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A unidirectional Hotelling model revisited

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Abstract

We consider a two-stage location-price Hotelling model where the consumers can only buy from one direction, as presented by Kharbach (2009, Economics Bulletin). We show that the equilibrium outcome derived by Kharbach does not constitute a subgame perfect Nash equilibrium.

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1. Introduction

Since Hotelling (1929), there have been many lines of work concerning strategic firm locations or, interpreted differently, product differentiation. In models where firms compete just in location (Hotelling) or location-then-quantity with firms providing shipping costs (Anderson and Neven (1991) and Hamilton et al. (1989)), firms agglomerate at the center of consumer support. When firms compete in prices after location with consumers paying for (shopping) transport cost (d'Aspremont, et al. (1979)), however, they locate at the two ends of the support. In this manner, the literature has dealt with conditions under which the firms locate in a manner consistent with the principle of minimum or maximum differentiation.

Kharbach (2009) noted that most of the rich spatial literature has only dealt with cases where the consumers can buy goods from either direction. He offered the following examples: highway and one way roads, and non-revertible flows in oil and gas pipelines. Another example may be time, as seen in TV scheduling problems as dealt by Cancian et al. (1995) and Nilssen and Sørgard (1998).[1] Thus the unidirectional setting is important as a counterpart to the well-analyzed bidirectional models.

As for the equilibrium, Kharbach (2009) claimed that if the consumers can only buy from one direction, neither minimum nor maximum differentiation occurs. That is, in a consumer space [0,1], if a consumer can buy goods from a firm located at a number larger than its own location, then the firms locate at 3/5 and 1.

The purpose of this work is to show that the equilibrium outcome given by Kharbach (2009) is incorrect. The main reason for this is that under the proposed equilibrium of Kharbach (2009), firm located to the right has an incentive to raise its price as high as the demand allows, as increasing its price does not affect the quantity demanded. The error in Kharbach (2009)'s method is that he assumed there is always a consumer indifferent between buying from either of the two firms. This is a standard mode of operation when solving the 'usual' bidirectional models, but this may not be adequate for the unidirectional model. Indeed, in the proposed equilibrium outcome the indifferent consumer is exactly at the location of the firm to the left, but such consumer does not exist once the deviation occurs.

This work proceeds as follows. Section 2 introduces the model. Section 3 examines the Kharbach (2009)'s outcome and then show why this is not an equilibrium. Finally, section 4 concludes the note and shows that subgame perfect equilibrium exists if the ratio of the consumer utility and the transport cost parameter is not too high.

2. The model of Kharbach (2009)

We follow the setting of Kharbach (2009). Two firms compete in a location-price competition game. In stage 1, the firms independently determine location in the segment [0,1] where consumers are distributed uniformly with density 1. The distances from the end point and the

[1] Kharbach (2009) and our work adopt the mill-pricing (shopping) setting where consumers travel. Colombo (2009) examined the unidirectional transport under the spatial discrimination (shipping) setting where firms transport the goods to each market in a consumer support. All three works deal with location-price competition.

location of firm A (B) are denoted by a (b), where $a \leq 1-b$. Thus firm A (B) is located at a ($1-b$). In stage 2, firms compete in prices, bearing marginal cost normalized to zero. A consumer must visit one of the firms in order to buy the good. In doing so, quadratic transport cost td^2 must be incurred, where t is a positive constant and d is the distance traveled. Consumers must also pay p_i when purchasing from firm i ($i = A,\ B$). By consuming the good, consumers obtain utility U, which is assumed to be large enough so all consumers want to purchase the good. Here, we assume that $U > t$, implying that a consumer located at 0 will find it beneficial to purchase the goods from firm at 1 when the price is zero. Thus this is the market serving condition.

Thus if the prices are not too large, a consumer located at x derives a disutility equal to $\min\{P_A + t(a-x)^2,\ P_B + t(1-b-x)^2\}$, depending on the firm from which she purchased the good.[2] There may be an indifferent consumer whose disutility level from buying from either firm is the same. We denote this consumer x_I and we have $P_A + t(a-x_I)^2 = P_B + t(1-b-x_I)^2$.

3. The proposed equilibrium and why it is incorrect

Here we derive Kharbach (2009)'s result and show why it is not an equilibrium.

This is a two-stage game, so we use subgame perfect Nash equilibrium. Thus the equilibrium strategy contains the original location in stage 1 and the price strategy for every location subgame pattern.

Kharbach (2009)'s result implicitly assumes that x_I is less than or equal to a. We derive the best response in prices in the second stage subgame and within this regime. The indifferent consumer is calculated to be at

$$x_I = \frac{P_B - P_A}{2t(1-b-a)} + \frac{1-b+a}{2},$$

and each firm's profit is $\Pi_A = P_A x_I$ and $\Pi_B = P_B(1-b-x_I)$. From the first order conditions, we have the best response functions

$$\begin{aligned} P_A &= (P_B + t(1-b-a)(1-b+a))/2 \\ P_B &= (P_A + t(1-b-a)^2)/2 \end{aligned}$$

from which we have the equilibrium candidate price levels. We also show the indifferent consumer and the profit levels here.

$$\begin{aligned} P_A &= t(1-b-a)(3-3b+a)/3 \\ P_B &= t(1-b-a)(3-3b-a)/3 \\ x_I &= (3-3b+a)/6 \\ \Pi_A &= t(1-b-a)(3-3b+a)^2/18 \\ \Pi_B &= t(1-b-a)(3-3b-a)^2/18 \end{aligned}$$

[2] In Kharbach (2009), Section 2 implicitly and erroneously assumes that consumers to the left of a buy from A and those located between a and $1-b$ buy from B. This assumption disappears from Section 3 on, as the indifferent consumer is shown to be located to the left of a.

In stage 1, each firm independently decides its location by solving the following first order conditions.

$$\frac{\partial \Pi_A}{\partial a} = -t(1-b+3a)(3-3b+a)/18 < 0$$
$$\frac{\partial \Pi_B}{\partial b} = -t(9-9b-7a)(3-3b-a)/18 < 0$$

Thus firm B wants to locate at $b = 0$ (the rightmost point). Firm A chooses to locate as far left as possible, maintaining the constraint $x_I \leq a$. Thus $a = 3/5$ is derived. The prices, quantities supplied, and profits are

$$P_A = 12t/25, \quad x_I = 3/5, \quad \Pi_A = 36t/125.$$
$$P_B = 8t/25, \quad 1-b-x_I = 2/5, \quad \Pi_B = 16t/125.$$

Thus far we have introduced Kharbach (2009)'s equilibrium outcome.

We now offer our main result.

Proposition 1 *The proposed equilibrium of Kharbach (2009) does not constitute a subgame perfect Nash equilibrium.*

This result is due to the existence of the following simple deviation by firm B. In the second stage subgame, firm B can raise its price higher than it is now, as this does not result in a loss of quantity supplied. This would strictly increase the profit for firm B, and thus the proposed outcome is not an equilibrium. Indeed, firm B can set its price at least as high as $U - t(1-a)^2$ without losing customers and it may set it higher if that leads to a higher level of profit.

The main failing in Kharbach (2009)'s analysis is that he assumed that there always is an indifferent consumer x_I, as is standard in the 'usual' bidirectional framework. Once there is a deviation as we have shown, there is no longer any indifferent consumers: All consumers at or to the left of firm A shop at firm A, and those to the right of firm A shop at firm B. This property is due to the unidirectional transport constraint. In the bidirectional models, firm B must always be wary of the pricing strategy by firm A and potentially losing its customer when raising its price P_B. In the unidirectional framework, there is no potential for business-stealing from B by firm A, leading to this deviation by firm B.

4. Conclusion

We have shown that the result derived in Kharbach (2009) is incorrect. The natural question is whether a subgame perfect equilibrium indeed exists. We show here that it does, if the ratio of U to t is sufficiently low. In such an equlibrium, the market should be divided into two regions, as shown in Figure 1. Also, if firm A supplies to market 0, it sets its price high enough so that the consumer at 0 is indifferent between buying and not buying. Thus firm A sets its price so that $P_A = U - ta^2$. Thus we have the following.

$$\begin{aligned} \Pi_A &= P_A a = aU - ta^3, \\ \frac{\partial \Pi_A}{\partial a} &= U - 3ta^2. \end{aligned}$$

Thus if $a = \sqrt{U/(3t)}$, this becomes the optimal market size for firm A. A similar approach can be done for firm B.

If this length is less than or equal to $1/2$, each firm can obtain local monopoly profits. This holds if $U/t \leq 3/4$.[3] In fact, there is a continuum of equilibria in terms of locations, when $U/t < 3/4$. Thus we have shown that if the ratio of consumer utility U and the transport cost parameter t (a proxy for much the consumer is willing to pay transport cost to purchase the good) is low enough, there is a subgame perfect equilibrium in that each firm locates away from each other and achieves local monopoly.

As for when U/t is large, the analysis becomes much more complicated. This is because there is a strategic incentive for the firms to either change locations or change prices. For example, firm A may have an incentive to move right in order to capture more market, but in doing so risks firm B's possible relocation to the left of A. Also, in the subgames where the locations are already determined, firm A may have an incentive to lower its price from the monopoly level (i.e., $U - ta^2$) in order for discourage firm B from engaging in a price war. These factors lead to much more complicated analyses that would be much too long for *Economics Bulletin* and are thus left for future research.

[3] We have assumed in section 2 that $U > t$ holds. This assumption is made often in the spatial economics literature in order to facilitate calculation. However, we can show that the general result holds even if $U \leq t$ holds. The analysis becomes a little more complicated and we do not include it in this note.

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