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Copula based Dynamic Hedging Strategy with Futures

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Abstract

We present in this paper a dynamic hedging strategy for futures based exclusively on copula functions. We develop an algorithm based on numerical simulations from estimated copula and marginal probability function to obtain innovations. We illustrate our approach through an empirical example with Crude Oil and Gold. OLS static estimate showed itself improper and the proposed algorithm obtained very good results in spot/future variance reduction strategy.

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1. Introduction

Stock futures markets provide a channel for stock holders potentially transfer risks. Effectiveness of such a hedging strategy relies heavily on the accuracy of hedge ratio estimation. Empirical literature on optimal hedge ratio calculation is predominated by variance-minimization approaches. This strategy consists in to choose an optimal position in the futures market in order to minimize spot-futures portfolio variance.

Early research usually employs constant hedging models, such as the one-to-one ratio or the ordinary least squares (OLS) method (for example, Benet, 1992). However, constant models are criticized because they assume constant variance and covariance between spot and futures returns over time. Thus, nowadays, the general agreement is that time-varying hedge ratios based on bivariate Generalized Auto-Regressive Conditional Hesteroscedastic - GARCH - class models are superior to the constant models, as performed by Choudhry (2003), for instance.

This kind of models is estimated under the assumption of multivariate normality or based some mixture of elliptical distributions. However, this assumption is unrealistic, as evidenced by numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent (Login and Solnik, 2001; Embrechets et al., 2003). These difficulties can be treated as a problem of copulas. A copula is a function that links univariate marginals to their multivariate distribution.

Regarding to dynamic hedging strategies, recent studies linked copula joint distributions with multivariate GARCH models, generally the Dynamic Conditional Correlation (DCC). This kind of models is referred as copula based GARCH models in literature. Hsu et al. (2008) and Lai et al. (2009) apply this type of construction for dynamic hedging strategies with futures.

Nonetheless, despite flexibility, copula based models still rely on multivariate GARCH specifications, as well parameter estimation which can lead to some liberty loss. Thus, even having flexible tools one frequently faces model risk. In this sense, we present in this paper a dynamic hedging strategy for futures based exclusively on copula functions, without need for a bivariate GARCH model estimation and validation. Thus, the main contribution of this paper to empirical finance literature is the presentation of a new tool for dynamic optimal hedging with futures. Also, this is a new approach for copula methods in finance, once there is no such kind of research in this topic based purely on copula functions.

2. Copula Methods Background

For ease of notation we restrict our attention to bivariate case. The extensions to *n*-dimentional case are straightforward. A function $C : [0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \;\; C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \tag{2}$$

Property (1) means uniformity of the margins, while (5), the *n-increasing property* means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X,Y) with distribution function *C*.

In the seminal paper of Sklar (1959), it was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (3) defines a distribution function *F* with marginal F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \;\; (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function F with marginal F_1 and F_2, there exists a copula C satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every $(u, v) \in [0,1]^2$:

$$C(u, v) = F\left(F_1^{-1}(u), F_2^{-1}(v)\right). \quad (4)$$

In (4), F_1^{-1} and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2. Regarding to the estimation, the dominant methods are traditional maximum likelihood (ML), pseudo-maximum likelihood (PML), proposed by Genest et al., 1995, and inversion of dependence measures like Spearman's Rho and Kendall's Tau. Chen and Fan (2006) developed an extension of the pseudo-maximum likelihood to markovian time series.

3. Copula based Dynamic Optimal Hedge Ratio

The logic of the variance minimization hedging strategy is to invest in the amount of futures, β, that minimizes the variance of the returns of a portfolio, consisting of the spot and futures position. Let r_t^P the return on the portfolio given by $r_t^P = r_t^S - \beta r_t^F$. Applying the variance operator we have:

$VAR(r_t^P) = VAR(r_t^S - \beta r_t^F)$, from variance properties we get:

$VAR(r_t^P) = VAR(r_t^S) + \beta^2 VAR(r_t^F) - 2\beta COV(r_t^S, r_t^F)$, Minimizing this expression with respect to β, we get

$2\beta VAR(r_t^F) - 2COV(r_t^S, r_t^F) = 0$, isolating β, we get the closed expression for minimal variance hedge ratio, or optimal hedge ratio, conform (5).

$$\beta = \frac{COV(r_t^S, r_t^F)}{VAR(r_t^F)}. \quad (5)$$

In a static point of view, β is the slope OLS estimator of a linear regression. On dynamic approach, there is a β for each point of time, conform $\beta_t = COV_t(r_t^S, R_t^F)/VAR_t(r_t^F)$. In bivariate GARCH models, or even copula based GARCH models, terms in numerator and denominator comes from conditional covariance matrix. By this reason, in these approaches, the optimal hedge ration lies on model risk, even on most flexible copula based GARCH estimates.

Thus, we propose the following dynamic hedging strategy based only on copula methods. For given estimated copula C:

1. Simulate samples $u_{t,n}^S$, $u_{t,n}^F$ with length *n* for each forecast time *t* (day, week, etc.) on out-sample period;
2. Convert $u_{t,n}^S$, $u_{t,n}^F$ to $z_{t,n}^S$, $z_{t,n}^F$ samples through inversion of marginal probability function *F* as $z_{t,n}^S = F^{-1}(u_{t,n}^S)$ and $z_{t,n}^F = F^{-1}(u_{t,n}^F)$;
3. From marginal, compute spot and future returns mean and variance for each forecast time *t*: μ_t^S, μ_t^F, σ_t^S, σ_t^F (note that mean and variance can be static or dynamic conform marginal behavior);
4. Compute spot and future returns as $r_{t,n}^S = \mu_t^S + z_{t,n}^S \sigma_N^S$ and $r_{t,n}^F = \mu_t^F + z_{t,n}^F \sigma_t^F$;
5. Compute the optimal hedge ratio for forecast time *t* as $\beta_t = \frac{COV(r_{t,n}^S, r_{t,n}^F)}{VAR(r_{t,n}^F)}$;

As generally occurs with numerical salutations, results improve with larger values for *n*. For strategy effectiveness measurement, it is usually verified the variance reduction obtained with portfolio $r_t^P = r_t^S - \beta_t r_t^F$ over variance of the non-hedged position of r_t^S.

4. Empirical Illustration

We collected daily data of WTI crude oil and Gold as well their respective future front contract from June 2009 to June 2012, totalizing 741 observations. We chose this period to avoid sub-prime crisis effects in estimations. Data was obtained on Commodity Exchange (COMEX) of the New York Mercantile Exchange (NYMEX). Figure 1 exhibits collected data. Vertical line represents the cut for out-sample data. Data referent to year 2012, 112 observations, was reserved for out-sample analysis. This out-sample period is a turbulent one, where is most required an efficient hedging strategy.

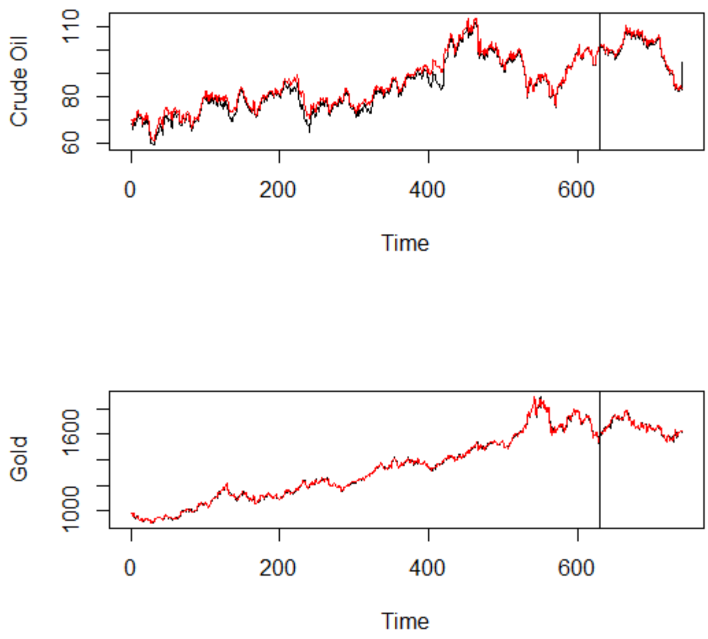

Figure 1 – Daily prices of Crude Oil and Gold from June 2010, to June 2012. Black represents spot contracts, while red is for future contracts. Vertical line represents division for out-sample data.

We calculated daily log-returns as prices logarithmic difference. We modeled returns marginal through ARMA (*m*,*n*) – GARCH (*p*,*q*) models with skewed student innovations. We chose model lag order and validated it through the verification of serial correlation on linear and squared standardized residuals through Q statistic. We standardized marginal residuals into pseudo-observations through ranks. Copulas estimation was made through ML which allows for the estimation of families with one or two parameters. For selection of the copula family we used AIC, which is directly linked with ML estimator. Candidate families were Normal, Student's t, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7 and BB8. These families are presented in Joe (1997), for example.

To determine if selected copula properly fits data, we applied a rank-based version of Cramér–von Mises statistic, discussed in Genest et al. (2009). The null hypothesis is data is fitted by $C_{\theta n}$, a copula with vector of parameters θ. With the estimated copulas, we follow the algorithm proposed in section 3 to obtain the dynamic optimal hedge ratio for Crude Oil and Gold. Results of marginal and copula estimation are presented in Table 1. Complementing, Figure 2 exhibits hedge ratio plots.

Table 1 – Results of marginal and copula models estimation for Crude Oil and Gold spot and future daily log-returns.

Commodity	**Crude Oil**				**Gold**			
Model	**Spot**		**Future**		**Spot**		**Future**	
Marginal	Est.	Sig.	Est.	Sig.	Est.	Sig.	Est.	Sig.
μ	0.0006	0.4753	0.0008	0.3017	0.0010	0.0000	0.0009	0.0357
ϕ_1	-	-	-0.8288	0.0001	0.9582	0.0000	-0.8606	0.0000
θ_1	-	-	0.8160	0.0001	-0.9999	0.0000	0.8752	0.0000
ω	0.0000	0.6366	0.0000	0.3208	0.0000	0.0812	0.0000	0.1040
α_1	0.0379	0.3728	0.0446	0.0962	0.0641	0.0009	0.0472	0.0032
β_1	0.9067	0.0000	0.9239	0.0000	0.9141	0.0000	0.9346	0.0000
Skew	0.9128	0.0000	0.8677	0.0000	0.9508	0.0000	0.8367	0.0000
Shape	7.2621	0.0002	10.4972	0.0097	9.5297	0.0160	6.2183	0.0001
$Q(10)$ Linear	4.2462	0.8343	5.3812	0.7163	7.0422	0.5321	10.1822	0.2527
$Q(10)$ Squared	7.2371	0.5113	9.3322	0.3151	9.3602	0.3128	7.6923	0.4641
Copula								
Family	Student *t*				BB1			
Parameter 1	0.9763	0.0000			0.3226	0.0001		
Parameter 2	1.7419	0.0000			1.6685	0.0000		
GoF test	0.4348	0.9911			0.0433	0.5813		
OLS ratio	1.0366				0.6288			
Var. Red. (%)	90.3212				14.3854			

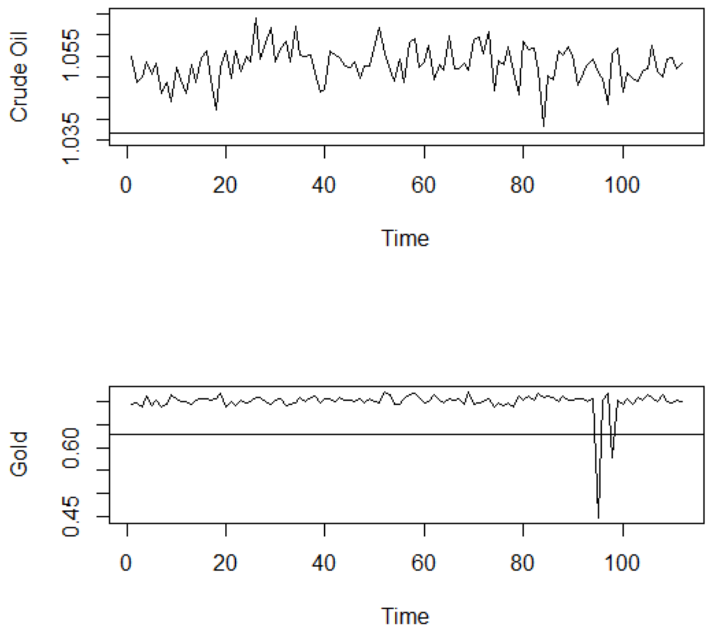

Figure 2 – Crude Oil and Gold dynamic optimal hedge ratios during out-sample period. Horizontal line represents OLS optimal hedge ratio.

Analyzing the marginal pattern is out of paper scope. However, we note marginal models proper fit data, as pointed by Q statistics and significant innovations distribution parameters. Regarding to dependence structure, Crude Oil spot and future returns have joint behavior conform Student copula, while Gold follows a BB1 family. Both Crude Oil and Gold copula parameters obtained statistical significance. Further, copulas were validated by goodness of fitness test, indicating adherence to data.

About hedging strategy, there is difference in optimal ratio magnitude between two commodities. Crude Oil obtained a bigger value, indicating one needs more resources to properly hedge a Crude Oil spot position than a Gold spot position. Figure 2 indicates OLS hedge ratio underestimate real optimal position because dynamic copula strategy, with exception for a short period of Gold out-sample series, always overcome OLS ratio. Crude Oil is more volatile than Gold as it has more oscillation in its dynamic ratio. This corroborate with distinction on variance reduction, which reached 90% for Crude Oil and 14% for Gold. This variance reduction is the goal of any hedging strategy. It indicates dynamic proposed approach can significantly reduce variability intrinsic to spot position, even on a turbulent period as our chosen out-sample. Thus, one can properly protect its spot position from variability derived from shocks during interest period.

5. Conclusion

We present in this paper a dynamic hedging strategy for futures based exclusively on copula functions. We developed an algorithm based on numerical simulations from estimated copula and marginal probability function to obtain innovations. With this innovations associated with mean and variance from marginal, we get simulations for spot and future returns. So, one directly get an optimal hedge ratio for each day on forecast period based on variance and covariance of these simulations. We illustrate our approach with an empirical example with Crude Oil and Gold. In sum, OLS static estimate showed itself improper and the proposed algorithm obtained very good results in spot/future variance reduction strategy.

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