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Closed Form of Fiscal Multipliers in a DSGE model with Liquidity-Constrained households

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Abstract

In this paper, we obtain the closed form of fiscal multipliers in a dynamic stochastic general equilibrium model with liquidity-constrained households. The closed form indicates that the first-period multiplier depends monotonically on the fraction of liquidity-constrained households over all the households. When this fraction is one, the maximum value of the multiplier is obtained as the inverse of one minus the labor share. This result indicates that our model has the traditional Keynesian fiscal multiplier (the inverse of one minus the marginal propensity to consume) as a special case.

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1. Introduction

Recent analyses of the effects of macroeconomic policies have often been conducted using a dynamic stochastic general equilibrium (DSGE) model. In DSGE-type models, the agents' behavior is usually depicted in a forward-looking manner and the models comprise stochastic difference equations. Therefore, it is often difficult to obtain the closed form of policy effects, even if the model is linear, and there is a need for numerical simulation. Against this background, we attempt to obtain the closed form of policy effects, with focus on fiscal policy, at the cost of omitting capital stock variation.[1] Instead of this cost, we obtain the closed form of fiscal multipliers. We believe that the closed-form expression would be particularly useful for policy makers having limited accessibility to full-blown DSGE modeling, like Christiano et al. (2005).

To construct a DSGE model for investigating fiscal-policy effects, we must carefully examine whether the model can replicate empirical results. Empirical results using the vector autoregression model often indicate that the consumption response to fiscal spending shocks is positive.[2] Therefore, the model must at least replicate this fact. However, a typical DSGE model generates a negative consumption response.[3] Fortunately, several remedies have been suggested: we need to assume (i) a "deep habit" (Ravn et al., 2006), (ii) a utility function that strengthens the complementarity between consumption and labor (Linnemann, 2006; Monacelli et al., 2010), or (iii) liquidity-constrained (non-Ricardian) households (Galí et al., 2007). In this paper, we assume the existence of liquidity-constrained households, which is frequently assumed in fiscal studies. We use this assumption for the sake of simplicity and because it seems to be plausible and realistic.

The following are the contributions of this paper. First, we obtain a reduced-form autoregressive moving average (ARMA) representation for output. Next, we obtain its moving average (MA) representation. This MA representation enables one to obtain the closed form of fiscal multipliers. Using this method, policy planners can easily calculate the effects of fiscal policy on output. The closed form indicates that the first-period multiplier depends monotonically on the fraction of liquidity-constrained households over all the households. When this fraction is one, the maximum value of the multiplier is obtained as the inverse of one minus the labor share. This result indicates that our model has the traditional Keynesian fiscal multiplier (the inverse of one minus the marginal propensity to consume) as a special case.

This paper is organized in the following manner. Section 2 presents the DSGE model used in the paper. Section 3 describes the ARMA representation for output that is used to present the closed form of fiscal multipliers. Section 4 presents the numerical implications of the multiplier. Section 5 concludes this paper.

2. The Model

In this section, we set up the DSGE model. We assume that the economy includes firms, Ricardian households, liquidity-constrained (non-Ricardian) households, and a government. For analytical simplicity, we omit capital stock; this simplification is needed to obtain the

[1]Note that in the estimated DSGE models, investment response is usually negative. However, as shown by Baxter and King (1993) and Aiyagari et al. (1992), if the persistency in government expenditure is sufficiently high, investment can increase.

[2] A positive consumption response for fiscal expansion has been empirically confirmed by Blanchard and Perotti (2002) and Mountford and Uhlig (2009) using US data. Beetsma and Giuliodori (2011) have confirmed this response in EU countries.

[3] For example, see Baxter and King (1993).

closed-form multiplier.

2.1 Firms

Suppose that the firms in our economy produce final goods using the following production function:

$$y_t = z(h_t)^{\alpha},$$
$$0 < \alpha < 1,$$

where Y_t denotes the real output, z, the deterministic technology level, and h_t, the hours worked.[4] Suppose further that the firms hire labor such that the profits $Y_t - w_t h_t$ are maximized and the households are paid the excess profits. Note that we normalize the number of employees to unity for all *t*.

2.2 Ricardian Households

Ricardian (or optimizing) households receive wages, excess profits from firms, and income from financial assets. In our economy, the only financial assets available are government bonds. The income that households receive is spent for consumption and bonds, after paying a lump-sum tax. The budget constraint for households can be defined in the following manner:

$$b_{t+1} = R_{t-1} b_t + w_t h_t + \Pi_t - c_t^o - t_t, \tag{1}$$

where b_t denotes a government bond, R_t, the gross real interest rate, c_t^o, the real consumption for Ricardian households, Π_t, excess profits from firms, and t_t, the real tax.

Assuming that the temporal utility function is $log c_t^o + \theta \log(1 - h_t)$, our maximization for Ricardian households can be expressed in the following manner:

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^{-t} log c_t^o + \theta \log(1 - h_t),$$
$$\text{s.t.}\, b_{t+1} = R_{t-1} b_t + w_t h_t - c_t^o - t_t.$$

As an additional constraint, we impose the no-Ponzi-game condition. From the above maximization problem, we obtain the following first-order conditions:

$$-\theta \frac{1}{1 - h_t} + \frac{w_t}{c_t^o} = 0, \tag{2}$$

$$1 - E_t \left[\beta R_t \frac{c_t^o}{c_{t+1}^o} \right] = 0. \tag{3}$$

2.3 Liquidity-constrained Households (Non-Ricardian Households)

This paper assumes that there are households that face liquidity constraints and that they therefore consume only up to their income level. Letting the consumption level of the households be c_t^l, we assume the following:

$$c_t^l = w_t h_t - t_t.$$

Further, we assume that liquidity-constrained households function in the same manner as Ricardian households, as in Galí et al. (2007).

Denoting the number of liquidity-constrained households by ω^l, total consumption c_t can be expressed as $c_t = \omega^l c_t^l + (1 - \omega^l) c_t^o$.

2.4 Government

The government collects a lump-sum tax from households and utilizes it to purchase final goods. If the government cannot collect a sufficient amount of tax to finance its expenditure, it issues government bonds. Therefore, the government's budget constraint is expressed in the

[4] The total number of households is normalized and fixed to one for all periods.

following manner:

$$b_{t+1} = R_{t-1}b_t + g_t - t_t, \tag{4}$$

where g_t denotes the real government expenditure. We assume that the government implements fiscal reforms if its debt increases. This assumption yields the following fiscal rules:

$$t_t = t(b_t), \tag{5}$$

$$g_t = g(b_t)e^{e_t}, \tag{6}$$

where $\frac{dt(b_t)}{db_t} \geq 0$ and $\frac{dg(b_t)}{db_t} \leq 0$. Furthermore, the government's spending shock is assumed as the following AR (1) process:

$$e_t = \rho e_{t-1} + \varepsilon_t,$$

where ε_t is an i.i.d. random variable with mean 0. These formulations for government behavior must be considered in order to avoid debt instability.

2.5 General Equilibrium

Equations (2) to (8) and the market clearing condition $y_t = c_t + g_t$ constitute the general equilibrium. We linearize the model around the non-stochastic steady state in order to obtain its closed form. The linearized model is expressed in the following manner:

$$\hat{y}_t - \hat{c}_t^o - \frac{\hat{h}_t}{1-h} = 0, \tag{7}$$

$$\hat{y}_t = \alpha \hat{h}_t, \tag{8}$$

$$\hat{y}_t = [\omega \hat{c}_t^l + (1-\omega)\hat{c}_t^o]\frac{c}{y} + \hat{g}_t\frac{g}{y}, \tag{9}$$

$$\hat{c}_t^l c = \alpha \hat{y}_t y - \hat{t}_t t, \tag{10}$$

$$\hat{t}_t = \eta_{tax}\hat{b}_t, \tag{11}$$

$$\hat{g}_t = -\eta_g \hat{b}_t + e_t, \tag{12}$$

$$\hat{R}_t = E_t[\hat{c}_{t+1}^o - \hat{c}_t^o], \tag{13}$$

$$\hat{b}_{t+1} = (\hat{R}_{t-1} + \hat{b}_t)R + \hat{g}_t\frac{g}{b} - \hat{t}_t\frac{t}{b}, \tag{14}$$

where $\omega = \omega^l c^l / c$;, and $\eta_{tax} = \frac{dt}{db}\frac{b}{t}$ and $\eta_g = \frac{dg}{db}\frac{b}{g}$ represent the elasticity of tax and government expenditure with respect to debt, respectively. In the above equations, "^" denotes the deviation from the steady-state value and the variables without the time subscript denote the steady-state value.

3. The Closed Form of Fiscal Multipliers

In this section, we first derive an ARMA representation of output from the linearized DSGE model. Next, we convert the ARMA representation to the MA representation in order to obtain the closed form of fiscal multipliers.

To derive the ARMA representation of output, we first use Equations (7) to (12) to obtain

$$\hat{y}_t = x_1\hat{b}_t + x_2 e_t, \tag{15}$$

where

$$x_1 = -\left(\omega\eta_{tax}\frac{t}{y} + \eta_g\frac{g}{y}\right)m, \qquad x_2 = \frac{g}{y}m,$$

$$m = \left[1 - \omega\alpha - (1-\omega)\frac{c}{y}\left[1 - \frac{1}{\alpha(1-h)}\right]\right]^{-1}.$$

Equation (15) and the fact that $m > 0$ imply that the output is decreasing for $\hat{b}_t$ and increasing for e_t. Note that m represents the fiscal multiplier and that a non-zero value of

ω is necessary to obtain a first-period multiplier larger than 1. Next, Equation (15) implies that

$$\hat{c}_t^o = x_3 \hat{b}_t + x_4 e_t, \tag{16}$$
$$x_3 = \left[1 - \frac{1}{\alpha(1-h)}\right] x_1, \qquad x_4 = \left[1 - \frac{1}{\alpha(1-h)}\right] x_2.$$

Note that x_4 represents the crowding-out effects of the Ricardian households' consumption level. Equations (15) and (16) are obtained in a linear static framework because we omit capital investment; therefore, the state variables $\hat{b}_t$ and e_t uniquely pin down the movement of jump variables. In this case, if the state variables are stable, so are the jump variables.

For stability of the model, the key variable is debt. Equations (13), (14), and (16) yield the following equation:

$$\hat{b}_t = x_5 \hat{b}_{t-1} + x_6 \hat{b}_{t-2} + \frac{g}{b} e_{t-1} + x_8 e_{t-2}, \tag{17}$$
$$x_5 = \left[x_3 R - \eta_g \frac{g}{b} - \eta_{tax} \frac{t}{b} + R\right],$$
$$x_6 = -x_3 R, \qquad x_7 = \frac{g}{b}, \qquad x_8 = x_4 (1-\rho) R.$$

As shown in this equation, debt evolves in the manner of ARMA (3,2).[5] Equivalently, Equation (15) yields the following ARMA (3,3) representation for output:

$$\hat{y}_t = x_5 \hat{y}_{t-1} + x_6 \hat{y}_{t-2} + x_2 e_t + x_9 e_{t-1} + x_{10} e_{t-2}, \tag{18}$$
$$x_9 = x_1 x_7 - x_2 x_5,$$
$$x_{10} = x_1 x_8 - x_2 x_6.$$

Using the time series analysis technique, we reduce the above ARMA representation to the following MA representation:[6]

$$\hat{y}_t = \psi_0 e_t + \psi_1 e_{t+1} + \psi_2 e_{t-2} \cdots,$$

where the coefficients satisfy the following system:

$$\psi_0 = x_2,$$
$$\psi_1 = x_1 x_7,$$
$$\psi_2 = x_1 x_8 + x_1 x_5 x_7,$$
$$\psi_i = x_5 \psi_{i-1} + x_6 \psi_{i-2}. \, (\forall i \geq 3)$$

Therefore, the effects of government spending shocks on output can be calculated by

$$\frac{d\hat{y}_{t+i}}{de_t} = \sum_{j=0}^{i} \psi_j \, \rho^{i-j}. \tag{19}$$

In our model—since $y_t = (1 + \hat{y}_{t+i}) y$, —the following are the fiscal (impact) multipliers, which are defined as how much output increases when there is one unit of fiscal expansion:

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[5] The stability conditions are $\left|\left(x_5 \pm \sqrt{(x_5)^2 - 4x_6}\right)/2\right| < 1$. If setting $\eta_g = \eta_{tax} = 0$ implies that the roots are 0 and R, then the stability conditions are never satisfied. Therefore, η_g and/or η_{tax} must be positive. This is rather intuitive because if the government does nothing for managing debt, debt is accumulated at the rate R.

[6] The details of this are as follows. Using the lag operator L, Equation (18) can be written in the following manner:

$$(1 - x_5 L - x_6 L^2) \hat{y}_t = (x_2 + x_9 L + x_{10} L^2) e_t.$$

In this equation, we define the following lag polynomial:

$$\psi(L) = \psi_0 + \psi_1 L + \psi_2 L^2 + \psi_3 L^3 + \cdots.$$

If $(1 - x_5 L - x_6 L^2)$ is invertible and therefore stable, we obtain $\hat{y}_t = \Psi(L) e_t$. Then, the following identity holds:

$$(1 - x_5 L - x_6 L^2)(\psi_0 + \psi_1 L + \psi_2 L^2 + \psi_3 L^3 + \cdots) = x_2 + x_9 L + x_{10} L^2.$$

The method of unidentified coefficients yields ψ_i.

$$\lim_{g_t \to g} \frac{y_{t+i} - y}{g_t - g} = \frac{d\hat{y}_{t+i}}{de_t}\frac{y}{g} = \frac{y}{g}\sum_{j=0}^{i} \psi_j \rho^{i-j}. \tag{20}$$

Note that $\lim_{g_t \to g} \frac{y_{t+i}-y}{g_t-g} = \frac{\Delta y_{t+i}}{\Delta g_t}$ because of the system's linearity. For convenience of researchers' applications, we provide the following explicit expression for multipliers:

the first-period multiplier= $\frac{y}{g}\psi_0$,

the second-period multiplier= $\frac{y}{g}(\psi_0\rho + \psi_1)$,

the third-period multiplier= $\frac{y}{g}(\psi_0\rho^2 + \psi_1\rho + \psi_2)$,

and so on.

4. Quantitative Analysis

This section presents quantitative implications using the closed form of the multipliers (20). First, we focus on the first-period multiplier denoted by $d_0 = \frac{d\hat{y}_t}{de_t}\frac{y}{g}$, that is,

$$d_0 = \left[1 - \omega\alpha - (1-\omega)\frac{c}{y}\left[1 - \frac{1}{\alpha(1-h)}\right]\right]^{-1}.$$

From this expression, it can be shown that $\omega = 0$ implies that $0 < d_0 < 1$, because α, h, and c/y are less than 1. On the other hand, a strictly positive ω can generate $d_0 \geq 1$. Note that as long as $d_0 \geq 1$, consumption positively responds to fiscal policy shocks.[7] In addition, it is evident that d_0 is an increasing function of ω. Figure 1 shows how d_0 depends on ω, fixing the typical value of α at 2/3 and under several values of the steady-state hours worked.[8] With the given value of ω, −multipliers negatively depend on h. The intuition is that in the steady state, a higher h implies a lower θ, which represents the weight of the temporal utility on leisure. Therefore, a higher h forces households to work less.

Insert Figure 1
Insert Table 1

Furthermore, it can be confirmed that $\lim_{\omega \to 1} d_0 = [1-\alpha]^{-1}$. This can be easily understood by considering the case of $\omega = 1$, where the model reduces to the traditional Keynesian model and α can be interpreted as the marginal propensity to consume (MPC). Note that if $\omega = 1$, then the aggregate consumption is equal to the aggregate wage income because of liquidity constraints. In turn, the wage income is equal to αY; therefore, the MPC

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[7] In Galí et al. (2007), the sticky price assumption is necessary to generate a positive consumption response. With price stickiness, an increase in real interest rate becomes moderate in implementing fiscal expansion. This dampens a decrease in consumption for Ricardian households. Furthermore, counter-cyclical markup movements shift the labor demand curve up, and this increases output. In our model, a positive consumption response is obtained without the sticky price assumption. This is because investment is omitted in our model. To consider why our model can generate such a response, first note that there is a pressure of a decrease in future output that is inevitable to guarantee the solvency of debt. This future output decrease can occur only through decreasing the hours worked, since our model does not include investment. This negative effect on future income generates a larger negative wealth effect, forcing the Ricardian households to work more and consume less; note that $[\alpha - 1/(1-h)]\hat{h}_t = \hat{c}_t^o$, from Equations (7) and (8). The increase in hours worked raises the current income and consumption for liquidity-constrained households, compensating for the decrease in Ricardian households' consumption. Thus, consumption in our model generates positive responses to fiscal expansion without price stickiness.

[8] For convenience, we provide the numerical values of the multipliers in Table 1.

is α. Thus, $\lim_{\omega \to 1} d_0$ can be interpreted as the traditional Keynesian fiscal multiplier. Finally, from the above expression of d_0, it is evident that if ω is sufficiently large, a higher labor income share α generates large multipliers. The reason is straightforward: a higher α forces non-Ricardian households to consume more.

The effects of fiscal policy after the second period are affected by debt dynamics due to the increase in lump-sum tax. Assuming $\rho = 0$ for simplicity, we can easily confirm the second-period multiplier in the following manner:

$$d_1 = -\left(\omega \eta_{tax} \frac{t}{b} + \eta_g \frac{g}{b}\right) \left[1 - \omega\alpha - (1-\omega)\frac{c}{y}\left[1 - \frac{1}{\alpha(1-h)}\right]\right]^{-1} < 0.$$

Therefore, in the second period, fiscal policies have a negative effect on the output when $\rho = 0$. The costs for debt to converge represented by η_{tax} and η_g have a negative effect on the output through a decrease in final demand. Although it would be preferable to set both η_{tax} and η_G at zero in order to offset the negative effect, this policy rule would generate an unstable debt process. Thus, the negative effect of fiscal policy is inevitable when $\rho = 0$. On the other hand, in the case of $\rho > 0$, since there is some amount of fiscal expansion in the second period and this has a positive effect on output, d_1 can be positive.

Finally, it is worth noting that if the primary balance is held in the steady state ($g - t = 0$), raising η_{tax} would incur less cost than raising η_g, because such a policy mitigates the negative impact on d_1. The intuition behind this is that although a one unit increase in η_g directly decreases demand by 1, a one unit increase of η_{tax} indirectly decreases demand by ω through an increase in the tax on non-Ricardian households.

5. Concluding Remarks

In this paper, we obtain the closed form of fiscal multipliers in a DSGE model in which there are liquidity-constrained households. This is obtained through the MA representation of output. The closed form indicates that the first-period multiplier depends monotonically on the fraction of liquidity-constrained households over all the households, and that the maximum value of the multiplier is obtained when the share of liquidity-constrained households is close to unity.

References

Aiyagari, S. R., Christiano, L. J., and Eichenbaum, M. (1992) “The Output, Employment, and Interest Rate Effects of Government Consumption” *Journal of Monetary Economics* **30**, 73-86.

Baxter, M. and King, R. G. (1993) “Fiscal Policy in General Equilibrium” *American Economic Review* **92**, 571-589.

Beetsma, R. and Giuliodori M. (2011) “The Effects of Government Purchases Shocks: Review and Estimates for the EU” *Economic Journal* **121**, F5-F32.

Blanchard, O. and Perotti, R. (2002) “An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output” *Quarterly Journal of Economics* **117**, 1329-1368.

Christiano, L., Eichenbaum, M., and Evans, C. (2005) “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy” *Journal of Political Economy* **113**, 1-45.

Galí, J., Valles, J., and Lopez-Salido, J. D. (2007) “Understanding the Effects of Government Spending on Consumption” *Journal of the European Economic Association* **5**, 227-250.

Linnemann, L. (2006) “The Effect of Government Spending on Private Consumption: A Puzzle?” *Journal of Money, Credit, and Banking* **38**, 1715-1735.

Monacelli, T., Perotti, R., and Trigari, A. (2010) “Unemployment Fiscal Multipliers,” *Journal* *of Monetary Economics* **57**, 531-553.

Mountford, A. and Uhlig, H. (2009) "What Are the Effects of Fiscal Policy Shocks?" *Journal of Applied Econometrics* **24**, 960-992.

Ravn, M. O, Schmitt-Gorohe, S., and Uribe, M. (2006) "Deep Habits" *Review of Economic Studies* **73**, 195-218.

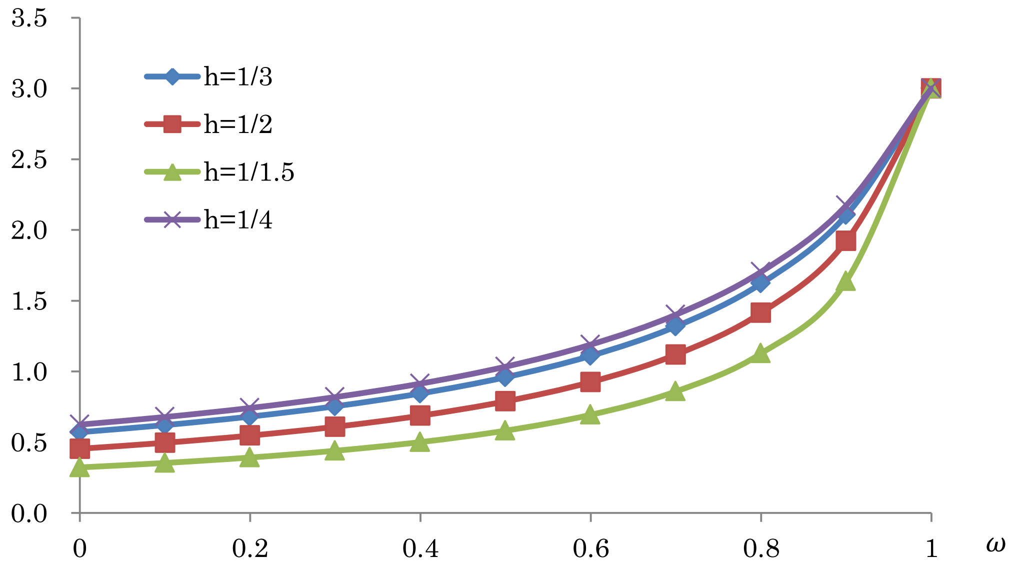

Figure 1. Dependency of the fiscal multiplier on the share of liquidity-constrained households under several levels of steady-state hours worked.
Note: Each line depicts the impact of fiscal multipliers.

Table 1. Fiscal multipliers under several levels of steady-state hours worked.

ω	h=1/4	h=1/3	h=1/2	h=1/1.5
0.0	0.63	0.57	0.45	0.32
0.1	0.68	0.62	0.50	0.35
0.2	0.74	0.68	0.55	0.39
0.3	0.82	0.75	0.61	0.44
0.4	0.91	0.85	0.69	0.50
0.5	1.03	0.96	0.79	0.58
0.6	1.19	1.11	0.93	0.69
0.7	1.40	1.32	1.12	0.86
0.8	1.70	1.62	1.42	1.13
0.9	2.17	2.11	1.92	1.64
1.0	3.00	3.00	3.00	3.00