under-capacity only if products are substitutes and it chooses over-capacity if products are complements.

All of the above papers assume that all firms are domestic. Yet, many real oligopolies include foreign firms, and this consideration is important because their presence changes the objective function of the public firm. In this paper we fill this gap in the literature. We follow Nishimori and Ogawa (2004) and Ogawa (2006) approach and examine a model where firms make capacity choices before competing in quantities but, unlike these authors, we consider a mixed duopoly in which the private competitor of the public firm is not a domestic firm, but a foreign one. Importantly, we allow for the possibility that this private firm is only partially owned by foreign investors, as in Ogawa and Sanjo (2007) and Wang and Chen (2011), where a private firm may have foreign capital and also some domestic capital, and this translates into a fraction of the private firm's profits being included in the social welfare function<sup>1</sup>. In our analysis we also allow for product differentiation.

We obtain that the private firm chooses over-capacity, as in Nishimori and Ogawa (2004) and Ogawa (2006), except if this firm is completely foreign-owned. In this polar case, the private firm chooses the technically efficient capacity scale. On the other hand, the change in the nationality of the private firm does not essentially alter the public firm's choice, since the result in Ogawa (2006) is preserved: the public firm chooses under-capacity if products are substitutes and over-capacity if they are complements.

---

<sup>1</sup> Fjell and Pal (1996) also suggest situations that lead to this possibility.

We also examine how the extent of over-capacity or under-capacity varies with the degree of foreign ownership of the private firm. We find that as the share of foreign capital in the private firm increases, the extent of over-capacity that this firm chooses diminishes. Similarly, a higher share of foreign capital leads to less under-capacity (over-capacity) in the public firm choice when products are substitutes (complements).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes firms' output and capacity decisions and Section 4 concludes.

## 2. The Model.

We consider a duopolistic market in which a foreign private firm (firm 1) competes with a public firm (firm 2). The inverse demands functions for these firms' products are given by:

$$p_i = a - q_i - bq_j \quad (1)$$

where  $q_i$  represents firm  $i$ 's output and  $p_i$  its price ( $i, j = 1, 2, i \neq j$ ). If  $b \in (0, 1]$  products are substitutes<sup>2</sup> and if  $b \in (-1, 0)$  they are complements.

These demand functions are derived from the maximization of an utility function for the representative consumer assumed to be quadratic, strictly concave and symmetric in  $q_1$  and  $q_2$ , as in Vives (1986), Furth and Kovenock (1993) and Bárcena-Ruiz and Garzón (2007):

$$U(q_1, q_2) = a(q_1 + q_2) - (q_1^2 + 2bq_1q_2 + q_2^2) / 2$$

Inserting the demand functions in (1) into the consumer surplus

$$CS = U(q_1, q_2) - p_1q_1 - p_2q_2, \text{ we obtain}^3$$

$$CS = (q_1^2 + q_2^2) / 2 + bq_1q_2 \quad (2)$$

<sup>2</sup> When  $b=1$ , the two products are in fact perfect substitutes, a case which is worth considering, has received special attention in the literature and can also be analyzed within this framework.

<sup>3</sup> Notice that when  $b = 1$ , CS reduces to the expression  $(q_1+q_2)^2/2$ , as expected in the homogeneous product case.

Firm  $i$ 's technology is represented by the cost function  $C_i(q_i, x_i)$ , where  $q_i$  is the firm's output and  $x_i$  is the firm's production capacity. Following Vives (1986), Horiba and Tsutsui (2000) and Nishimori and Ogawa (2004), we assume that the cost function takes the form

$$C_i(q_i, x_i) = m_i q_i + (q_i - x_i)^2 \quad (3)$$

Under this cost function, the long-run average cost is minimized when quantity equals production capacity, and both excess capacity and under-capacity are inefficient.

Firm 1's objective function is its profit:

$$\pi_1 = p_1 q_1 - m_1 q_1 - (q_1 - x_1)^2 \quad (4)$$

In contrast, the objective function of firm 2 is social welfare, defined as the sum of consumer surplus, the profit of the public firm, and a fraction  $\theta$  of the profit of the private firm:

$$SW = CS + \pi_2 + \theta \pi_1 \quad (5)$$

where  $\pi_2 = p_2 q_2 - m_2 q_2 - (q_2 - x_2)^2$ , and  $\theta \in [0, 1]$  represents the share of domestic capital in the private firm. When  $\theta = 1$ , the private firm is completely domestic and the model coincides with that in Ogawa (2006). When  $\theta = 0$ , the private firm is wholly owned by foreign shareholders and its profits are completely excluded from the social welfare function, which is a common assumption in models analyzing foreign competition in mixed markets. But, the social welfare function in (5) also allows for situations in which foreign investors own only a fraction smaller than one of the private firm, a fact that translates into this firm's profits entering into the social welfare function with a smaller weight than the public firm's profit, as in Ogawa and Sanjo (2007).

We consider the following two-stage game: in the first stage, each firm chooses its production capacity. In the second stage, each firm chooses its output level knowing both firms' capacity choices.

### 3. Results.

To look for a subgame perfect equilibrium, we solve the game backwards and examine first the last stage of the game.

In the second stage of the game, given the production capacities  $x_1$  and  $x_2$ , when firm 1 chooses  $q_1$  to maximize its profit  $\pi_1$  as given in (4) and firm 2 chooses  $q_2$  to maximize social welfare as given in (5) we obtain, after solving for  $q_1$  and  $q_2$ :

$$q_1 = \frac{(3-b)a - 3m_1 + bm_2 + 6x_1 - 2bx_2}{12 - \theta b^2} \quad (6)$$

$$q_2 = \frac{a(4-b\theta) - 2\theta bx_1 + 8x_2 + \theta bm_1 - 4m_2}{12 - \theta b^2} \quad (7)$$

In the first stage of the game, each firm  $i$ ,  $i=1, 2$ , chooses its production capacity  $x_i$ .

Maximization of firm 1's profit  $\pi_1$  anticipating  $q_1$  and  $q_2$  as given by (6) and (7) yields

$$x_1 = \frac{12(3-b)a - 24bx_2 - 36m_1 + 12bm_2}{b^4\theta^2 - 24b^2\theta + 72} \quad (8)$$

Similarly, maximization of social welfare, as given in (5), with respect to the public firm's production capacity  $x_2$ , given the output choices in (6) and (7) yields

$$x_2 = \frac{(b^3\theta^2 - 4\theta b(b+3) - 3b + b^2 + 48)a - 2b(12\theta - b^2\theta^2 + 3)x_1}{b^4\theta^2 - 16b^2\theta - 2b^2 + 48} + \frac{(12\theta b + 3b - b^3\theta^2)m_1 + (4\theta b^2 - b^2 - 48)m_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 48} \quad (9)$$

Solving for  $x_1$  and  $x_2$  in (8) and (9) yields

$$x_1 = \frac{12(1-b)a - 12m_1 + 12bm_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (10)$$

$$x_2 = \frac{(24 - 3b + b^3\theta^2 - 12b\theta - 4b^2\theta + b^2)a + (12b\theta + 3b - b^3\theta^2)m_1}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} + \frac{(4\theta b^2 - b^2 - 24)m_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (11)$$

Replacing (10) and (11) in (6) and (7) we obtain<sup>4</sup>

$$q_1 = \frac{(12 - b^2\theta)(1 - b)a - (12 - b^2\theta)m_1 + (12 - b^2\theta)bm_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (12)$$

$$q_2 = \frac{(b^3\theta^2 - 4b^2\theta - 12b\theta - 2b + 24)a + (2b + 12b\theta - b^3\theta^2)m_1 + (4b^2\theta - 24)m_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (13)$$

We thus have, from (10) - (13):

$$x_1 - q_1 = b^2\theta \frac{(1 - b)a - m_1 + bm_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (14)$$

$$x_2 - q_2 = -b \frac{(1 - b)a - m_1 + bm_2}{b^4\theta^2 - 16b^2\theta - 2b^2 + 24} \quad (15)$$

And from (14) and (15) we obtain the following result

**Proposition.** In the mixed duopoly with a foreign private firm, the public firm chooses under-capacity ( $x_2 < q_2$ ) when products are substitutes, and it chooses over-capacity ( $x_2 > q_2$ ) when products are complements. In both cases, the private firm chooses over-capacity ( $x_1 > q_1$ ) if the share of domestic capital is positive ( $\theta > 0$ ), and it chooses the capacity level that minimizes long-run average cost ( $x_1 = q_1$ ) in the polar case in which the firm is completely foreign-owned ( $\theta = 0$ ).

*Proof:* Notice first that the term  $[(1 - b)a - m_1 + bm_2]$  in the numerator in equations (14) and (15) is strictly positive because it has the same sign as  $q_1$  in equation (12). Since the term  $[24 + b^4\theta^2 - 16b^2\theta - 2b^2]$  in the denominator in these two equations is also strictly positive, we have that: i)  $(x_2 - q_2)$  in equation (15) is strictly negative if  $b > 0$  and strictly positive if  $b < 0$ , and ii)  $(x_1 - q_1)$  is strictly positive (zero) if  $\theta$  is strictly positive (zero).  $\square$

Since the public firm cares about consumer surplus, it tries to induce the private firm to increase its output. We see from (6) that the relationship between the private firm's output  $q_1$  and the public firm's capacity level  $x_2$  is negative when  $b > 0$  and positive when  $b < 0$ .

<sup>4</sup> We assume that  $a$  is sufficiently large in relation to  $b$ ,  $m_1$  and  $m_2$ , that  $x_1$ ,  $x_2$ ,  $q_1$  and  $q_2$  in (10)-(13) are all strictly positive. If  $b=1$ , we also need that  $m_1 < m_2$  for  $x_1$  and  $q_1$  to be strictly positive.

Therefore, to induce an increase in the private firm's output the public firm reduces its capacity level in the first case and increases it in the second case.

To understand the private firm's capacity choice notice from (7) that, when  $\theta > 0$ , the public firm reduces its output level  $q_2$  in response to an increase in the private firm's capacity level  $x_1$ . Therefore, the private firm increases its capacity scale above the cost-minimizing level ( $x_1 > q_1$ ) to make the public firm reduce its output level. In contrast, when  $\theta = 0$  (which means that the private firm is completely foreign-owned) equation (7) shows that the public firm output level is unaffected by the private firm capacity level  $x_1$ . In this polar case, the private firm has no reason to deviate from the cost-minimizing capacity choice ( $x_1 = q_1$ ).

We now examine the effect on  $(x_1 - q_1)$  and  $(x_2 - q_2)$  of a change in the nationality of the private firm –as measured by  $\theta$ .

Differentiating  $(x_1 - q_1)$  in (14) we obtain:

$$\frac{\partial(x_1 - q_1)}{\partial\theta} = \frac{b^2[(1-b)a - m_1 + bm_2][24 - 2b^2 - \theta^2b^4]}{[b^4\theta^2 - 16b^2\theta - 2b^2 + 24]^2} \quad (16)$$

Which is positive since both  $[(1-b)a - m_1 + bm_2]$  and  $[24 - 2b^2 - \theta^2b^4]$  are positive. Thus, as the share of foreign capital  $(1-\theta)$  increases, the extent of over-capacity  $(x_1 - q_1)$  decreases and, as we stated above, completely vanishes in the polar case of a wholly foreign-owned private firm ( $\theta = 0$ ).

Differentiating  $(x_2 - q_2)$  in (15) yields

$$\frac{\partial(x_2 - q_2)}{\partial\theta} = \frac{b^3[(1-b)a - m_1 + bm_2][-16 + 2b^2\theta]}{[b^4\theta^2 - 16b^2\theta - 2b^2 + 24]^2} \quad (17)$$

Which is positive if  $b < 0$  and negative if  $b > 0$ . Thus, when products are complements ( $b < 0$ ) and, therefore,  $(x_2 - q_2) > 0$ , the extent of over-capacity  $(x_2 - q_2)$  diminishes as the share of foreign capital  $(1-\theta)$  increases. Similarly, when products are substitutes ( $b > 0$ ) and, thus,  $(x_2 - q_2) < 0$ , as the share of foreign capital  $1-\theta$  increases the public firm also moves closer to the cost-minimizing capacity choice.

#### 4. Conclusion.

In this paper we have examined firms' capacity choices in a mixed duopoly where the private competitor of the public firm is a foreign firm, allowing for the possibility of partial foreign ownership. We have found that the private firm chooses over-capacity, as in previous literature focusing on the domestic case, unless it is completely foreign-owned, in which case it chooses the cost-minimizing capacity scale instead. On the other hand, the public firm capacity choice is not essentially changed by the nationality of the private firm: just as in the case of a domestic competitor (Ogawa, 2006), the public firm chooses under-capacity when products are substitutes and it chooses over-capacity when products are complements. We have also shown that, as the share of foreign capital in the private firm increases, the extent of over-capacity that this firm chooses diminishes. Similarly, the extent of under-capacity that the public firm chooses when products are substitutes (or over-capacity when products are complements) diminishes as the share of foreign capital in the private firm increases.

#### References

Bárcena-Ruiz, J.C. and M.B Garzón (2007) "Capacity Choice in a Mixed Duopoly under Price Competition", *Economics Bulletin* 12, 26, 1-7.

Brander, J. A. and B. J. Spencer (1983) "Strategic Commitment with R&D: the Symmetric Case" *Bell Journal of Economics* 14, 225-235.

Dixit, A. (1980) "The Role of Investment in Entry Deterrence" *Economic Journal* 90, 95-106.

Fjell, K., and D. Pal (1996) "A Mixed Oligopoly in the Presence of Foreign Private Firms" *Canadian Journal of Economics* 29, 737-743.

Furth, D. and D. Kovenock (1993) "Price Leadership in a Duopoly with Capacity Constraints and Product Differentiation" *Journal of Economics* 57, 1-35.

Horiba, Y. and S. Tsutsui (2000) "International Duopoly, Tariff Policies and the Case of Free Trade" *Japanese Economic Review* 51, 207-220.

Lu, Y. and S. Poddar (2005) "Mixed Oligopoly and the Choice of Capacity" *Research in Economics* 59, 365-374.

Nishimori, A. and H. Ogawa (2004) "Do Firms always Choose Excess Capacity" *Economics Bulletin* 12, 2, 1-7.



Ogawa, H. (2006) "Capacity Choice in the Mixed Duopoly with Product Differentiation" *Economics Bulletin* 12, 8, 1-6

Ogawa, H. and Y. Sanjo (2007) "Location of Public Firm in the Presence of Multinational Firm: A Mixed Duopoly Approach" *Australian Economic Papers* 46, 191-203.

Vives, X. (1986) "Commitment, Flexibility, and Market Outcomes" *International Journal of Industrial Organization* 4, 217-229.

Wang, L. and T. Chen (2001) "Mixed Oligopoly, Optimal Privatization, and Foreign Penetration" *Economic Modelling* 28, 1465-1470.

Wen, M. and D. Sasaki (2001) "Would Excess Capacity in Public Firms be Socially Optimal?" *Economic Record* 77, 283-290.