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An advice game with reputational and career concerns

Kunio Tsuyuhara
University of Calgary

Abstract

I analyze a two-period advice game in which the decision maker chooses to retain or replace the advisor after the first period. The potential replacement creates career concerns for the advisor and thus creates incentives to misinform the decision maker. When the career concern is sufficiently strong, the advisor always lies. I characterize the condition on which the decision maker can induce truthful report by committing to a stochastic retention rule. I show that the decision maker's expected payoff is decreasing with the advisor's level of career concern.

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Contact: Kunio Tsuyuhara - kunio.tsuyuhara@ucalgary.ca.
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1. Introduction

In a multi-period cheap-talk game where the decision maker is uncertain about the state of the world as well as the type of the advisor, the advisor may have a reputational incentive to misinform even if his preferences over actions are identical to those of the decision maker.[1] Even worse, if the decision maker has an option after the first period to replace the advisor, given the report and the realized state, the advisor may also have a career incentive to lie.[2] This paper analyzes the incentives of the advisor in this scenario in a two-period cheap-talk game and analyzes how the commitment to a specific retention rule can alleviate the excessive motive to lie.

I analyze the game with two variations; one where there is no commitment by the decision maker, and the other where she can commit to a retention rule, as a function of the first-period report and the realized state. With commitment, if the advisor's career concern is not too large, the decision maker can induce truthful report even when it is not possible without commitment, and enhance welfare of the decision maker.

2. An Advice Game: A Benchmark Case

There are two types of agents: a decision maker and advisors. The decision maker faces a two-period decision problem. Time is indexed by $t = 1, 2$. Her life-time payoffs are given by

$$-(a_1 - \omega_1)^2 - (a_2 - \omega_2)^2,$$

where $a_t \in [0, 1]$ is an action she takes, and $\omega_t \in \{0, 1\}$ is a stochastic state in each period. Each state occurs with equal probability, and its probability distribution is independent over time. With this form of preferences, the statistically optimal decision for a_t equals the probability that she assigns to $\omega_t = 1$. The advisors observe a signal, $s_t \in \{0, 1\}$, about the state, which is correct with probability γ, i.e., $P(\omega_t = s_t) = \gamma \in (\frac{1}{2}, 1)$. When hiring an advisor, the decision maker knows that there are two types of advisors in the market: unbiased and biased. $\lambda_1 \in (0, 1)$ is the fraction of unbiased advisors in the market. I call the belief (prior or posterior) that the advisor is an unbiased type *reputation* and use these terms interchangeably depending on the context.

When hired, the advisor gives a report $r_t \in \{0, 1\}$. Given this report, the decision maker takes an action. After an action is taken, the true state is publicly observed, and respective payoffs are realized. After the first period, the decision maker updates her belief about the type of the advisor based on the report and the realized state.

[1] See Morris (2001). See Ely and Välimäki (2003) and Ely, Fudenberg, and Levine (2008) for more general discussion on negative implications of reputational concerns

[2] Prat (2005) is the closely related study of career concern due to replacement. See Dewatripont, Jewitt, and Tirole (1999), Gibbons and Murphy (1992), and Holmstrom (1999) for other models of career concerns.

Depending on these inferences, the decision maker either replaces or retains the first-period advisor for the second period. An unbiased advisor's life-time utility is given by

$$1-(a_1-\omega_1)^2+\mathbb{I}_{(r_1,\omega_1)}\cdot\delta\left[1-(a_2-\omega_2)^2\right],$$

where $\mathbb{I}_{(r_1,\omega_1)}$ is the indicator function, which takes 1 if he is retained in the second period and 0 if otherwise. The preferences are perfectly aligned with the decision maker in each period, but the second period payoffs are conditional on being retained. The coefficient $\delta\in\mathbb{R}_+$ is a level of career concern and measures how much the advisor cares about the second period payoff relative to that of the first period. I assume that the biased advisor is a behavioural type; i.e., he has a myopic preferences $-(a_t-1)^2$ whenever hired and does not consider the influence of his report on the future outcome.

I will analyze the game by backward induction and characterize the sequential equilibrium. Suppose the decision maker enters the second period with an advisor whose reputation is λ_2. Let $\Gamma(r,\lambda_2)$ denote the decision maker's updated belief that the state is 1, given the report r and the reputation λ_2. Let $\Pr(r,\omega)$ be the probability that the decision maker receives report r when the underlying state is ω. Then, the decision maker's expected payoff of the second period is:

$$V(\lambda_2)=-\sum_{\omega}\sum_{r}\Pr(r,\omega)\left(\Gamma(r,\lambda_2)-\omega\right)^2.$$

On the other hand, let $\Pr(\omega|s)$ be the posterior probability that the state is ω when the advisor observes the signal s. The unbiased advisor truthfully reports the signal $(r=s)$ in the second period. Then, the expected payoff of the unbiased advisor is

$$v(\lambda_2)=1-\frac{1}{2}\sum_{\omega}\sum_{s}\Pr(\omega|s)(\Gamma(s,\lambda_2)-\omega)^2.$$

I do not provide the specific parametric expressions of these value functions in this note, but it can be shown that both functions are increasing in λ_2. With higher reputation, the decision maker trusts the advisor's report more and chooses a statistically better action based on the advisor's signal. Therefore, the decision maker is better off entering the second period with the advisor with higher reputation. This argument implies the following lemma.

Lemma 1. *In any equilibrium, the decision maker replaces the advisor in the first period if and only if his posterior reputation is lower than the market average,* λ_1.

With this lemma, reporting 0 is the strictly dominating action if the advisor receives a signal 0 in the first period. However, if he receives a signal 1, he faces the following tradeoff: reporting 1 induces the statistically optimal decision but negatively affect his reputation, while reporting 0 secures the second period payoff but adversely affect the current period decision. Given this tradeoff, it is without loss to restrict attention to strategies where the unbiased advisor reports 1 (truthful report) with probability η when he receives the signal 1 and reports 0 (lie) with probability $1-\eta$.

The decision maker calculates the posterior reputation based on the advisor's report and the realized state in the first period. Since the biased advisors always report 1 regardless of signal, the decision maker knows the advisor is unbiased type if she receives the report 0; however, if she receives the report 1, Bayes' rule implies that the advisor's posterior reputation is strictly below λ_1 regardless of the realized state. Combining this observation and Lemma 1 implies the following proposition.

Proposition 1. *In equilibrium, the decision maker replaces the advisor in the first-period if and only if she receives the report 1, regardless of the realized state.*

We are interested in the condition under which the advisor reports truthfully despite the fact that reporting 1 results in the termination of the relationship. The following proposition characterizes the conditions for such equilibria.

Proposition 2. *There exists a strictly positive level of career concern, $\bar{\delta} > 0$, such that*

1. *for any $\delta > \bar{\delta}$, $\eta = 0$ is the unique equilibrium, and*
2. *for any $\delta \in [0, \bar{\delta}]$, there are three equilibria where $\eta = 0$, $\eta = 1$, and $\eta^m \in (0,1)$.*

The proof is standard and omitted in this note. I call the equilibrium with $\eta = 1$ *truthful equilibrium* and the equilibrium with $\eta = 0$ *politically correct equilibrium*, following Morris (2001). Note that, for any $\delta \geq 0$, there always exists a politically correct equilibrium because when the decision maker's belief about the advisor's truthful reports is sufficiently small, reporting 0 is indeed the pure-strategy best response by the advisor to that inference. Since reporting 0 is always dominant action when the signal is 0, this equilibrium satisfies the intuitive criterion. When the truthful equilibrium is realized, however, it is informationally efficient, in the sense that the decision maker can take a statistically optimal action. Therefore, the decision maker is willing to play this equilibrium whenever possible.

3. The Game with Partial Commitment

This section considers the possibility of retention/replacement mechanism that can recover the truthful equilibrium when $\delta > \bar{\delta}$.

Definition 1. *A* retention rule *specifies the probability that the advisor is retained after the first period as a function of the report and the realized state.*

Assume that the decision maker can commit to such a rule.

Definition 2. The optimal retention rule *maximizes the decision maker's expected payoff, and recovers the truthful equilibrium whenever profitable.*

In principle, the decision maker chooses four retention probabilities, each of which corresponds to a combination of reports and realized states. However, since receiving report 0 provides sufficient information for identifying the advisor's type, conditioning the retention based on the realized state after receiving report 0 is a punishment for misreporting by an unbiased advisor. It has only an indirect effect for inducing truthful report while it incurs loss of information about the advisor's type. On the other hand, conditioning the retention based on the realized state after receiving report 1 is a reward and provides direct incentive with an unbiased advisor to truthfully report the signal. When the decision maker has these two instruments, it is not optimal (minimizing cost) to *simultaneously use the punishment and reward.* Therefore, in search of the optimal retention rule, I will focus on the following simple rule.[3] If the decision maker receives a report 0, the advisor is retained with probability one regardless of the realized state. On the other hand, if she receives a report 1, the advisor is retained with probability ϕ_0 if the realized state is 0 and ϕ_1 if the realized state is 1. Let $\boldsymbol{\phi} = \{(\phi_0, \phi_1) \in [0, 1]^2\}$.

Proposition 3. *The optimal retention rule must satisfy $\phi_0 = 0$ unless $\phi_1 = 1$.*

This result is intuitive; When there are two instruments for inducing the truthful report, the decision maker first rewards the advisor if the report coincides with the realized state. She rewards the advisor even when the report turns out to be incorrect only if $\phi_1 = 1$ and it is still profitable to induce the truthful report by increasing ϕ_0.

The following proposition establishes the condition under which there exists the optimal retention rule that induces the truthful report.

Proposition 4. *For any (λ_1, γ), there exists a threshold $\delta_{\lambda_1,\gamma} > \bar{\delta}$ such that, for any $\delta \in (\bar{\delta}, \delta_{\lambda,\gamma}]$, the optimal retention rule induces the truthful report.*

Sketch of the Proof. The supplementary appendix provides the complete proof. Here, I outline the key arguments. First, define the following functions. Let $E(r, \eta)$ denotes the advisor's expected life-time payoff from reporting r given the strategy η and signal received is 1, under no commitment case. Let $\Upsilon(\boldsymbol{\phi})$ denote the expected extra payoff from reporting truthfully when signal received is 1, under the retention rule $\boldsymbol{\phi}$. Finally, let $\Pi(\boldsymbol{\phi}, \eta)$ denote the decision maker's life-time expected payoff given the retention rule $\boldsymbol{\phi}$ and the advisor's strategy η.

[3] If rewarding the truthful report does not provide enough incentive for reporting truthfully, the decision maker, in addition, need to use a punishment to induce truthful report of the unbiased advisor. However, in such a case, the decision maker cannot screen out biased advisors for the second period. Moreover, the case where the decision maker still finds it profitable is sufficiently small, if it exists at all. I thank the referee for raising an issue of general retention rule.

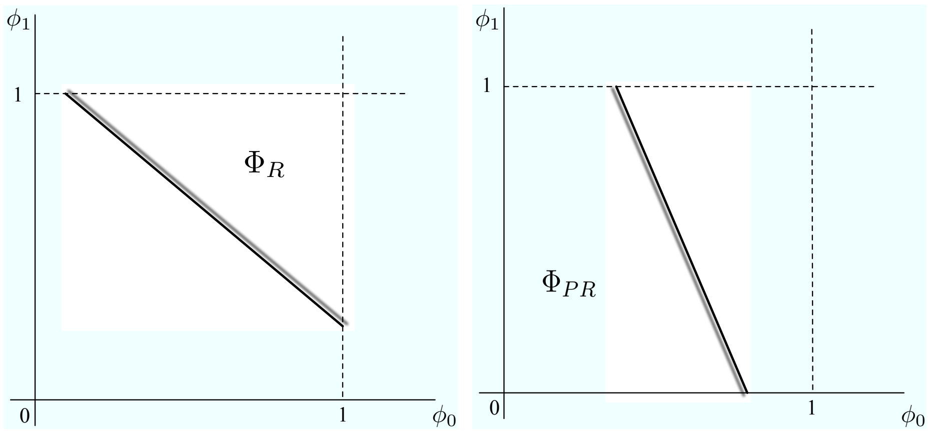

Figure 1: The set of retention rules $\boldsymbol{\phi}$ that satisfy the conditions (1) and (2)

When $\delta > \bar{\delta}$, $E(0,1) > E(1,1)$ and the advisor will misreport in the benchmark model; however, the advisor would have an incentive to truthfully report given the retention rule $\boldsymbol{\phi}$ if $E(0,1) \leq E(1,1) + \Upsilon(\boldsymbol{\phi})$. At the same time, the decision maker is better off by inducing the truthful report given the retention rule $\boldsymbol{\phi}$ if $\Pi(\mathbf{0},0) \leq \Pi(\boldsymbol{\phi},1)$. Define the following sets of retention rules:

$$\Phi_R \equiv \{\boldsymbol{\phi} : E(0,1) \leq E(1,1) + \Upsilon(\boldsymbol{\phi})\} \tag{1}$$

and

$$\Phi_{PR} \equiv \{\boldsymbol{\phi} : \Pi(\mathbf{0},0) \leq \Pi(\boldsymbol{\phi},1)\}. \tag{2}$$

Figure 1 shows a particular case of the respective set. The property of Φ_{PR} depends on the parameters λ_1 and γ while that of Φ_R depends also on δ. The existence of the optimal retention rule that recovers the truthful equilibrium and makes the decision maker better off is equivalent to the existence of nonempty intersection $\Omega \equiv \Phi_R \cap \Phi_{PR}$.

Lemma 2. *The boundary of Φ_{PR} is steeper than that of Φ_R.*

This lemma suggests that the candidates of retention rules (the intersection of those sets) should appear on the upper left corner of the intersection of two lines if it exists (Figure 2). I can also show that the cost minimizing retention rule minimizes ϕ_0. This implies that ϕ_0 is positive only if ϕ_1 is bounded at 1, which implies Proposition 3.

Now, fixing the parameters λ_1 and γ, i.e., fixing the boundary of Φ_{PR}, increasing δ shifts the boundary of Φ_R outward, shrinking the size of the intersection if exists. Therefore, for any (λ_1, γ), I can construct the threshold δ that guarantees the existence of the intersection and denote it by $\delta_{\lambda_1,\gamma}$. □

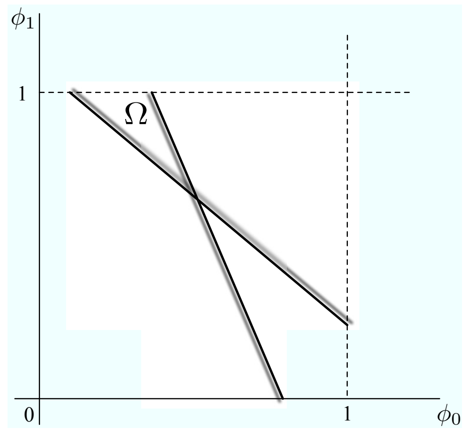

Figure 2: The intersection Ω

Corollary 1. *If the degree of career concern is sufficiently high, i.e.,* $\delta > \delta_{\lambda,\gamma}$*,* $\boldsymbol{\phi} = (0,0)$ *is the optimal retention rule, and the decision maker replaces the advisor in the first-period if and only if she receives the report 1.*

Even if a retention rule can induce the truthful report, it is straightforward to show that the necessary retention probability increases as the advisor's career concern increases. Since the decision maker's payoff decreases with the necessary retention probability, her expected payoff also decreases with the advisor's career concern. The following proposition summarizes the effect of advisor's career concern on the decision maker's welfare.

Proposition 5. *The decision maker's expected payoff monotonically decreases with the advisor's level of career concern.*

4. Conclusion

This paper examines a variation of Morris (2001) where the decision maker has an option to replace the advisor after the first period. Allowing commitment to a retention rule at the outset, the decision maker can induce truthful equilibrium even when it is not possible without commitment and enhance welfare of the decision maker. Two areas for further research are to investigate what kind of institution would lead to optimal retention rule and how the optimal retention rule evolves as the relationship continues for more than two periods.

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