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Delegated versus Nondelegated Decision Making

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Abstract

This paper analyzes the value of menus of contracts in a context of delegated expertise with nondelegated decision making. Firms delegate to managers the task of obtaining information about strategic decisions, and they choose whether or not to delegate the actual decision-making process to these managers. A delegated decision-making process involves offering the expert a contract that stipulates different payoffs contingent on verifiable results, whereas a nondelegated decision-making process involves a menu of contracts from which the expert can choose. We find that menus of contracts are valuable because they induce efficient decision making at lower cost.

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1 Introduction

The role of managers is not only to run the organization in line with the board's direction but also to present the board with recommendations useful for making strategic decisions. The problem is that to acquire and process information about different decisions is costly for managers. Because effort and information are nonverifiable, directors use contracts to provide the manager with appropriate incentives. Once directors has delegated the acquisition of information to a manager, they can choose between a nondelegated and a delegated decision-making process. In nondelegated decision making, the board delegates to a manager the task of acquiring information about different decisions under a contract designed to provide the manager with incentives to exert effort in obtaining information and make truthful recommendations to directors, who make the final decisions. In delegated decision making, board of directors delegate to the manager the task of choosing among different decisions and use the design of incentive contracts to provide the manager with incentives not only to exert effort to acquire information but also to make efficient decisions contingent on that information.

The usual analytical framework for analyzing executive compensation is the principal–agent model with hidden action. However, that framework is of limited usefulness in the environments contemplated in this paper. In particular, with nondelegated decision making, the problem for shareholders is not only how to provide incentives for the manager to learn which decision is optimal but also how to provide incentives to make truthful recommendations. In this case, an incentive problem arises between principals and agents (managers) that combines moral hazard and adverse selection. The principal offers to the agent a menu of contracts from which the agent can choose after exerting effort and learning about the profitability of different decisions.

The objective of this paper is to compare both types of decision making in terms of compensation contracts. To make a comparison with an environment of delegated decision making we refer to Celentani et al. (2010), who propose a *delegated expertise* and *decision-making* model to analyze executive compensation. Given that reports are verifiable, the first question we address is whether or not compensation should depend on the manager's report. We then use a numerical example to show that it is less costly to secure effort and optimal decisions when the manager need only obtain information and make reports than when the manager must use this information to make strategic decisions. In line with Moers (2006), we find that menus of contracts yield better solutions to incentive problems stemming from the quality of the contract's performance measures. Two works that assess —similar to this paper's— the value of menus are Lambert (1986) and Zermeño (2011). These authors find that menus of contracts are valuable because they do not distort decision making.

2 The model

We consider a firm with a risk-neutral owner who delegates to a risk-averse manager the task of investigating the return distributions of two alternative projects. The projects may represent the consequences of acquiring (or not) a potential target, or of investigating the development of two alternative technologies. The returns of project $i \in \{A, B\}$, denoted by r_i, can take two values: S (success) and F (failure). These returns are independently distributed, with p_A and p_B denoting the success probability of projects A and B, respectively, which are common knowledge. Without loss of generality we assume that $p_A > p_B$.

Assumption 1. *Project A is ex ante optimal: $p_A > p_B$.*

This assumption reflects that, generically, prior information will suggest one of the two projects as being more likely to succeed. To obtain additional information about the return distributions of both projects, the manager must exert costly effort e, which the owner cannot observe. Absent exerting the necessary effort ($e = 0$), the manager obtains no additional information about the probability distributions of r_A and r_B, and by expending the necessary effort ($e = 1$), the manager incurs in a cost of effort g and obtains a pair of private and independent noisy signals, $\sigma = (\sigma_A, \sigma_B)$, of the respective returns r_A and r_B. We assume that each signal σ_i can take one of two values, $\sigma_i \in \{H, L\}$, and has a probability distribution given by:

$$\Pr(\sigma_i = H \mid (r_i, r_j)) = \begin{cases} 1 - \varepsilon & \text{if } r_i = S \\ \varepsilon & \text{if } r_i = F \end{cases}, \; i \in \{A, B\}.$$

Parameter $\varepsilon \in \left[0, \frac{1}{2}\right]$ represents the precision of the signals, with a lower ε indicating greater precision. Our assumptions about the distribution of (r_A, r_B) and (σ_A, σ_B) ensure that each signal σ_i provides information only about the realization of returns of the corresponding project i: H is a favorable signal about r_i and L is an unfavorable signal. We let $\Sigma \equiv \{L, H\} \times \{L, H\}$, let $\sigma \in \Sigma$, and let π_σ be the unconditional probability of the manager receiving signal pair σ after exerting effort. We use the following notation for the probability of success conditional on the realization of the signal pair σ and a decision rule $d(\sigma)$:

$$q_\sigma^{d(\sigma)} = \Pr(S \mid \sigma, d(\sigma)).$$

Conditional on obtaining the signal pair σ, project i is optimal if $q_{\sigma_i}^i > q_{\sigma_j}^j$. It follows immediately from $p_A > p_B$ that if $\varepsilon < \frac{1}{2}$ then $q_H^A > q_L^A > q_L^B$ and $q_H^A > q_H^B > q_L^B$. Thus, for the signals to have decision value they must be precise enough that $q_L^A < q_H^B$ hence the optimal project conditional on $\sigma = LH$ is B. To ensure that $q_L^A < q_H^B$, we make the following technical assumption.

Assumption 2. $\varepsilon < K$.

If we let $\delta^*(\sigma)$ be the optimal decision conditional on the signals, then Assumption 2 implies that $\delta^*(LH) = B$ and $\delta^*(\sigma) = A$ otherwise. We assume that the owner prefers to implement the optimal decision rule.

At the contracting stage, we assume that the firm's owner makes a take-it-or-leave-it offer to the manager that consists of a menu of contracts, one for each realization of the vector of signals $\sigma \in \Sigma$. Each contract specifies a decision rule, $d(\sigma)$, and a wage schedule, w_σ, where $w_\sigma = \left(w_\sigma^S, w_\sigma^F\right)$. By expending effort, the manager obtains a signal pair (before the return of project i is realized) and then chooses an element from the menu of contracts. We assume that this selection is observable but that the manager cannot commit to making honest recommendations. We assume that the manager has reservation utility $\overline{U}$ and preferences described by a Bernoulli utility function $u(w) - g(e)(g(1) - g(0) = g > 0)$, $u(\cdot)$ continuously differentiable, with $u'(\cdot) > 0$ and $u''(w) < 0$.

The extensive form of the game is summarized as follows:

1. The principal makes a take-it-or-leave-it contract offer $w = (d(\sigma), w_\sigma)$, $\sigma \in \Sigma$. Each pair $(d(\sigma), w_\sigma)$ represents a decision rule and a salary payment for each of the possible public histories following the acceptance of the contract by the manager (combinations of a report and a realization of return).[1]
2. The manager accepts or rejects.
   (a) If the manager rejects, the game ends and he obtains reservation utility $\overline{U}$.
   (b) If the manager accepts, he is hired.
      i. The manager chooses whether to exert effort, $e = 1$, or not, $e = 0$.
      ii. Nature privately chooses the vector of return realizations $r = (r_A, r_B) \in \{F, S\}^2$.
      iii. If the manager has not exerted effort (information set 0), he chooses a pair $(d(\sigma), w_\sigma)$ of the contract.
      iv. If the manager has exerted effort, he receives a signal pair $\sigma \in \Sigma$ and chooses a pair $(d(\sigma), w_\sigma)$ of the contract.
      v. The principal makes a decision $d(\sigma) = \{A, B\}$.
      vi. The return realization is publicly observed.
      vii. The principal pays the manager the salary associated with the realized public history.

3 The optimal contracting problem

Following Grossman and Hart (1983), we rewrite the problem of finding the vector of payments with the minimum cost as the problem of finding the vector of utility levels for the agent that satisfy the agent's incentive compatibility and participation constraints at minimum cost to the principal. We define $s_\sigma = u\left(w_\sigma^S\right)$, $f_\sigma = u\left(w_\sigma^F\right)$, $u_\sigma = (s_\sigma, f_\sigma)$, and $\upsilon = u^{-1}$. We regard $u_\sigma = (s_\sigma, f_\sigma)$ as the principal's control variables, and the principal's problem of implementing decision rule $d(\sigma)$ can be written as:

[1] In the case of delegated decision making, a contract specifies a salary payment for every possible public history following the acceptance of the contract, i.e., combinations of a decision and a realization of return: $w = (w_{AF}, w_{AS}, w_{BF}, w_{BS})$.

$$\min_{u} \quad \Sigma_\sigma \pi_\sigma \left[q_\sigma^{d(\sigma)} \upsilon\left(s_\sigma\right) + \left(1 - q_\sigma^{d(\sigma)}\right) \upsilon\left(f_\sigma\right)\right] \tag{P}$$

$$\text{s.t.} \quad \Sigma_\sigma \pi_\sigma \left[q_\sigma^{d(\sigma)} s_\sigma + \left(1 - q_\sigma^{d(\sigma)}\right) f_\sigma\right] - g \geq \overline{U} \tag{PC}$$

$$\Sigma_\sigma \pi_\sigma \left[q_\sigma^{d(\sigma)} s_\sigma + \left(1 - q_\sigma^{d(\sigma)}\right) f_\sigma\right] - g \geq p_{d(\sigma)} s_\sigma + \left(1 - p_{d(\sigma)}\right) f_\sigma \quad \forall \sigma \in \Sigma \tag{IC\_σ}$$

$$q_\sigma^{d(\sigma)} s_\sigma + \left(1 - q_\sigma^{d(\sigma)}\right) f_\sigma \geq q_\sigma^{d(\sigma')} s_{\sigma'} + \left(1 - q_\sigma^{d(\sigma')}\right) f_{\sigma'} \quad \forall \sigma' \in \Sigma, \ \forall \sigma \in \Sigma, \ \sigma \neq \sigma' \tag{$\sigma - \sigma'$}$$

(P) is a simple optimization problem: minimize a convex function subject to a finite number of linear constraints, after which the Kuhn–Tucker theorem yields necessary and sufficient conditions for optimality. Inequality (PC) is the agent's participation constraint. Inequalities (IC_σ) are a group of four ex ante incentive compatibility constraints. Each of these constraints guarantees that the agent prefers to exert effort and choose the contract specified for each signal over not exerting effort and choosing one of the contracts. For example, IC_{LH} ensures that the agent does not want simply to choose contract u_{LH} and not exert effort:

$$\Sigma_\sigma \pi_\sigma \left[q_\sigma^{d(\sigma)} s_\sigma + \left(1 - q_\sigma^{d(\sigma)}\right) f_\sigma\right] - g \geq p_B s_{LH} + \left(1 - p_B\right) f_{LH}.$$

Finally, inequalities $(\sigma - \sigma')$ are a group of twelve interim incentive compatibility constraints. Each of these constraints guarantees that, for each signal, the agent prefers to choose the contract specified for that signal over any other contract. For instance, HH-LH guarantees that the agent, after observing $\sigma = HH$, chooses contract u_{HH} instead of contract u_{LH}:

$$q_H^A s_{HH} + \left(1 - q_H^A\right) f_{HH} \geq q_H^B s_{LH} + \left(1 - q_H^B\right) f_{LH}.$$

4 Results

Lemma 1. *$w_{HH}^S = w_{HL}^S$ and $w_{HH}^F = w_{HL}^F$.*

Proof. HH-HL and HL-HH imply

$$q_H^A s_{HH} + \left(1 - q_H^A\right) f_{HH} = q_H^A s_{HL} + \left(1 - q_H^A\right) f_{HL}. \tag{1}$$

Assume $s_{HH} > s_{HL}$. By (1), $f_{HH} < f_{HL}$ and $s_{HH} - f_{HH} > s_{HL} - f_{HL}$. But if $s_{HH} = s_{HL}$ and $f_{HH} = f_{HL}$, then all the constraints still hold and the expected cost of the contract is reduced because of risk aversion. □

Lemma 1 states that, if two signal pairs lead to the same probability of success conditional on making the optimal decision, then the two contracts must be the same.

Proposition 1. $w^F_{HH} < w^S_{HH}$, $w^F_{LH} < w^S_{LH}$, *and* $w^F_{LL} > w^S_{LL}$.

Proof. Appendix. □

Proposition 1 shows that it is generically optimal to make the manager's pay depend on reports even when additional constraints on contracting must be imposed to ensure that the manager behaves honestly. This result contrasts with Gromb and Martimort's (2007) result that, when reports are manipulable, contract terms must be based on the project's actual outcome and not on the reports. Moreover, it shows that managers are rewarded more when their reports are in line with performance, which motivates each manager to learn and report truthfully. In particular, a manager who reports on a pair of unfavorable signals will be paid *more* in the case of failure.

Proposition 2. $\max\left\{w^F_{HH}, w^F_{LH}, w^S_{LL}\right\} < \min\left\{w^S_{HH}, w^S_{LH}, w^F_{LL}\right\}$.

Proof. Appendix □

With delegated decision making, there is a unique payoff per decision and pay is increasing in returns: $\max\left\{w^F_A, w^F_B\right\} < \min\left\{w^S_A, w^S_B\right\}$.[2] This guarantees that an agent who exerts effort has incentives to make optimal decisions. However, Proposition 5 shows that with nondelegated decision making, i.e., with menus, different state-contingent payoffs can lead to the same decision. In particular, $\delta^*(LL) = \delta^*(HH) = A$ but $w^S_{LL} \neq w^S_{HH}$ and $w^F_{HH} \neq w^F_{LL}$. That is, a manager will be paid *more*, conditional on success (failure), if he reports on a pair of favorable (unfavorable) rather than unfavorable (favorable) signals, even when both reports lead to the firm to choose A, because success (failure) and a pair of favorable (unfavorable) signals are more informative about how well the agent's task was performed. So, for providing the manager with the corresponding incentives,[3] the measures of performance used in a nondelegated decision-making process (reports and outcomes) are more informative that the measures of performance used in a delegated decision-making process (decisions and outcomes), which results in a less risky contract. Since the manager is risk averse, this reduces the cost of the contract to the principal.

Example 1. Suppose that $U(w) = \frac{w^{1-\sigma}}{1-\sigma}$, $\sigma = 1.2$, $p_A = 0.7$, $p_B = 0.6$, $\varepsilon = 0.3$, $\overline{U} = \frac{3^{1-\sigma}}{1-\sigma}$, and $g = 0.0273$. Without menus, the optimal contract that provides incentives to exert effort to acquire information and make efficient decisions is

$$w^F_A = 2.2323 < w^F_B = 2.3757 < w^S_A = 3.4241 < w^S_B = 3.4908,$$

and the expected payment made by the principal is $w^e = 3.1606$.

In contrast, the optimal menu of contracts that provides appropriate incentives to exert effort and make truthful recommendations is

$$w^F_{HH} = 2.5433 < w^F_{LH} = 2.6719 < w^S_{LL} = 2.8641 < w^S_{HH} = 3.2255 < w^S_{LH} = 3.2457 < w^F_{LL} = 3.3484.$$

Here the expected payment is lower, $w^e = 3.1167$.

[2] See Proposition 2 in Celentani et al. (2010).

[3] To acquire information and make optimal decisions in a delegated environment, and to acquire information and report it truthfully in a nondelegated environment.

5 Conclusions

We have studied the incentive problem that arises when firms delegate to managers the task of investigating the return distributions of different risky projects. The firm can observe the reports made by the manager, but this report is fully manipulable. Moreover, the firm does not observe whether the manager exerted the necessary effort to acquire information about the projects. The firm's problem is to design a contract that provides incentives for the manager both to exert effort in obtaining information (moral hazard problem) and to report that information truthfully (adverse selection problem). In this context, we find that a menu of contracts is valuable because it provides the right incentives at a lower cost than the contract that arises when the owner delegates decision making to the manager and offers a unique contract contingent on observables. The reason is that menus of contracts set different state-contingent payoffs for the same decision, which leads to more informative signals and then, to less risky contracts.

6 References

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Appendix

We denote by λ, μ_σ, and $\gamma^{\sigma}_{\sigma'}$ the corresponding multipliers. Define $\mu \equiv \Sigma_\sigma \mu_\sigma$.

The first-order conditions are:

$$v'(s_{HH}) = (\lambda+\mu) - \frac{\mu_{HH} p_A}{q_H^A \pi_{HH}} + \frac{\left(\gamma^{HH}_{LH} + \gamma^{HH}_{LL} + \gamma^{HL}_{LH}\right)}{\pi_{HH}} - \frac{\gamma^{LL}_{HH} q_L^A}{q_H^A \pi_{HH}}, \tag{A.1}$$

$$v'(f_{HH}) = (\lambda+\mu) - \frac{\mu_{HH}(1-p_A)}{(1-q_H^A)\pi_{HH}} + \frac{\left(\gamma^{HH}_{LH} + \gamma^{HH}_{LL} + \gamma^{HL}_{LH}\right)}{\pi_{HH}} - \frac{\gamma^{LL}_{HH}\left(1-q_L^A\right)}{(1-q_H^A)\pi_{HH}}, \tag{A.2}$$

$$v'(s_{LH}) = (\lambda+\mu) - \frac{\mu_{LH} p_B}{\pi_{LH} q_H^B} - \frac{\left(\gamma^{HH}_{LH} - \gamma^{LH}_{LL}\right)}{\pi_{LH}} - \frac{\left(\gamma^{LL}_{LH} + \gamma^{HL}_{LH}\right) q_L^B}{\pi_{LH} q_H^B}, \tag{A.3}$$

$$v'(f_{LH}) = (\lambda+\mu) - \frac{\mu_{LH}(1-p_B)}{\pi_{LH}(1-q_H^B)} - \frac{\left(\gamma^{HH}_{LH} - \gamma^{LH}_{LL}\right)}{\pi_{LH}} - \frac{\left(\gamma^{LL}_{LH} + \gamma^{HL}_{LH}\right)\left(1-q_L^B\right)}{\pi_{LH}(1-q_H^B)}, \tag{A.4}$$

$$v'(s_{LL}) = (\lambda+\mu) - \frac{\mu_{LL} p_A}{\pi_{LL} q_L^A} - \frac{\gamma^{HH}_{LL} q_H^A}{\pi_{LL} q_L^A} - \frac{\left(\gamma^{LH}_{LL} - \gamma^{LL}_{HH} - \gamma^{LL}_{LH}\right)}{\pi_{LL}}, \tag{A.5}$$

$$v'(f_{LL}) = (\lambda+\mu) - \frac{\mu_{LL}(1-p_A)}{\pi_{LL}(1-q_L^A)} - \frac{\gamma^{HH}_{LL}\left(1-q_H^A\right)}{\pi_{LL}(1-q_L^A)} - \frac{\left(\gamma^{LH}_{LL} - \gamma^{LL}_{HH} - \gamma^{LL}_{LH}\right)}{\pi_{LL}}. \tag{A.6}$$

A.1 Proof of Proposition 4

Lemma 2. $s_{HH} \geq f_{HH}$.

Proof. Immediate from (A.1) and (A.2). □

Lemma 3. $s_{LH} \geq f_{LH}$

Proof. Immediate from (A.3) and (A.4). □

Lemma 4. *The interim constraint HL-LH is not binding*

Proof. By Lemma 3 and $q_H^B > q_L^B$, if HL-LH were binding then HH-LH would not hold. □

Lemma 5. $s_{LL} \leq f_{LL}$.

Proof. Immediate from (A.5) and (A.6). □

Corollary 1. $s_{LL} = f_{LL} = w_{LL}$ *implies the following statements:*

(i) $\mu_{LL} = \gamma_{LL}^{HH} = 0$;

(ii) $s_{HH} > f_{HH}$ *and* $s_{LH} > f_{LH}$;

(iii) Either $\mu_{HH} > 0$ *or* $\gamma_{HH}^{LL} > 0$;

(iv) Either $\mu_{LH} > 0$ *or* $\gamma_{LH}^{LL} > 0$.

Lemma 6. *If* $s_{LL} = f_{LL} = w_{LL}$, *then* LH-LL *cannot be binding* $\left(\gamma_{LL}^{LH} = 0\right)$.

Proof. Suppose LH-LL binding, then equation LH-LL and $q_H^B > q_L^B$ imply LL-LH nonbinding $\left(\gamma_{LH}^{LL} = 0\right)$. By Corollary 1 (iv), IC_{LH} is binding, then,

$$p_B s_{LH} + (1 - p_B) f_{LH} \geq w_{LL}.$$

This expression contradicts LH-LL, given that $q_H^B > p_B$ and $s_{LH} \geq f_{LH}$. □

Lemma 7. $s_{LL} < f_{LL}$

Proof. Suppose $s_{LL} = f_{LL} = w_{LL}$. By Lemma 6, LH-LL is nonbinding, then

$$q_H^B s_{LH} + \left(1 - q_H^B\right) f_{LH} > w_{LL}. \tag{A.8}$$

However, from $\gamma_{LL}^{LH} = 0$, (A.3) and (A.5), $w_{LL} > s_{LH}$, which contradicts (A.8). Thus, $s_{LL} \neq f_{LL}$ and by Lemma 5 the result follows. □

Corollary 2. *Either* $\mu_{LL} > 0$ *or* $\gamma_{LL}^{HH} > 0$ *(or both)*

Proof. Immediate from Lemma 7, (A.5) and (A.6). □

Lemma 8. $s_{HH} > f_{HH}$.

Proof. Suppose $s_{HH} = f_{HH} = w_{HH}$. This implies HH-LH and LH-LL binding,

$$w_{HH} = q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL}. \tag{A.9}$$

Because $s_{LL} < f_{LL}$ and $\left(1 - q_H^A\right) < \left(1 - q_L^A\right)$,

$$q_H^A s_{LL} + \left(1 - q_H^A\right) f_{LL} < q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL},$$

which implies HH-LL nonbinding $\left(\gamma_{LL}^{HH} = 0\right)$. By Corollary 2, IC_{LL} is binding, and then

$$p_A s_{LL} + (1 - p_A) f_{LL} \geq w_{HH}. \tag{A.10}$$

Because $s_{LL} < f_{LL}$ and $(1 - p_A) < \left(1 - q_L^A\right)$,

$$p_A s_{LL} + (1 - p_A) f_{LL} < q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL}. \tag{A.11}$$

From (A.10) and (A.11),

$$w_{HH} < q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL},$$

which contradicts (A.9) □

Lemma 9. $s_{LH} > f_{LH}$.

Proof. Suppose $s_{LH} = f_{LH} = w_{LH}$. From LH-LL and LL-LH,

$$w_{LH} = q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL}.$$

Now, because $s_{LL} < f_{LL}$ and $(1 - p_A) < \left(1 - q_L^A\right)$,

$$w_{LH} > p_A s_{LL} + (1 - p_A) f_{LL},$$

which implies IC_{LL} nonbinding ($\mu_{LL} = 0$). Since $s_{LL} < f_{LL}$ and $\left(1 - q_H^A\right) < (1 - p_A)$,

$$w_{LH} > q_H^A s_{LL} + \left(1 - q_H^A\right) f_{LL}. \tag{A.12}$$

Combining HH-LH and (A.12),

$$q_H^A s_{HH} + \left(1 - q_H^A\right) f_{HH} > q_H^A s_{LL} + \left(1 - q_H^A\right) f_{LL},$$

which implies HH-LL nonbinding ($\gamma_{LL}^{HH} = 0$), in contradiction to Corollary 2. □

A.2 Proof of Proposition 5

Corollary 3. *Either $\mu_{HH} > 0$ or $\gamma_{HH}^{LL} > 0$ (or both)*

Proof. Immediate from Lemma 8, (A.1) and (A.2). □

Corollary 4. *Either $\mu_{LH} > 0$ or $\gamma_{LH}^{LL} > 0$.*

Proof. Immediate from Lemma 9, (A.3) and (A.4). □

Lemma 10. *HH-LL is not binding.*

Proof. Suppose HH-LL binding, then

$$q_H^A s_{HH} + \left(1 - q_H^A\right) f_{HH} = q_H^A s_{LL} + \left(1 - q_H^A\right) f_{LL}. \tag{A.13}$$

By Lemmas 7, 8 and $q_H^A > q_L^A$, (A.13) yields

$$q_L^A s_{HH} + \left(1 - q_L^A\right) f_{HH} < q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL},$$

which implies LL-HH nonbinding $\left(\gamma_{HH}^{LL} = 0\right)$. Since $\mu_{HH} > 0$, by Corollary 3, IC_{HH} is binding, so

$$p_A s_{HH} + (1 - p_A) f_{HH} \geq p_A s_{LL} + (1 - p_A) f_{LL}. \tag{A.16}$$

By Lemmas 7, 8 and $q_H^A > p_A$, (A.16) yields

$$q_H^A s_{HH} + \left(1 - q_H^A\right) f_{HH} > q_H^A s_{LL} + \left(1 - q_H^A\right) f_{LL},$$

which contradicts HH-LL binding. □

Lemma 11. *IC_{LL} is binding.*

Proof. Immediate from Corollary 2 and Lemma 10. □

Lemma 12. *IC_{HH} is binding.*

Proof. Assume IC_{HH} nonbinding. By Corollary 3, $\gamma_{HH}^{LL} > 0$, which implies LL-HH binding:

$$q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL} = q_L^A s_{HH} + \left(1 - q_L^A\right) f_{HH}. \tag{A.19}$$

Since IC_{LL} is binding and IC_{HH} is not,

$$p_A s_{LL} + (1 - p_A) f_{LL} \geq p_A s_{HH} + (1 - p_A) f_{HH}; \tag{A.20}$$

By Lemmas 7, 8 and $p_A > q_L^A$, (A.19) yields

$$p_A s_{LL} + (1 - p_A) f_{LL} < p_A s_{HH} + (1 - p_A) f_{HH},$$

in contradiction to (A.20). □

Lemma 13. *IC_{LH} is binding.*

Proof. Suppose IC_{LH} nonbinding. By Corollary 4, $\gamma_{LH}^{LL} > 0$, which implies LL-LH binding,

$$q_L^A s_{LL} + \left(1 - q_L^A\right) f_{LL} = q_L^B s_{LH} + \left(1 - q_L^B\right) f_{LH}. \tag{A.23}$$

Since IC_{LH} is nonbinding and IC_{LL} is,

$$p_A s_{LL} + (1 - p_A) f_{LL} \geq p_B s_{LH} + (1 - p_B) f_{LH}. \tag{A.24}$$

By Lemmas 7, 9, $p_A > q_L^A$ and $p_B > q_L^B$, (A.23) yields

$$p_A s_{LL} + (1 - p_A) f_{LL} < p_B s_{LH} + (1 - p_B) f_{LH},$$

which contradicts (A.24). □

Lemma 14. $\max\left\{w_{HH}^F, w_{LH}^F, w_{LL}^S\right\} < \min\left\{w_{HH}^S, w_{LH}^S, w_{LL}^F\right\}$.

Proof. Immediate from Lemmas 7, 8, 9 and 11–13. □