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Wage Inequality, R&D Labor and R&D Productivity

Toshihiro Okada
School of Economics, Kwansei Gakuin University

Abstract

This paper examines the transitional dynamics of an R&D-based endogenous growth model with heterogeneous labor and explains the post-war comovement of three variables in the U.S. economy: the skill premium, the share of labor devoted to R&D and the growth rate of labor productivity. This paper provides a complete dynamic analysis of the model. We argue that the changing distribution of high skilled workers between sectors may have played an important role in explaining the U.S. skill premium movement, and we show that the transitional dynamics initiated by the structural change (possibly the decrease in R&D productivity in the late 1960s) can explain the comovement of the three variables.

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Contact: Toshihiro Okada - toshihiro.okada@kwansei.ac.jp.
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1. Introduction

In this paper, we attempt to explain some important features of the post-war U.S. economy by exploring the transitional dynamics of an R&D-based endogenous growth model. We focus on medium-term fluctuations in three variables of the U.S. economy: the relative wage of high-skilled labor (i.e., the college skill premium), the share of labor devoted to R&D and the growth rate of labor productivity.

The data on wages show that significant fluctuations have occurred in the U.S. skill premium during the past several decades. Figure 1 shows that the U.S. skill premium fell dramatically during the 1970s and grew throughout the 1980s and 1990s. In explaining this dynamic, the recent work on the skill premium pays significant attention to skill-biased technological change, (e.g., Acemoglu (1998, 2000 and 2003) and Galor and Moav (2000)). Acemoglu (1998) presents a model of directed technological change and argues that the large exogenous rise in the number of U.S. college graduates in the 1970s (i.e., the rise in the relative supply of skilled labor) first reduced the skill premium but then induced the development of skill-biased technology, which increased the skill premium in the subsequent period.[1] [2]

In addition to the relationship between the skill premium and relative supply of skilled labor, there is an interesting but rarely discussed fact regarding the post-war U.S. economy. In the U.S., the share of labor devoted to R&D, the number of R&D workers, and the labor productivity growth rate show dynamics that are very similar to those of the skill premium. As with the skill premium, the share (and also number) of R&D workers and the productivity growth rate fell dramatically in the 1970s and then gradually increased back toward previous levels in the 1980s and 1990s (see Figures 2 and 3). Skill-biased technological change does not explain this phenomenon.

This paper provides another explanation for the dynamics of the U.S. skill premium and explains the comovement of the skill premium, the share of R&D workers and the productivity growth in the U.S. during the post-war period. Toward this end, it builds a simple Romer-Jones type R&D-based endogenous growth model that is extended to include labor heterogeneity. The model is a knowledge-driven horizontal R&D growth model.[3] Because the purposes of this paper is to show that a simple R&D-based endogenous growth model without directed technological change can explain the recent inverted U-shaped time paths of the skill premium and the other variables mentioned above, the

[1] Other frequently cited papers in the directed technological change literature include Kiley (1999), Acemoglu (2000 and 2003), Acemoglu and Zilibotti (2001), and Caselli and Coleman (2006).

[2] Within the framework of directed technological change Afonso (2006 and 2008) also explains the inverted U-shaped time path of the skill premium. The explanation is however significantly different from that of standard models of directed technological change. The standard models emphasize the dominance of the market-size effect over the price effect with regard to technology development and show that this dominance leads to the development of technology which is biased in favor of abundant type of labor. Afonso (2006 and 2008) however shows that the dominance of the market-size effect is not necessary for explaining the recent skill premium dynamics once other important factors (e.g., a technology adoption effect) are introduced.

[3] Another frequently used specification that is employed in R&D-based endogenous growth models is the lab-equipment specification, in which all that is required for R&D is output (i.e., investment in equipment or laboratories) and labor (i.e., scientists and enginers) is not considered. In contrast, the knowledge-driven specification assumes that labor is the only R&D input. Because this paper attempts to show the dynamics of R&D workers, the lab-equipment specification is not suited to our analysis. Another reason for our choice of specification is that as Romer (1990) argues, R&D is a labor-intensive activity.

model is kept as simple as possible.[4]

This paper argues that the structural change, the (unexpected) sharp decrease in R&D productivity around 1970, first pushed the U.S. economy away from its steady state; then, the economy gradually moved back toward the steady state. The paper shows that the transitional dynamics initiated by the structural change can explain the correlated movement of the variables.

2. The Basic Setup of the Model

The model extends the studies by Romer (1990), Jones (1995b) and Barro and Sala-i-Martin (ch.6, 2004) by allowing for labor heterogeneity (high- and low-skilled labor). We assume that an economy has two sectors: final and intermediate goods sectors. Intermediate goods firms require a design (i.e., a blueprint) to produce the goods, and they themselves undertake research on new designs.

The economy produces homogenous final goods, Y. The production function of Y at time t is given by

$$Y(t) = L_L(t)^{\alpha} L_{HY}(t)^{\beta} \int_0^{N(t)} X_j(t)^{1-\alpha-\beta} dj, \; 0 < \alpha + \beta < 1. \tag{1}$$

Final goods are produced under perfect competition. Firms in the final goods sector employ low-skilled labor L_L and high-skilled labor L_{HY} and use nondurable intermediate goods to produce Y. Equation (1) shows that both low- and high-skilled workers are essential to the production of final goods. The assumption that is made here is that low-skilled labor can only perform simple tasks and firms in the final goods sector need skilled labor who can perform complicated tasks. As we will discuss later, the share of skilled labor in total labor is assumed to take such a value that the equilibrium level of the relative wage of high- to low-skilled workers (i.e., the skill premium), is greater than 1. Given this assumption, no high-skilled worker wishes to work as a low-skilled worker. X_j is the jth type of nondurable intermediate goods and N is the number of available types of nondurable intermediate goods.

Normalizing the price of Y to 1, the final goods firm's profit is shown by

$$Y(t) - \int_0^{N(t)} p_j(t) X_j(t) dj - w_L(t) L_L(t) - w_{HY}(t) L_{HY}(t),$$

where p_j is the price of nondurable intermediate good j, w_L is the low-skilled labor wage and w_{HY} is the high-skilled labor wage. Assuming that the final goods market is competitive, we can obtain the usual relationships between factor prices and marginal products as follows (the time argument is dropped below):

$$p_j = L_L^{\alpha} L_{HY}^{\beta} (1-\alpha-\beta) X_j^{-\alpha-\beta}, \tag{2}$$

$$w_L = \alpha L_L^{\alpha-1} L_{HY}^{\beta} \int_0^N X_j^{1-\alpha-\beta} dj \text{ and } w_{HY} = \beta L_L^{\alpha} L_{HY}^{\beta-1} \int_0^N X_j^{1-\alpha-\beta} dj. \tag{3}$$

[4]Horizontal R&D-based growth models (expanding variety models) are much simpler than vertical R&D-based growth models (Schumpeterian growth models). Furthermore, horizontal R&D models have an advantage in terms of tractability. With the knowledge-driven specification, vertical R&D models become even more complicated.

Once the intermediate goods firm invents a new design, it retains perpetual monopoly power over the use of this design. The firm itself conducts R&D in pursuit of the invention. The production of one intermediate good requires η units of forgone final output. Therefore, the flow of operational profit of the monopolist at a point of time is given by

$$\pi_j = p_j X_j - \eta X_j.$$

The present value of the returns from the operation is, then, given by

$$V_j = \int_t^\infty \pi_j(v) e^{-\int_t^v r(\omega)d\omega} d\nu = \int_t^\infty \left[p_j(v)X_j(v) - \eta X_j(v)\right] e^{-\int_t^v r(\omega)d\omega} d\nu, \tag{4}$$

where r is the interest rate. The monopolist facing the demand curve (2) solves the following problem:

$$\max \int_t^\infty \left[p_j(v)X_j(v) - \eta X_j(v)\right] e^{-\int_t^v r(\omega)d\omega} d\nu, \quad s.t.\ p_j = L_L^\alpha L_{HY}^\beta (1-\alpha-\beta) X_j^{-\alpha-\beta}.$$

Solving the problem yields $X_j = \overline{X} = \left(\frac{L_L^\alpha L_{HY}^\beta (1-\alpha-\beta)^2}{\eta}\right)^{\frac{1}{\alpha+\beta}}$ and $p_j X_j = \overline{p} = \frac{\eta}{1-\alpha-\beta}$. These equations show that each monopolist in the intermediate goods sector produces the same amount of intermediate goods and charges the same price. This therefore implies that the present value of the monopoly operational profits is the same across the firms: $V_j = \overline{V} = \int_t^\infty \overline{\pi}(v) e^{-\int_t^v r(\omega)d\omega} d\nu$ where $\overline{\pi}(t) = \overline{p}\overline{X}(t) - \eta \overline{X}(t)$.

We assume that R&D requires a certain amount of high-skilled labor and that intermediate goods firms require η/N^ϕ $(0 < \phi < 1)$ units of high-skilled labor to invent a new design. These assumptions imply the following: (i) the existing stock of designs spills over (designs are non-rival goods), and (ii) a higher level of the existing stock of designs implies that a lower the level of high-skilled labor is required for the invention. The invention cost is then given by

$$Z(t) = \frac{\eta}{N(t)^\phi} w_{HN}(t), \tag{5}$$

where w_{HN} is the wage for high-skilled labor engaged in R&D.

We assume free entry into R&D; that is, any firm can pay Z to secure the present value of the monopoly profits. In equilibrium, therefore, $V = Z$ must be satisfied. This condition yields the following:

$$\int_t^\infty \overline{\pi}(v) e^{-\int_t^v r(\omega)d\omega} d\nu = Z(t). \tag{6}$$

Differentiating both sides of equation (6) with respect to t yields:

$$r(t) = \frac{\overline{\pi}(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)}. \tag{7}$$

In addition, because η/N^ϕ units of skilled labor are required to invent one new design, the aggregate amount of high-skilled labor devoted to the R&D is given by $L_{HN}(t) =$

$\dot{N}(t)(\eta/N(t)^{\phi})$. We then can obtain

$$\dot{N}(t) = \frac{1}{\eta}\ L_{HN}(t)\ N(t)^{\phi}. \tag{8}$$

In equilibrium, the high-skilled labor employed in the final goods sector should receive the same wage as the high-skilled labor employed in R&D. Therefore, $w_{HY}(t) = w_{HN}(t)$ must hold. We denote this common wage rate for high-skilled labor as w_H. Based on expression (3), the skill premium, w_H/w_L, is then given by

$$\frac{w_H(t)}{w_L(t)} = \frac{\beta L_L(t)}{\alpha L_{HY}(t)}.$$

Denoting s and u as the share of high skilled labor in the total population and the share of R&D workers in the high-skilled labor population, respectively, the skill premium is then

$$\frac{w_H(t)}{w_L(t)} = \frac{\beta}{\alpha}\frac{1-s}{s}\frac{1}{1-u(t)}, \tag{9}$$

where s is constant and exogenously given. We assume that s takes a value such that $s < \frac{\beta}{\alpha+\beta}$. Therefore, $\frac{w_H}{w_L} > 1$ holds for any value of u between 0 and 1. This condition implies that no high-skilled worker wishes to work as a low-skilled worker. Consequently, $L_{HY}(t) = (1-u(t))sL(t)$, $L_{HN}(t) = u(t)sL(t)$, and $L_L(t) = (1-s)L(t)$ where L is the total population, which grows at a constant rate: $\dot{L}(t) = nL(t)$.

Finally, we consider a representative household's utility maximization problem. Because we have assumed that s is constant, the household's composition in terms of the skill levels of its members, i.e., the ratio of high-skilled members to low-skilled members, is constant. Normalizing the number of members of the household at time 0 to 1, the household wishes to maximize overall utility U as given by

$$U = \int_0^{\infty} \frac{c(t)^{1-\theta}-1}{1-\theta} e^{(n-\rho)t} dt, \tag{10}$$

where $c = C/L$, C is total consumption, ρ is the rate of time preference, and $\rho > 0$.[5] The flow budget constraint for the household is given by

$$\dot{a}(t) = (1-s)w_L(t) + sw_H(t) + r(t)a(t) - c(t) - na(t), \tag{11}$$

where $a = A/L$ and A represents total assets. From the first order conditions, we can obtain the following Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\theta}. \tag{12}$$

The transversality condition is $\lim_{t\to\infty} \lambda(t)a(t) = 0$ where $\lambda(t)$ denotes the shadow price in the present-value Hamiltonian. Because the aggregate financial assets of the households equal the total market value of the firms, we can represent the total assets per person as $a(t) = Z(t)N(t)/L(t)$.[6] Therefore, the transversality condition can be rewritten as

[5] We assume that $n - \rho < 0$ so that U is bounded.

[6] Because the final goods firms earn zero profits, the market value of the firms in the economy equals

follows:

$$\lim_{t\to\infty}\left[\lambda(t)\frac{Z(t)N(t)}{L(t)}\right]=0. \tag{13}$$

3. Transitional Dynamics

This section explores the dynamics of the model in detail. We show that the economy has a unique steady state and that its saddle path is stable (i.e., there is a unique and monotonic path converging towards the steady state). We also show that the stable saddle-path is the only possible equilibrium.

Appendix A shows that we can analyze the dynamics of the economy by using the following two differential equations:

$$\frac{(1-\dot{u(t)})}{(1-u(t))}=\frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta}\,G_N(t)T(t)+(1-\phi-\theta)G_N(t)-\rho\right) \tag{14}$$

and

$$\begin{aligned}\frac{\dot{G_N}(t)}{G_N(t)}&=\frac{-(1-\alpha-\beta)\,(\alpha+\beta)^2}{(\beta\,\theta+\alpha)\,\beta}\,G_N(t)\,T(t)^2\\&-\frac{(\alpha+\beta)\,(1-\phi-\theta)}{(\beta\,\theta+\alpha)}\,G_N(t)\,T(t)+\frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\rho\,T(t)+n-(1-\phi)\,G_N(t)\,,\end{aligned} \tag{15}$$

where $G_N(t)\equiv\dot{N}(t)/N(t)$ and $T(t)\equiv(1-u(t))/u(t)$. Because T increases (decreases) when $(1-\dot{u(t)})$ is positive (negative), we can draw a phase diagram in the $(T,\,G_N)$ space using equations (14) and (15). The $(1-\dot{u(t)})=0$ and $\dot{G}_N(t)=0$ loci are given by[7]

$$G_N(t)=\frac{\beta\,\rho}{(1-\alpha-\beta)\,(\alpha+\beta)\,T(t)+(1-\phi-\theta)\,\beta} \tag{16}$$

and

$$\begin{aligned}&G_N(t)\\&=\frac{(\alpha+\beta)\,\beta\,\rho\,T(t)+(\beta\,\theta+\alpha)\,\beta\,n}{(1-\alpha-\beta)\,(\alpha+\beta)^2\,T(t)^2+(1-\phi-\theta)\,(\alpha+\beta)\,\beta\,T(t)+(\beta\,\theta+\alpha)\,(1-\phi)\,\beta}\end{aligned}\;. \tag{17}$$

Equations (16) and (17) represent the $(1-\dot{u(t)})=0$ locus and the $\dot{G}_N(t)=0$ locus, respectively. Solving equations (16) and (17) gives the steady state values of T and G_N:[8]

$$T^*=\frac{\beta\;(\rho\,(1-\phi)+n(\theta+\phi-1))}{n\,(1-\alpha-\beta)\,(\alpha+\beta)} \tag{18}$$

and

$$G_N^*=\frac{n}{1-\phi}\;. \tag{19}$$

At this unique steady state, y, c and N grow at the rate given by equation (19).[9] T^* and G_N^* are both positive based on the assumed parameter values.

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the number of monopolists in the intermediate goods sector, N, multiplied by the invention cost, Z.

[7] $(1-\dot{u(t)})=0$ is also satisfied if $u=1$ that is, along the vertical axis in Figure 4 and $\dot{G}_N=0$ is also satisfied if $G_N=0$ that is, along the horizontal axis in Figure 4.

[8] We assume that the transversality condition (13) is satisfied at the steady state. This implies that $n-\rho(1-\phi)-n(\theta+\phi-1)>0$ holds.

[9] As in Jones (1995), the counterfactual scale effect does not exist.

We can draw three possible phase diagrams, (a), (b) and (c), as shown in Figure 4, depending on the values of the parameters. The curves denoted by 1 show the $(1 - \dot{u(t)}) = 0$ loci, and the curves denoted by 2 show the $\dot{G}_N(t) = 0$ loci. The $(1 - \dot{u(t)}) = 0$ and $\dot{G}_N(t) = 0$ loci intersect only once. The thick curves with arrows are saddle paths.

The important point shown in Figure 4 is that there exists a unique stable saddle-path towards the steady state in each phase diagrams. Starting from a low (high) level of $u(t)$ on the saddle path (i.e., a high (low) level of $T(t)$), both $u(t)$ and $G_N(t)$ monotonically increase (decrease) towards their steady state levels. This stable saddle path is also the only possible equilibrium. When the economy is not initially on the saddle path, it can take two types of paths. The first is the path that eventually hits the vertical axis, and the second is the path that asymptotically reaches the point where $u = 0$ and $G_N = 0$. The former violates the Euler equation (12), and the latter violates the labor constraint (see Appendix B for the proof).

Next, we consider the determination of the economy's starting point. Let $t = 0$ be the beginning of the planning period. $N(0)$ is predetermined (i.e., historically given), but $G_N(0)$ and $u(0)$ (i.e., $T(0)$) are not predetermined. From equation (8), we can obtain

$$G_N(0) = \frac{1}{\eta} u(0) s N(0)^{\phi - 1}, \tag{20}$$

where $L(0)$ is normalized to 1 (see also equation (a-6)in Appendix A). Therefore, with a given $N(0)$, any pair of $G_N(0)$ and $u(0)$ that satisfies equation (20) indicates possible starting points for the economy. Using equation (20) then allow us to draw in the $(T,\, G_N)$ space the locus that provides the possible starting points for the economy for a given value of $N(0)$. We call this locus *the* $N(0)$ *locus*.

The phase diagram in Figure 5 shows the $(1 - \dot{u(t)}) = 0$ and $\dot{G}_N(t) = 0$ loci with the $N(0)$ locus.[10] The $(1 - \dot{u(t)}) = 0$ locus and $\dot{G}_N(t) = 0$ locus are denoted as 1-1 and 2-2, respectively. The solid and dashed curves show the saddle path and $N(0)$ locus, respectively. Appendix C shows that there exists at least a range of $N(0)$ that guarantees that the $N(0)$ locus intersects with the saddle path only once. We assume that $N(0)$ is in that range. In Figure 5, the $N(0)$ locus intersects with the saddle path at point A. Because the saddle path is, as we have shown, the only possible equilibrium, point A represents the economy's starting point.

Note that because $N(0)$ is a predetermined variable, N cannot change discontinuously in the event of any unexpected structural change, i.e., a decrease or increase in the model's parameter value. Other variables, such as $u(t)$, $c(t)$ and $G_N(t)$, can adjust discontinuously in the event of an unexpected structural change.

4. Skill Premium, R&D Labor and Productivity Growth

As mentioned previously, during the post-war periods in the U.S., the skill premium, the share of R&D workers, and the growth rate of GDP per worker have exhibited closely related fluctuations. As shown in Figures 1, 2 and 3, we can observe the medium-term comovement of the three variables, i.e., the inverted U-shaped time paths of the variables.

In this section, we attempt to explain the comovement using the analysis presented in the previous section. We assume that a structural change took place in the U.S. economy

[10] We choose phase diagram (a) in Figure 4. The choice is not important because all three phase diagrams in Figure 4 have similar dynamics.

in the late 1960s and reduced the productivity in the R&D performing sector. In the model, this structural change is shown by an increase in η (the productivity parameter) because the intermediate goods firm requires $\frac{\eta}{N(t)^{\phi}}$ units of high-skilled labor to invent a new design. Figure 6 presents the time paths of the ratios of yearly TFP (total factor productivity) changes to the number of R&D scientists and engineers and to the R&D spending in the period 1957-2002.[11] The figure shows that a rapid decline in the R&D productivity did occur from the mid-1960s to the early 1970s, which is about the same period in which the sharp decline in the share of labor devoted to R&D occurred (see Figure 2).[12] [13] Explaining the rapid change in R&D productivity is beyond the scope of this paper, but several explanations for the decline in R&D productivity have been advanced, e.g., Evenson (1984): technology exhaustion, and Caballero and Jaffe (1993): the expansion of markets which leads to more competition in R&D.

The phase diagram presented in Figure 7 shows the effects of the unexpected permanent rise in η. In Figure 7, curve 1-1 shows the $(1 - \dot{u}(t)) = 0$ locus, and curve 2-2 shows the $\dot{G}_N(t) = 0$ locus. The thick curve with arrows indicates the saddle path. Assume that the economy is initially at point A on the saddle path where $G_N = G_N^{'}$ and $T = T^{'}$. Curve 3-3 shows the $N(0)$ locus.

An unexpected increase in η leads to a downward shift in the $N(0)$ locus according to equation (20). The shifted $N(0)$ locus is presented by curve 3'-3'. Note that because η is not included in equations (16) and (17), the $(1 - \dot{u}(t)) = 0$ and $\dot{G}_N(t) = 0$ loci do not shift. Because the saddle path is the only equilibrium, the economy jumps from point A to point B. Therefore, T jumps up from T' to T'', and G_N jumps down from G_N' to G_N'', both at the time of the increase in η. The increase in T implies the decrease in u. Because the skill premium is given by $\frac{w_H}{w_L}(t) = \frac{\beta}{\alpha}\frac{1-s}{s}\frac{1}{1-u(t)}$, the decrease in u leads to the decrease in the skill premium.[14] In the subsequent period, T gradually decreases back toward T^* because the economy is on the stable saddle-path at point B. This behavior implies that u and $\frac{w_H}{w_L}$ also gradually increase back toward their steady state levels.[15]

The intuitive explanation for the initial drop in the skill premium is as follows. The structural change represented by the unexpected increase in the productivity of the R&D performing firms reduces the amount of high-skilled labor demanded in R&D. This reduction leads to a decrease in high-skilled labor demand relative to low-skilled labor demand, i.e., a decrease in the share of high-skilled R&D labor in the high-skilled labor population (and in the total labor population). This decrease in the relative demand for high skilled labor then reduces the skill premium.

[11] To obtain changes in TFP, we use the TFP growth rates calculated by the Federal Reserve Bank of San Francisco. The dataset is carefully adjusted for variations in factor utilization (see Fernald (2010) for the methodology used) to isolate the effects of demand shocks. Thus, the data are likely to be reliable measures of technology change. We also use the TFP calculated by Jones (2002) for the initial TFP value (the value in 1956).

[12] This finding is consistent with those of Perron (1997), who argues that there was a break around 1970 in the post-war macroeconomic time series for the U.S. and the other G7 countries.

[13] From equation ((13), $1/\eta = \dot{N}(t)/(L_H(t)\ N(t)^{\phi})$, $0 < \phi < 1$ where $\dot{N}(t)/L_H(t)$ represents the R&D productivity in the figure. Because TFP (i.e. N) increased in this period, the decline in $1/\eta$ should be even more rapid than is indicated in the figure.

[14] The behavior of c at the time of the shock is shown in Appendix D.

[15] Equation (20) shows that even if s (the relative supply of skilled labor) rises, as long as η increases more than s, the economy jumps down towards point B. Thus, the transitional dynamics presented here is compatible with the large increase in the relative supply of skilled labor that occurred from the mid-1960s to the early 1970s.

The change in η also has a significant effect on the dynamics of the rate of labor productivity growth. We consider the effect below. We can write the growth rate of output per labor as (see equation (a-8) in Appendix A):

$$\frac{\dot{y}(t)}{y(t)} = \frac{\beta}{\alpha+\beta}\,\frac{(1-\dot{u}(t))}{(1-u(t))} + G_N(t)\,. \tag{21}$$

Substituting equation (14) into equation (21), we can write the growth rate as: [16]

$$\frac{\dot{y}(t)}{y(t)} = \frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta\,\theta+\alpha}\,\Lambda(t)\,\frac{1}{\eta} + \frac{(\alpha+\beta)^2-\beta\,\phi}{\beta\,\theta+\alpha}\,\Lambda(t)\,\frac{1}{\eta}\,u(t) - \frac{\beta}{\beta\,\theta+\alpha}\,\rho\,, \tag{22}$$

where $\Lambda(t) = s\,e^{n\,t}N(t)^{\phi-1}$. Equation (22) shows that assuming $(\alpha+\beta)^2 - \beta\,\phi \geq 0$, the unexpected increase in η and the resulting decrease in u first reduce the growth rate of output per labor. The growth rate of output per labor drops because the decrease in u implies a decrease in the number of high-skilled workers in R&D, which in turn reduces the number of newly produced designs (and thus the growth rate of N) and the growth rate of output per labor. Thus, in Figure 7, the growth rate of output per labor at point B is less than it is at point A. Because the economy gradually reverts to the steady state (i.e., u increases), the growth rate of output per labor increases back toward G_N^* .[17]

Summarizing the above, the increase in η first reduces the share of R&D workers in the total labor population, the skill premium and the growth rate of output per labor. These values then gradually increase back toward their steady-state levels. The transitional dynamics of the model can thus explain the findings regarding the post-war U.S. economy that were described in the beginning of this section.

5. Conclusion

This paper examines the transitional dynamics of an R&D-based endogenous growth model with heterogeneous labor and explains the post-war comovement of three variables in the U.S. economy: the skill premium, the share of R&D workers in total labor and the growth rate of labor productivity. The paper argues that the changing distribution of high-skilled workers between sectors could play an important role in explaining the U.S. skill premium movement and shows that the transitional dynamics initiated by structural change (possibly the shock to R&D productivity) can explain the comovement of the variables. Although the paper does not empirically identify the suspected structural change, R&D appears to be an important factor in explaining the trends in the post-war U.S. economy.

The most important direction for future work is to do a numerical analysis of the transitional dynamics by calibrating the model. A second direction for future work is to introduce human capital into the model, as this factor has been considered important to economic growth (e.g., Nelson and Phelps (1966) and Lucas (1988)). For example, because the effectiveness of R&D is likely to be affected by the level of human capital in

[16] Substituting $T = \frac{1-u}{u}$ and equation (a-6) into equation (14) gives

$$\frac{(1-\dot{u}(t))}{(1-u(t))} = \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta}\,\frac{s\,e^{n\,t}\,(1-u(t))}{\eta\,N(t)^{1-\phi}} + \frac{s\,e^{n\,t}\,u(t)}{\eta\,N(t)^{1-\phi}}(1-\phi-\theta) - \rho\right)\,.$$

Substituting this into equation (21), we obtain equation (22).

[17] $\Lambda(t)$ is constant at the steady state. Because $\Lambda(t) = s\,e^{n\,t}N(t)^{\phi-1}$, $\Lambda(t)$ increases over time because the growth rate of $\Lambda(t)$ is positive at every point on the saddle path where $T > T^*$.

an economy, considering human capital could generate some interesting findings about cross-country differences in the relationship between the skill premium and R&D.

Appendix A

To analyze the dynamics of the economy, we must derive equations that explain the dynamics of $c(t)$, $u(t)$ and $N(t)$. One differential equation is derived from the Euler equation (12) together with the expression for the rate of return given by equation (7). Substituting equation (7) into equation (12) yields

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left(\frac{\pi(t)}{Z(t)} + \frac{\dot{Z}(t)}{Z(t)} - \rho\right). \tag{a-1}$$

Because $\pi(t) = \overline{p}\overline{X}(t) - \eta\overline{X}(t)$, $X_j = \overline{X} = \left(\frac{L_L^{\alpha}L_{HY}^{\beta}(1-\alpha-\beta)^2}{\eta}\right)^{\frac{1}{\alpha+\beta}}$ and $p_jX_j = \overline{p} = \frac{\eta}{1-\alpha-\beta}$ from Section 2, the monopolist's operational profits can be rewritten as

$$\pi(t) = \eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}}(\alpha+\beta)(1-\alpha-\beta)^{\frac{2-\alpha-\beta}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{\beta}{\alpha+\beta}}(1-u(t))^{\frac{\beta}{\alpha+\beta}}L(t). \tag{a-2}$$

Using expression (3) and equation (5) yields (in equilibrium $w_{HY} = w_{HN}$)

$$Z(t) = \frac{\eta}{N(t)^{\phi}}\beta L_L(t)^{\alpha}L_{HY}(t)^{\beta-1}\int_0^{N(t)} X_j^{1-\alpha-\beta}dj.$$

Because $X_j = \overline{X} = \left(\frac{L_L^{\alpha}L_{HY}^{\beta}(1-\alpha-\beta)^2}{\eta}\right)^{\frac{1}{\alpha+\beta}}$, $L_{HY}(t) = (1-u(t))sL(t)$ and $L_L(t) = (1-s)L(t)$ from Section 2, this expression can be rewritten as:

$$Z(t) = \beta\eta^{\frac{2(\alpha+\beta)-1}{\alpha+\beta}}(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}(1-s)^{\frac{\alpha}{\alpha+\beta}}s^{\frac{-\alpha}{\alpha+\beta}}(1-u(t))^{\frac{-\alpha}{\alpha+\beta}}N(t)^{1-\phi}. \tag{a-3}$$

Taking logs and differentiating with respect to time on both sides of the above equation yield

$$\frac{\dot{Z}(t)}{Z(t)} = (1-\phi)\frac{\dot{N}(t)}{N(t)} - \frac{\alpha}{\alpha+\beta}\frac{(1-\dot{u}(t))}{(1-u(t))}. \tag{a-4}$$

Substituting equations (a-2), (a-3) and (a-4) into equation (a-1) gives:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[\frac{(1-\alpha-\beta)(\alpha+\beta)}{\beta}G_N(t)T(t) + (1-\phi)G_N(t) - \frac{\alpha}{\alpha+\beta}G_T(t) - \rho\right], \tag{a-5}$$

where $G_N(t) \equiv \frac{\dot{N}(t)}{N(t)}$, $G_T(t) \equiv \frac{(1-\dot{u}(t))}{(1-u(t))}$, and $T(t) \equiv \frac{1-u(t)}{u(t)}$.

The second differential equation, which shows the dynamics of N, is derived from equation (8). Based on equation (8), the growth rate of $N(t)$ is given by

$$G_N(t) = \frac{1}{\eta}u(t)\,s\,L(t)\,N(t)^{\phi-1}. \tag{a-6}$$

Thus, the growth rate of G_N is shown as follows:

$$\frac{\dot{G}_N(t)}{G_N(t)} = -G_T(t)\,T(t) - (1-\phi)\,G_N(t) + n. \tag{a-7}$$

Finally, we derive the equation that describes the motion of u. Because $X_j = \overline{X} = \left(\frac{L_L^{\alpha} L_{HY}^{\beta} (1-\alpha-\beta)^2}{\eta}\right)^{\frac{1}{\alpha+\beta}}$, $L_{HY}(t) = (1-u(t))sL(t)$ and $L_L(t) = (1-s)L(t)$, using equation (1) we can write output per labor, $y(t)$, as follows:

$$y(t) = \eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}} (1-s)^{\frac{\alpha}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} (1-u(t))^{\frac{\beta}{\alpha+\beta}} N(t) \,. \tag{a-8}$$

Taking logs and differentiating with respect to time on both sides of the above equation and rearranging it, we obtain

$$G_T(t) = \frac{\alpha+\beta}{\beta} \left(\frac{\dot{y}(t)}{y(t)} - G_N(t) \right) . \tag{a-9}$$

Because $C(t) = Y(t) - \eta\, N(t) \bar{X}(t)$, consumption per labor is given by

$$c(t) = y(t) - \frac{\eta\, N(t)\, \bar{X}(t)}{L(t)} \,. \tag{a-10}$$

Because $X_j = \overline{X} = \left(\frac{L_L^{\alpha} L_{HY}^{\beta} (1-\alpha-\beta)^2}{\eta}\right)^{\frac{1}{\alpha+\beta}}$, $L_{HY}(t) = (1-u(t))sL(t)$ and $L_L(t) = (1-s)L(t)$, we have

$$\begin{aligned} \frac{N(t)\, \bar{X}(t)}{L(t)} &= \eta^{\frac{-1}{\alpha+\beta}} (1-\alpha-\beta)^{\frac{2}{\alpha+\beta}} (1-s)^{\frac{\beta}{\alpha+\beta}} s^{\frac{\beta}{\alpha+\beta}} (1-u(t))^{\frac{\beta}{\alpha+\beta}} N(t) \\ &= y(t) \left(\eta^{-1} (1-\alpha-\beta)^{\frac{2(\alpha+\beta)}{\alpha+\beta}} \right) . \end{aligned}$$

Substituting this equation into equation (a-10) yields

$$c(t) = y(t) \left(1 - (1-\alpha-\beta)^2 \right) . \tag{a-11}$$

This result leads to $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$. Therefore, equation (a-9) is rewritten as

$$G_T(t) = \frac{\alpha+\beta}{\beta} \left(\frac{\dot{c}(t)}{c(t)} - G_N(t) \right) . \tag{a-12}$$

Equations (a-5), (a-7) and (a-12) together describe the dynamics of c, u and N and thus the economy. The above equations can be reduced to the following two equations:

$$\frac{\dot{(1-u(t))}}{(1-u(t))} = \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)} \left(\frac{(1-\alpha-\beta)\,(\alpha+\beta)}{\beta} G_N(t) T(t) + (1-\phi-\theta) G_N(t) - \rho \right) \tag{a-13}$$

and

$$\begin{aligned} \frac{\dot{G_N(t)}}{G_N(t)} &= \frac{-(1-\alpha-\beta)\,(\alpha+\beta)^2}{(\beta\,\theta+\alpha)\,\beta} G_N(t)\, T(t)^2 \\ &- \frac{(\alpha+\beta)\,(1-\phi-\theta)}{(\beta\,\theta+\alpha)} G_N(t)\, T(t) + \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)} \rho\, T(t) + n - (1-\phi)\, G_N(t) \,. \end{aligned} \tag{a-14}$$

Appendix B

Assume that the economy is on a path that eventually hits the vertical axis in finite time in Figure 1. When it hits the vertical axis, $u = 1$, which implies $y = 0$ according to equation (a-8): all high skilled workers are employed in R&D. Therefore, because $\frac{\dot{c}(t)}{c(t)} = \frac{\dot{y}(t)}{y(t)}$, c must jump downward to 0 at the time when the economy reaches the vertical axis. This behavior violates the Euler equation (12). Therefore, the path cannot be an equilibrium.

Next, we assume that the economy is on a path that asymptotically reaches at the point where $u = 0$ and $G_N = 0$. If the economy is on this path, u and G_N will monotonically decrease after some point of time. By using equation (14), one can write the growth rate of $(1-u)$ as follows:

$$\begin{aligned}\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))} &= \frac{(1-\alpha-\beta)\,(\alpha+\beta)^2}{(\beta\,\theta+\alpha)\,\beta}\,\frac{s\;e^{n\,t}\,(1-u(t))}{\eta\,N(t)^{1-\phi}} \\ &\quad + \frac{(\alpha+\beta)\,(1-\phi-\theta)}{(\beta\,\theta+\alpha)}\,G_N(t) - \frac{(\alpha+\beta)}{(\beta\,\theta+\alpha)}\rho\,. \end{aligned} \tag{b-1}$$

Equation (b-1) shows that $\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))}$ will monotonically increase towards infinity after some point of time (i.e., $\lim_{t\to\infty}\frac{\dot{\overline{(1-u(t))}}}{(1-u(t))} = \infty$ holds on this path). This behavior violates the labor constraint ($0 \leq 1-u \leq 1$). Therefore, the path can not be an equilibrium.

As a result, the saddle path in Figure 1 is the only equilibrium in the model.

Appendix C

To show that there is at least a range of $N(0)$ that guarantees that the $N(0)$ locus intersects with the saddle path only once, we first consider the $\dot{\overline{(1-u(t))}} = 0$ locus. Substituting equation (20) into equation (16) and solving for $u(0)$ yields

$$u(0) = \frac{\beta\eta\rho - s\,(1-\alpha-\beta)(\alpha+\beta)N(0)^{\phi-1}}{s\,N(0)^{\phi-1}(\beta(1-\phi-\theta)-(1-\alpha-\beta)(\alpha+\beta))} \tag{c-1}$$

The denominator in equation (c-1) is negative with the assumed parameter values. We assume that $N(0)$ is sufficiently low to satisfy $u(0) > 0$ in equation (c-1). Therefore, the $N(0)$ locus intersects with the $\dot{\overline{(1-u(t))}} = 0$ locus only once at a point where $T(0) > 0$ and $G_N(0) > 0$ in Figure 2. To the left (right) of the intersection point, the $N(0)$ locus is below (above) the $\dot{\overline{(1-u(t))}} = 0$ locus.

Because the $\dot{\overline{(1-u(t))}} = 0$ locus also intersects with the saddle path only once at the steady state, there is a value of $N(0)$ that makes the $N(0)$ locus go through the steady state. We define this value as $N(0)^*$. Therefore, the $N(0)$ locus and the saddle path have only one intersection point when $N(0) = N(0)^*$. This implies that at least when $N(0)$ is in the neighborhood of $N(0)^*$, the $N(0)$ locus intersects with the saddle path only once.

Appendix D

The behavior of c at the time of the structural change is revealed by equations (a-8) and (a-11). Substituting equation (a-8) into equation (a-11), consumption per labor at $t = 0$ can be given by

$$
\begin{aligned}
c(0) &= (1-(1-\alpha-\beta)^2)y(0) \\
&= \Omega\,\eta^{\frac{-(1-\alpha-\beta)}{\alpha+\beta}}\,(1-u(0))^{\frac{\beta}{\alpha+\beta}}\,N(0)\;,
\end{aligned}
$$

where $\Omega = (1-(1-\alpha-\beta)^2)(1-\alpha-\beta)^{\frac{2(1-\alpha-\beta)}{\alpha+\beta}}\,(1-s)^{\frac{\alpha}{\alpha+\beta}}\,s^{\frac{\beta}{\alpha+\beta}}$. Therefore, c can jumps up or down when η increases unexpectedly. The direction depends on the extent to which the increase in η reduces $u(0)$. Note that the discontinuous change in c does not imply a violation of the Euler equation given by equation (12) because the sudden increase or decrease in c is the optimal response to the new information.

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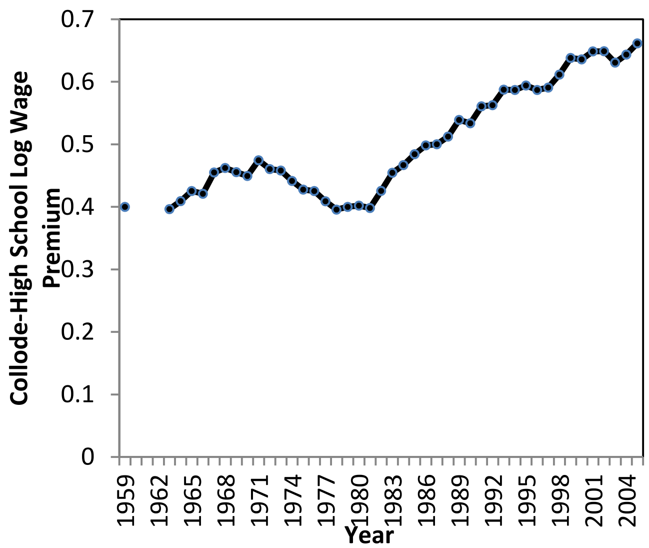
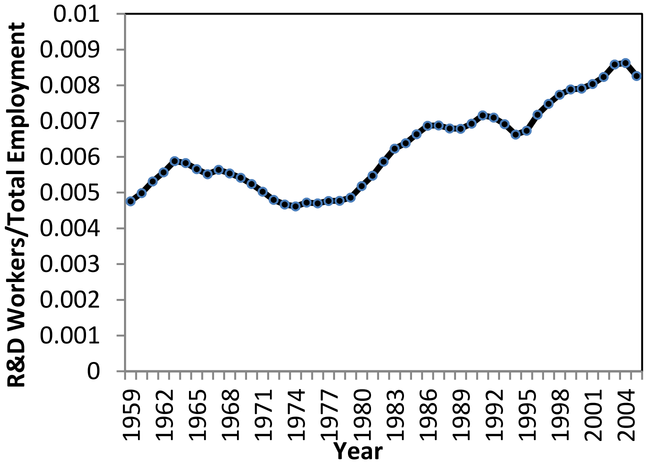
Figure 1: College skill premium: 1959–2005



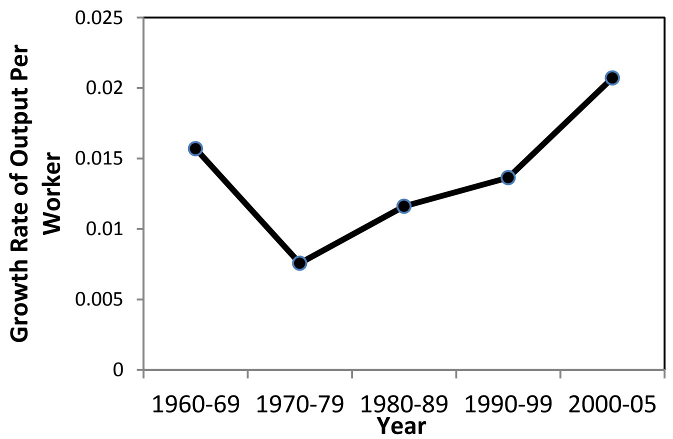
Note: Data from Autor, Katz and Kearney (2008).

Figure 2: R&D workers share: 1959–2005



Notes: Data on R&D workers (scientists and engineers engaged in R&D) are mainly from NSF/Division of Science Resources Statistics, Survey of Industrial Research and Development. Since the NSF dataset has some missing observations (1959–62, 1965–66, and 1969–70), we use Jones (1995b)'s data on R&D workers to fill the missing observations (we extend the R&D workers series in the NSF dataset by annual changes calculated in the Jones (1995b)'s data. Data on total employment come from BLS, Establishment Survey.

Figure 3: Labor productivity growth: 1960–2005



Notes: Data on Real GDP (Billions of Chained 2005 Dollars) are from NIPA. Data on labor (total employment) come from BLS, Establishment Survey.

Figure 4: Phase diagrams

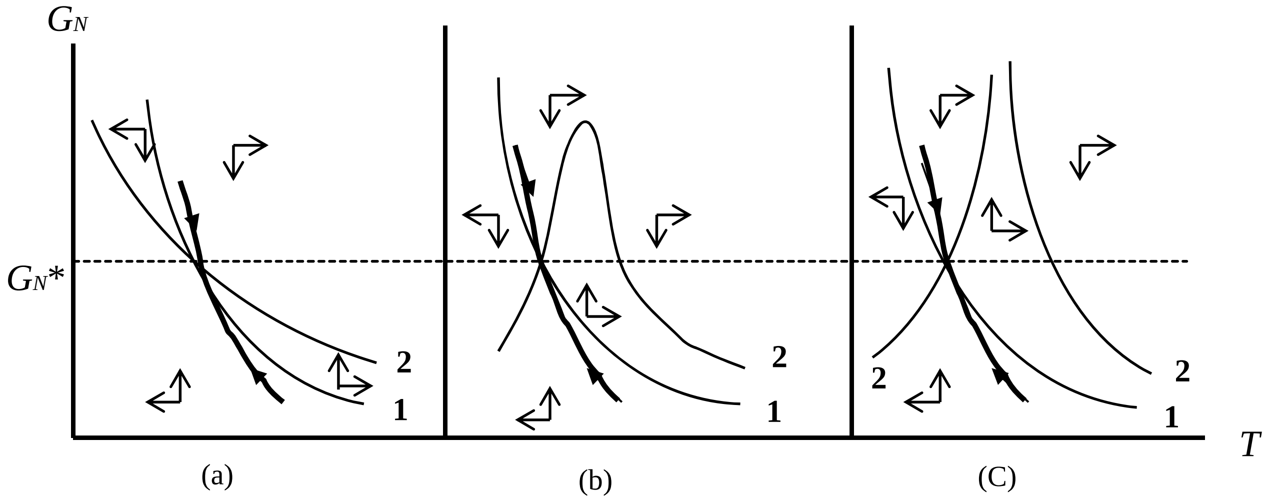


Figure 5: The N(0) locus

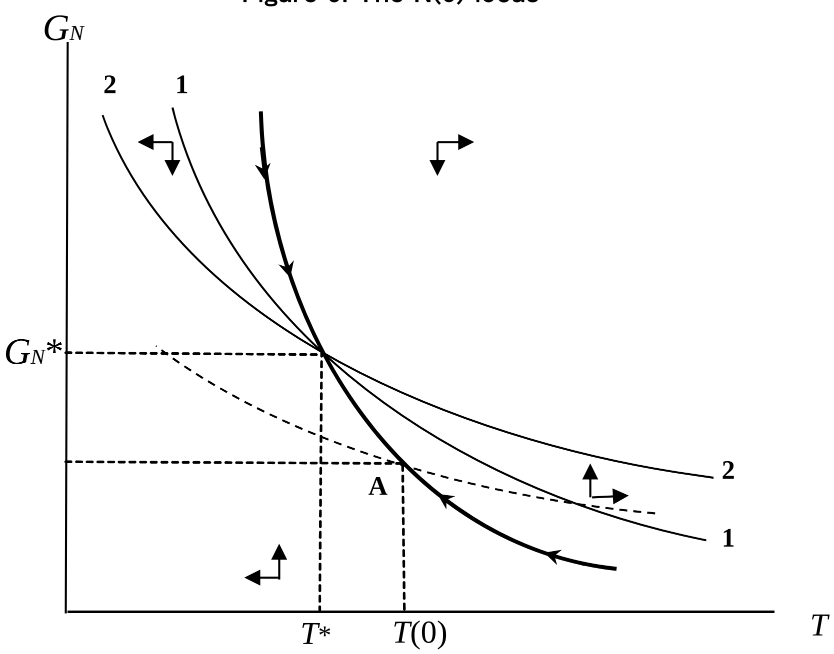
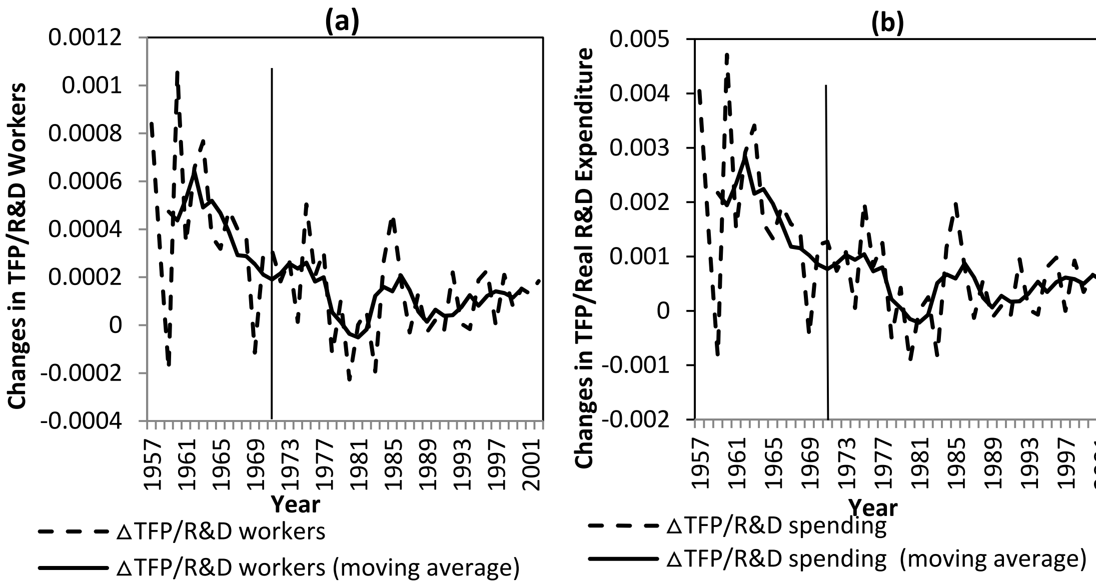


Figure 6: R&D productivity: 1957–2002



Note: Data on R&D workers (scientiss and engineers engaged in R&D) are the same as those used in figure 2. Real R&D data (1996$ price) are from NSF/Division of Science Resources Statistics, National Patterns of R&D Resources, Science & Engineering Indicators 2004. Yearly changes in TFP are calcuated by using the TFP growth rate data from Federal Reserve Bank of San Francisco and the initial year (1956) data from Jones (2002). The term 'moving average' indicates 5 year centered moving average.

Figure 7: The effects of an unexpected increase in η