Ownership concentration, monitoring, and optimal board structure

Clara Graziano  
University of Udine

Annalisa Luporini  
University of Florence

Abstract
We analyze the choice between a one-tier and a two-tier board structure in a firm with a large shareholder sitting on the board. The board has two tasks: project selection and monitoring the ability of the manager. In a one-tier structure, the sole board performs all tasks. In a two-tier structure, the management board is in charge of project selection and the supervisory board is in charge of monitoring. We show that such a two-tier structure can limit interference from the large shareholder and provide the manager with the incentive to exert effort to become informed on investment projects without reducing the large shareholder’s incentive for monitoring. This results in higher expected profits. If the increase in profits is high enough, the large shareholder prefers a two-tier board even if this implies that the manager selects his own preferred project.
1. Introduction

Boards of directors have several functions, from monitoring and disciplining the management to advising, screening and also firing it. Given such a complex role, the effectiveness of board action may be affected by potential conflicts between different tasks. An organizational design problem then arises, whose solution must provide proper incentives for the performance of different tasks. Adams and Ferreira (2007) have recently shown in a context of dispersed ownership that the monitoring role of the board may contrast with its advising role. They suggest that such a conflict may be overcome by separating the two tasks in a dual board structure.\(^1\) Despite the fact that in many countries concentrated ownership and family firms are still the norm,\(^2\) organizational design in settings with concentrated ownership has received less attention.\(^3\) Contrary to what happens in public companies with dispersed ownership, in companies where ownership is concentrated there often is an 'excessive' involvement of owners in the management of the firm. For example, the high turnover of the top executives in several Italian family firms, among which Benetton and Barilla, has been attributed to the difficult coexistence of family and non-family members in the firm.\(^4\) Along the same lines of reasoning, Franz Humer, CEO of the Swiss pharmaceutical firm Roche, stated that one of the strengths of Roche is that the Hoffman family, the controlling shareholder of Roche, never became involved in the firm's management.\(^5\) We explore whether an appropriately designed dual structure may be used to provide incentives in the presence of a large shareholder actively involved in the firm and sitting on the board.

Our focus is on the choice between one-tier (or sole) and two-tier (or dual) structures in a setting where the board performs two tasks: choosing investment projects and monitoring the ability of the manager. A large shareholder, having a high stake in the firm, has a high incentive in monitoring management (Shleifer and Vishny, 1986), but its presence may interfere with managerial initiative (Burkart, Gromb and Panunzi, 1997) which is necessary to gather information on possible projects. We consider a context where large shareholder’s and manager’s preferences over investment projects differ because of private benefits accruing

---

\(^1\)Other papers have investigated the difficult coexistence of diverse roles in boards of directors, e.g. Graziano and Luporini (2003), Hermelin (2005) and Dominguez et al. (2008). For a recent survey of the literature on boards see Adams et al. (2010).

\(^2\)See for example Bertrand and Schoar (2006), Faccio and Lang (2002), Holderness (2009). Since the seminal work of La Porta et al., (1999), the prevalence of ownership concentration has been linked to a legal context providing weak protection of minority shareholders (see e.g. Burkart, Panunzi and Shleifer for a theoretical rationale of these findings in terms of corporate governance). We examine the role of delegation in such a context, taking the legal environment as given. Consequences on the welfare of minority shareholders are pointed out in the conclusion.

\(^3\)A notable exception is Burkart, Panunzi and Shleifer (2003).

\(^4\) "Barilla, via al valzer dei manager" Corriere Economia page 6, Corriere della Sera, 10 November 2008,

to the manager and possibly also to the large shareholder. We investigate if the separation of tasks provided by an appropriate two-tier board can alleviate the conflict between large shareholder and manager allowing the former to maintain her stake in the firm and thus her incentive to monitor the ability of the latter.\(^6\)

Our main finding is that a two-tier structure, by always delegating project selection to the manager, can enhance his incentive to exert effort and collect information, without affecting the large shareholder’s incentive for monitoring. This in turn leads to higher expected profits. The increase in profits can be sufficiently high to induce the large shareholder to prefer a two-tier board even if this implies that the manager will choose his own favorite project.

The rest of the note is organized as follows. Section 2 presents the basic framework. The choice of monitoring intensity by the large shareholder is analyzed in Section 3. Section 4 and 5 illustrate the choice of effort by manager and board/large shareholder in a sole and in a dual structure. Section 6 compares the two structures and presents the main results. Finally, Section 7 concludes.

### 2. The model

A firm is run by a risk neutral manager (he) who is also necessary to gather information on risky projects. Firm ownership is concentrated in the hands of a large shareholder (LS, she) who holds a fraction \(\alpha\) of shares and sits on the board. The remaining \((1 - \alpha)\) of shares are dispersed among small investors not represented on the board.

The board has a dual role: it approves the choice of investment projects and it supervises the manager, deciding whether to retain or dismiss him. In the one-tier structure, both tasks are attributed to a sole board controlled by the large shareholder. In the two-tier structure, instead, the two tasks are separated: investment selection is attributed to a management board composed mainly of firm executives, while monitoring is attributed to a supervisory board controlled by the large shareholder. We assume that the objective of the management board is aligned to that of the manager\(^7\) and that the same person cannot sit on both boards.\(^8\)

\(^6\)In line with previous models of board behavior (see for example Hermelin and Weisbach (1998), Hirshleifer and Thakor (1998)) we use the word ‘monitoring’ to indicate the activity of the large shareholder aimed at discovering the ability of the manager. In the context of Burkart, Gromb and Panunzi as well as of Adams and Ferreira, ‘monitoring’ refers to the supervision of project choice. In their models there is no role for managerial ability.

\(^7\)This may be the case when the management board can enjoy managerial private benefits without reducing the personal benefit of the manager. For example, the manager/CEO may want to expand the firm beyond the optimal size for his personal prestige and power. However, all members of the management board benefit from the increased visibility of a larger firm.

\(^8\)Such an assumption corresponds to the usual law prescriptions (see e.g. the German and Italian laws regulating the dual structure or the regulations of the European Company).
Consequently, while in the sole board an informed large shareholder is able to impose her preferred project, in the dual structure the manager can enforce his own choice. Under both structures, the large shareholder controls the institution that is in charge of monitoring, and possibly firing, the manager. Given her stake in the firm, the large shareholder has the strongest incentive to engage in monitoring and we then assume that she performs such task. The main motivation for this assumption is that other board members, for example minority shareholders representatives, tend to free ride. Thus, the active individuals in our framework are the manager and the large shareholder.

The model develops over four periods. At $t = 0$, LS chooses the board structure and hires a manager, randomly selecting him from a pool composed of two types of individuals: high ($H$) and low ($L$) ability. The fraction of $H$ managers is $\lambda$. We assume that types are unobservable by LS and third parties. The manager himself learns his type only after joining the firm.

At $t = 1$, manager and large shareholder exert unobservable efforts to become informed about profitable projects. The firm faces $N$ investment projects, but only two of them are relevant. The other $N - 2$ projects (indexed from 3 to $N$) yield highly negative returns and private benefits so that neither the manager nor the large shareholder want to pick a project at random. Projects 1 and 2 yield profit $\pi > 0$ in the case of success, and zero profit in the case of failure. Both projects are successful with probability $p^H$ if the manager is high-ability and with probability $p^L$ if the manager is low-ability, where $p^H > p^L > 0$. Project 1 yields private benefits $B$ to LS and $b_1$ to the manager. Project 2, on the contrary, yields zero private benefits to LS and $b_2$ to the manager, with $b_2 > b_1$. Private benefits are obtained in all states of nature, even in the case of zero profits from the project.

Projects cannot be distinguished from one another without acquiring costly information. By exerting effort $e$, the manager becomes informed with probability $e$, at cost $e^2/2$. Also LS can become informed on project returns by exerting effort $\varepsilon$ at cost $\varepsilon^2/2$. However, in order to use such information, LS needs the information gathered by the manager: by exerting effort $\varepsilon$, LS becomes informed with probability $\varepsilon e$.

Large shareholder and manager have partially different objectives. The large shareholder aims at maximizing her share of expected profits plus private benefits $B$ net of her possible effort and monitoring costs. The manager instead wants to maximize his expected private benefits $b$ net of his effort costs. Both the manager’s reservation utility and the salary are

---

9 Alternatively they may collude with the manager as is often the case for executive directors whose career depends on the incumbent CEO.

10 For example, the benefit may consist in hiring a friend or a relative, or in doing business with a company controlled by a friend or relative. The presence of private benefits has been largely documented in the literature. For a discussion of the possible ways in which controlling shareholders may expropriate minority shareholders see for example Shleifer and Vishny (1997) or La Porta et al. (1999).

11 For simplicity, we do not allow for incentive pay as this would not alter our qualitative results. Moreover,
normalized to zero.

Given these objectives, it follows that in a sole board project 1 is chosen if both LS and the manager are informed, while project 2 is chosen when only the manager is informed. In the dual board instead, project 2 is chosen when the manager is informed. Project choice is not verifiable by third parties.

At $t = 2$, if either project 1 or 2 is undertaken, LS engages in monitoring and becomes informed on the ability of the manager with probability $M$, at cost $M^2/2$. Monitoring is aimed at finding out a low-ability manager in order to replace him and so raise expected profits.\(^\text{12}\) A new manager cannot change project.

At $t = 3$, profits are realized as well as private benefits.

We proceed by backward induction, examining first the large shareholder’s decision on monitoring and then the choice of effort levels. To simplify the algebra we only consider values of the parameters that ensure interior solutions for effort levels and monitoring intensity. We then assume that $p^H - p^L < 1/\alpha \pi \lambda (1 - \lambda)$, $b_2 < 1$, and $B < B^{Max}$ where $B^{Max}$ is the value that makes $\varepsilon = 1$, and is defined in Lemma 1. Our results however hold even when corner solutions are allowed.

3. Monitoring

Since expected profit is the same under both project 1 and 2, we can analyze monitoring independently of the chosen project. Recall that monitoring intensity $M$ allows the large shareholder to learn the ability of the manager with probability $M$, while with probability $(1 - M)$ she stays uninformed. An $H$ manager is retained while an $L$ manager is fired. When monitoring is unsuccessful, the manager is retained because the probability of a replacement being high-ability, $\lambda$, is the same as that of the incumbent being of type $H$. Monitoring intensity results from the maximization of the large shareholder’s share of expected profit, net of the monitoring costs $M^2/2$

$$\max_{M} \alpha \pi \{\lambda p^H + (1 - \lambda)[\lambda p^HM + (1 - \lambda)p^LM + (1 - M)p^L)]\} - M^2/2.$$ 

$\pi$ is obtained with probability $p^H$ when the incumbent is good (i.e. with probability $\lambda$) independently of monitoring, and when, following monitoring (i.e. with probability $M$) a

\(^{12}\)For simplicity, we abstract from firing costs. Introducing such costs, however, would not alter our results.
bad manager is replaced with a good one. Moreover, it is obtained with probability \( p^L \) when the replacement happens to be bad and when the incumbent is bad and monitoring is unsuccessful (i.e. with probability \( 1 - M \)). From the first-order condition, we obtain

\[
M^* = \alpha \pi (1 - \lambda) \lambda (p^H - p^L).
\]  

(1)

Optimal monitoring positively depends on the large shareholder’s fraction of shares \( \alpha \), and on the expected gain from replacing a bad manager, \( \pi (1 - \lambda) \lambda (p^H - p^L) \).

4. The choice of efforts in a sole board

In the sole board structure (denoted by subscript \( S \)), both the manager and the large shareholder exert effort at time 1 in order to identify the projects with probability \( e \) and \( e^S \), respectively. Since LS implements project 1 when she is informed, \( e^S \) also represents the probability of project 1 being selected. With probability \( e(1 - \varepsilon) \) only the manager is informed and then chooses project 2. Finally, with probability \( (1 - e) \) neither the manager nor LS is informed and no project is undertaken, yielding zero profits and private benefits.

The maximization problem of the manager

By the time effort is chosen, the manager knows his own type. Then, an \( H \) manager chooses the optimal level of effort \( e^H_* \) anticipating that he will always be retained, while an \( L \) manager chooses \( e^L_* \) anticipating that, if project 1 or 2 is selected, he will be retained with probability \( (1 - M^*) \). Let us define \( K_H \equiv 1 \) and \( K_L \equiv (1 - M^*) \). A manager of ability \( i = H, L \) then solves

\[
\max_{e} e [\varepsilon^*_S b_1 K_i + (1 - \varepsilon^*_S) b_2 K_i] - e^2 / 2,
\]

which yields

\[
e^i_* = [b_2 - \varepsilon^*_S (b_2 - b_1)] K_i.
\]

(2)

Managerial effort is negatively correlated to the effort of LS, \( \varepsilon^*_S \). The reason is that a higher value of \( \varepsilon^*_S \) reduces the probability of implementing project 2, the preferred project of the manager. Since \( K_H > K_L \), it immediately follows that \( e^H_* > e^L_* \), i.e. that an \( H \) manager exerts a higher level of effort than an \( L \) one. Given that an \( H \) manager is always retained, his effort does not depend on monitoring. The effort of an \( L \) manager, instead, negatively depends on monitoring, that is on the probability of dismissal, because \( K_L \) depends on \( M^* \).

The maximization problem of the large shareholder

When making her decision on the optimal level of effort, LS does not know the type of the manager. She then expects a level of managerial effort equal to \( e^*_S \equiv \lambda e^H_* + (1 - \lambda) e^L_* \) which is associated to the probability of high profit

\[
P_S = \lambda e^H_* p^H + (1 - \lambda)e^L_* [p^L (1 - M^*) + (p^H \lambda + p^L (1 - \lambda)) M^*]
\]

(3)
where it is taken into account that an $L$ manager, exerting effort $e^*_{Sl}$, is subsequently replaced with probability $M^*$, and that the replacement can either be high ability (with probability $\lambda$) or low ability (with probability $1 - \lambda$). Then, LS solves:

$$
\max_\varepsilon e^*_{S} \left[ \varepsilon (B + \alpha \pi P_S) + (1 - \varepsilon) \alpha \pi P_S - \frac{(M^*)^2}{2} \right] - \varepsilon^2 \frac{2}{2}.
$$

where the first term in the square bracket represents the expected profit plus the private benefit accruing from project 1 (with prob. $\varepsilon$), and the second term the profit from project 2 (with prob. $1 - \varepsilon$). The third term is the cost of monitoring, incurred only if project 1 or 2 is chosen (i.e. with prob. $e^*_S$). Finally, the last term of the objective function is the cost of exerting effort $\varepsilon$. From the first-order condition we obtain

$$
\varepsilon^*_S = B e^*_S.
$$

The effort level chosen by LS positively depends on the effort of the manager $e^*_S$, which in turn negatively depends on $e^*_S$. Considering $e^*_S \equiv \lambda e^*_H + (1 - \lambda)e^*_L$, by solving the system (2) – (4), we obtain:

$$
\varepsilon^*_S = \frac{B b_2 K}{1 + B (b_2 - b_1) K}, \quad e^*_i = \frac{b_2 K_i}{1 + B (b_2 - b_1) K}, \quad i = L, H.
$$

where $K \equiv \lambda K_H + (1 - \lambda) K_L$.

The effort of the large shareholder is positively related to the level of private benefits $B$. It is also positively related to the effort of the manager because the higher $e^*_S$, the higher is the marginal benefit of an increase in $\varepsilon^*_S$ in terms of increased probability of choosing project 1. This is the reason why $\varepsilon^*_S$ positively depends on $b_2$. The effort of the manager, however, is negatively related to that of the large shareholder and consequently reaches its maximum when $B$ tends to zero. The following lemma establishes the relation between efforts and $B$.

**Lemma 1:** The large shareholder’s effort $\varepsilon^*_S$ is continuously increasing in her private benefits $B$, ranging from $\varepsilon^*_S = 0$ when $B = 0$ to $\varepsilon^*_S \rightarrow 1$ when $B \rightarrow B^{Max} \equiv \frac{1}{b_1 K}$. The manager’s effort $e^*_i$ is continuously decreasing in $B$, ranging from $e^*_i = b_2 K_i$ to $e^*_i = b_1 K_i$, $i = H, L$.

**Proof:** The result immediately follows from $\frac{\partial \varepsilon^*_S}{\partial B} = \frac{b_2 K}{[1 + B (b_2 - b_1) K]^2} > 0$ and $\frac{\partial e^*_i}{\partial B} = \frac{-b_2 K_i (b_2 - b_1) K}{[1 + B (b_2 - b_1) K]^2} < 0$. $\square$

5. The choice of effort in a dual board

In a dual board (denoted by the subscript $D$), only the manager exerts effort at time 1. This can then be interpreted as a case where project choice is fully delegated to the manager,
who always chooses project 2 when informed, i.e. with probability $e$. Project 1 is never chosen. Recall that the probabilities of retaining an $H$ or $L$ manager are independent of the structure of the board, and remain $1$ and $(1 - M^*)$ respectively.

A manager of ability $i = H, L$ chooses the optimal level of effort $e_i^*$ by solving

$$
\max_e e [b_2 K_i] - e^2 / 2.
$$

which yields

$$
e_i^* = b_2 K_i.
$$

(5)

Since $K_H > K_L$, it follows that $e_H^* > e_L^*$. Moreover, the effort of an $H$ manager is again independent of monitoring, while that of an $L$ manager negatively depends on $M^*$.

6. One-Tier versus Two-Tier board

In order to compare the sole and the dual structures, consider first of all the level of effort. Comparing (2) with (5) we obtain the following lemma.

**Lemma 2:** The level of effort exerted by the manager is higher in the dual board structure independently of his type: $e_D^* \geq e_S^*$ with $e_D^* = e_S^*$ iff $B = 0$ implying $\varepsilon^*_S|_{B=0} = 0$.

Managerial effort is higher in the dual board because, by choosing project 2 when informed, the manager can appropriate a higher level of private benefits. Given that the information of the manager is necessary, Lemma 2 implies that the probability of generating information is always higher in a dual board. Only if $B = 0$ and LS exerts no effort, the two structures become equivalent.

The probability of high profit, $P_S$ has been defined for the sole board case in (3). Let now define the analogous probability for the dual board case:

$$
P_D = e_D^{H*} \lambda p^H + e_D^{L*} (1 - \lambda) [p^L (1 - M^*) + (p^H \lambda + p^L (1 - \lambda))M^*]
$$

It immediately follows from Lemma 2 that $P_D > P_S$, implying that the expected profits are higher under the dual structure,

$$
E(\Pi_D) = \pi P_D > \pi P_S = E(\Pi_S).
$$

The large shareholder, however, is also interested in her private benefits. Consequently, her preference between the two board structures depends on her expected gains rather than on the expected profits. Under the dual structure, her expected gains correspond to her fraction of the expected profits net of monitoring costs:

$$
E(G_D) = \alpha \pi P_D - e_D^*(M^*)^2 / 2.
$$

(6)
Under the sole structure instead, the large shareholder also obtains $B$ when project $1$ is undertaken (i.e. with probability $e_s^* e_S^*$). Her expected gains then are

$$E(G_S) = e_s^* e_S^* B + \alpha \pi P_S - (e_s^*)^2 / 2 - e_s^*(M^*)^2 / 2. \quad (7)$$

Denote with $E(G_S)_{B \rightarrow B_{\text{Max}}}$ the limit value of $E(G_S)$ when $B \rightarrow B_{\text{Max}}$. In the Appendix we prove the following

Proposition: Expected profits are higher under the dual board structure. Large shareholder preferences, however, depend on the size of her private benefits and we can distinguish two cases according to the value of $E(G_D)$. If $E(G_D) \geq E(G_S)_{B \rightarrow B_{\text{Max}}}$ the large shareholder always prefers the dual board structure; if instead $E(G_D) < E(G_S)_{B \rightarrow B_{\text{Max}}}$ there exists a threshold value $\widehat{B} > 0$ such that the large shareholder prefers the dual board structure iff $B < \widehat{B}$.

The higher effort exerted by the manager in the dual structure results in higher profits. As long as $B$ is not ‘too large’, this may lead the large shareholder to prefer such a structure to the sole one despite her forgone private benefits.

7. Concluding Remarks

When ownership is concentrated in the hands of a large shareholder, a two-tier board of directors, where the large shareholder sits on the upper-level board, can be a useful device to delegate decisions on investment projects to the manager. We show that the two-tier board has the advantage of leaving initiative to the lower level board (controlled by the manager). As a result, the manager exerts greater effort in gathering information on projects and this in turn leads to higher profits. It has been pointed out in the literature (Burkart, Gromb, Panunzi, 1997) that reducing the ownership stake (the value of $\alpha$ in our model) is a powerful means to enhance managerial initiative. This, however, would reduce the large shareholder’s incentive to monitor the ability of the manager. By resorting to a dual board, managerial initiative is promoted without reducing incentives for monitoring. The ‘price’ to be paid in order to enhance managerial initiative without interfering with ownership structure and monitoring is the exclusion of the large shareholder from the management board. However, the large shareholder may be willing to pay such a price because the increase in her share of profits may more than compensate for her loss of private benefits.

We have implicitly assumed that in the dual structure, the large shareholder does not indirectly exert pressure on the manager in order to influence his choices. Although limiting large shareholder activism in the real world can be a challenging task, it is more difficult for large shareholders to overrule or to interfere with management board decisions when the
functions of the two boards are separated and clearly defined by corporate charters. Thus, we believe that a two-tier board structure may be a valuable option to commit not to reverse managerial decisions.

Our model also suggests that a dual structure may reduce the conflict of interests between majority and minority shareholders which is a relevant issue worldwide (Villalonga and Amit, 2006). Indeed, the large shareholder, by restricting her interference in firm management, also restricts her ability to expropriate minority shareholders.
References


Graziano, C. and A. Luporini (2010) “Optimal Delegation when the Large Shareholder has Multiple Tasks” CESifo working paper number 3028.


Appendix. Proof of the Proposition

It is proved in the text (see p. 6) that the expected profits are always higher under the dual board structure.

To prove the part on expected gains, note that \( \hat{B} \) must be the value of \( B > 0 \), which equates (7) to (6). We have to prove that such value exists and is unique. Rewrite the expected gains of the large shareholder as follows:

\[
E(G_S) = \varepsilon^*_S B \varepsilon^*_S + \alpha \{ e^*_S H(1-\lambda)X_H + e^*_L(1-\lambda)X_L \} - (\varepsilon^*_S)^2 / 2 
\]

where \( X_H \equiv \pi p_H - (M^*)^2 / 2\alpha \), \( X_L \equiv \pi \left[ p_L (1-\lambda) + (p_H + p^H(1-\lambda))M^* \right] - (M^*)^2 / 2\alpha \).

Given (4), (8) can in turn be written as:

\[
E(G_S) = \alpha \left[ X_H \varepsilon^*_S + X_L (1-\lambda) e^*_S \right] + \frac{(\varepsilon^*_S)^2}{2} 
\]

Recall that \( \varepsilon^*_S = \frac{b_2 K_i}{1+B(b_2-b_1)K} \), \( e^*_i = \frac{b_2 K_i}{1+B(b_2-b_1)K} \), \( i = L, H \), with derivatives:

\[
\frac{\partial \varepsilon^*_S}{\partial B} = \frac{b_2 K}{[1+B(b_2-b_1)K]^2} > 0 \\
\frac{\partial e^*_i}{\partial B} = -\frac{b_2 K_i(b_2-b_1)K}{[1+B(b_2-b_1)K]^2} < 0.
\]

We know from Lemma 1 that \( \varepsilon^*_S = 0 \) when \( B = 0 \) and that it is increasing in \( B \), but never reaches 1. When \( \varepsilon^*_S = 0 \), \( \varepsilon^*_S = \varepsilon^*_i = b_2 K_i = e^*_i \), \( i = H, L \), and \( P_S = P_D \). It then immediately follows from (8) and (9) that \( E(G_S)_{B=0} = E(G_D) \). As \( \varepsilon^*_S \) approaches 1 for \( B \to B^{max} \), instead, \( e^*_S \) asymptotically tends to \( e^*_S = b_1 K_i \). Using these findings in (10), we obtain

\[
E(G_S)_{B=0} = \alpha \left[ X_H \varepsilon^*_S + X_L (1-\lambda) \varepsilon^*_L \right] = \alpha \left[ X_H b_2 K_H + (1-\lambda) X_L b_2 K_L \right] = E(G_D),
\]

\[
E(G_S)_{B \to B^{max}} = \alpha \left[ X_H \varepsilon^*_S + (1-\lambda) X_L \varepsilon^*_L \right] + \frac{1}{2} = \alpha \left[ X_H b_1 K_H + (1-\lambda) X_L b_1 K_H \right] + \frac{1}{2}.
\]

Taking the derivative of (10) with respect to \( B \), we obtain:

\[
\frac{\partial E(G_S)}{\partial B} = \alpha \left[ X_H \lambda \frac{\partial \varepsilon^*_S}{\partial B} + X_L (1-\lambda) \frac{\partial e^*_S}{\partial B} \right] + \varepsilon^*_S \frac{\partial \varepsilon^*_S}{\partial B} = \frac{b_2 K}{[1+B(b_2-b_1)K]^2} \{ -\alpha [X_H \lambda K_H + X_L (1-\lambda) K_L] (b_2-b_1) + \varepsilon^*_S \}.
\]
We then have two possible cases:

a) if \( \alpha [X_H \lambda K_H + X_L (1 - \lambda) K_L] (b_2 - b_1) \geq 1 \), \( \frac{\partial E(G_S)}{\partial B} \) is always negative, implying that \( E(G_S) \) is continuously decreasing from \( E(G_S)_{B=0} = E(G_D) \) to \( E(G_S)_{B=B_{Max}} \).

b) if \( \alpha [X_H \lambda K_H + X_L (1 - \lambda) K_L] (b_2 - b_1) < 1 \), \( \frac{\partial E(G_S)}{\partial B} \) is negative for low values of \( B \) when \( \varepsilon_S^* < \alpha [X_H \lambda K_H + X_L (1 - \lambda) K_L] (b_2 - b_1) \), and positive for higher values of \( B \) when \( \varepsilon_S^* > \alpha [X_H \lambda K_H + X_L (1 - \lambda) K_L] (b_2 - b_1) \), implying that \( E(G_S) \) is first continuously decreasing (starting from \( E(G_S)_{B=0} = E(G_D) \)) and then increasing up to \( E(G_S)_{B=B_{Max}} \).

Consequently, \( E(G_S) \) is maximized either for \( B = 0 \) when \( E(G_S)_{B=0} = E(G_D) \geq E(G_S)_{B=B_{Max}} \), or for \( B \rightarrow B_{Max} \) otherwise. Hence \( \hat{B} \) exists only when \( E(G_D) < E(G_S)_{B=B_{Max}} \), that is when \( \alpha [X_H \lambda K_H + X_L (1 - \lambda) K_L] (b_2 - b_1) \geq \frac{1}{2} \).