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Is there a nonlinear long-run relation in the U.S. interest rate and inflation?

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Abstract

Recent advances in nonlinear cointegration analysis find evidence for a nonlinear long-run relation between the U.S. interest rate and inflation. Employing the Breitung's (2001) rank tests for nonlinear cointegration, we find herein little evidence for cointegration in the U.S. data. We also provide simulation results regarding the performance of the rank tests for some plausible nonlinear models for the data.

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1 Introduction

The cointegration method has been broadly applied to many economic variables since its initial introduction by Engle and Granger about 25 years ago. Only linear cointegrating relations were considered in the seminar paper. Recently, a great deal of progress has also been made in the study of nonlinear relations. For example, Saikkonen and Choi (2004), Karlsen *et al.* (2007), and Wang and Phillips (2009) have developed an asymptotic theory for nonlinear cointegrated systems.

Whereas the famous Fisher equation specifies a linear relation between the interest rate and inflation, little evidence has been found in the U.S. data to suggest that they are cointegrated in a linear fashion. Recent studies have detected, however, a nonlinear cointegrating relation. See Christopoulos and León-Ledesma (2007) for detailed empirical results and additional references. Therefore, one may surmise that the interest rate and inflation are indeed nonlinearly cointegrated, if not linearly.

We provide herein a different finding regarding their long-run relation. Using rank tests for cointegration, we detect no evidence for a nonlinear cointegrating relationship. The tests were those proposed by Breitung (2001), and do not require that specific functional forms be specified *a priori*. This is a very attractive feature of the tests because in many applications the exact nonlinear relationships are unknown. We also conduct Monte Carlo simulations to evaluate the performance of the rank tests for some plausible nonlinear models for the U.S. data.

In the following section, we briefly review the Breitung's (2001) rank tests.

2 Rank tests for nonlinear cointegration

Assume that variables x_t and y_t exhibit the following relation:

$$u_t = g(y_t) - f(x_t) \tag{1}$$

for $t = 1, \ldots, T$, where $g(y_t) \sim I(1)$, $f(x_t) \sim I(1)$, and $u_t \sim I(0)$. T denotes the sample size. Given that $u_t \sim I(0)$, x_t and y_t are nonlinearly cointegrated.

Define the ranked series as $R_T(y_t) = \text{Rank of } [y_t \text{ among } y_1, \ldots, y_T]$ and construct $R_T(x_t)$ accordingly. The rank test is constructed by replacing $f(x_t)$ and $g(y_t)$ with the ranked series. Because a sequence of ranks is invariant to a monotonic transformation of the data, we have

$$R_T[g(y_t)] = R_T(y_t)$$

and

$$R_T[f(x_t)] = R_T(x_t).$$

Define the rank difference as

$$d_t = R_T(y_t) - R_T(x_t).$$

Consider the following distance measures:

$$\kappa_T = T^{-1} \sup_t |d_t| \tag{2}$$

and

$$\xi_T = T^{-3} \sum_{t=1}^{T} d_t^2. \tag{3}$$

When $f(x_t)$ and $g(y_t)$ move together, d_t should be small. Therefore, the null hypothesis of no (nonlinear) cointegration is rejected if the test statistics are too small.

When $f(x_t)$ and $g(y_t)$ are correlated, the test statistics are corrected with the estimated correlation coefficient of rank differences. For instance,

$$\kappa_T^* = \frac{\kappa_T}{\hat{\sigma}_{\Delta d}} \qquad \text{and} \qquad \xi_T^* = \frac{\xi_T}{\hat{\sigma}_{\Delta d}^2} \tag{4}$$

where

$$\hat{\sigma}_{\Delta d}^2 = T^{-2} \sum_{t=2}^{T} \left(d_t - d_{t-1}\right)^2.$$

Additionally, Breitung (2001) defines

$$\kappa_T^{**} = \frac{\kappa_T^*}{\lambda_\kappa^\alpha \left(E\rho_T^R\right)} \qquad \text{and} \qquad \xi_T^{**} = \frac{\xi_T^*}{\lambda_\xi^\alpha \left(E\rho_T^R\right)} \tag{5}$$

and suggests that $\lambda_\kappa^\alpha \left(E\rho_T^R\right)$ be approximated with $\lambda_\kappa^{0.05} \simeq 1 - 0.174 \left(\rho_T^R\right)^2$ and $\lambda_\xi^\alpha \left(E\rho_T^R\right)$ with $\lambda_\xi^{0.05} \simeq 1 - 0.462\rho_T^R$, where ρ_T^R is the correlation coefficient of the rank differences:

$$\rho_T^R = \frac{\sum_{t=2}^{T} \Delta R_T\left(x_t\right) \Delta R_T\left(y_t\right)}{\sqrt{\left(\sum_{t=2}^{T} \Delta R_T(x_t)^2\right) \left(\sum_{t=2}^{T} \Delta R_T(y_t)^2\right)}}.$$

It is also possible to test for the existence of cointegration among $k+1$ variables, $y_t, x_{1t}, \ldots, x_{kt}$. Let $R_T(\mathbf{x_t}) = [R_T(x_{1t}), \ldots, R_T(x_{kt})]'$ be a $k \times 1$ vector and $\tilde{\mathbf{b}}_T$ be the least squares estimate from a regression of $R_T(y_t)$ on $R_T(\mathbf{x_t})$. Using the residuals

$$\tilde{u}_t^R = R_T(y_t) - \mathbf{b_T'} R_T(\mathbf{x_t}) \tag{6}$$

a multivariate rank test statistic is obtained from the normalized sum of squares:

$$\Xi_T[k] = T^{-3} \sum_{t=1}^{T} \left(\tilde{u}_t^R\right)^2.$$

To account for a possible correlation between the series, a modified test statistic should be applied:

$$\Xi_T^*[k] = \frac{\Xi_T[k]}{\hat{\sigma}_{\Delta u}^2} \tag{7}$$

where $\hat{\sigma}_{\Delta u}^2 = T^{-2} \sum_{t=2}^{T} \left(\tilde{u}_t^R - \tilde{u}_{t-1}^R\right)^2$.

3 Main empirical results

3.1 Data

We employ, in this study, the U.S. interest rate, i_t, and inflation, π_t, data. For comparison to the previous findings in Christopoulos and León-Ledesma (2007), we elect to use the same data series as theirs. The sample period is 1960:Q1∼2004:Q4, with a total of 180 observations. We also find very close results with the monthly data series.

There is little evidence for linear cointegration between i_t and π_t with the Johansen's method. Christopoulos and León-Ledesma (2007) demonstrate, however, that there are nonlinearities between i_t and π_t, applying the testing procedure developed by Saikkonen and Choi (2004).

3.2 The rank tests for cointegration

Our main results are reported in Table I. Only the κ_T and ξ_T tests find some evidence for cointegration. However, these tests are based on the assumption that $f(i_t)$ and $g(\pi_t)$ are independent. Allowing for correlation, no evidence for cointegration is detected from other tests. In fact, none of the test statistics is significant even at the 10% significance level. In summary, the rank tests find no evidence for cointegration, linear or nonlinear. This is quite a different finding from that of Christopoulos and León-Ledesma (2007).

Table I: Rank tests for cointegration between U.S. interest rate and inflation

data frequency	T	κ_T	ξ_T	κ_T^*	ξ_T^*	κ_T^{**}	ξ_T^{**}	$\Xi_T^*[1]$
quarter	180	0.6111*	0.0393**	0.5145	0.0278	0.5241	0.0327	0.0288
month	358	0.6676	0.0508*	0.5349	0.0326	0.5355	0.0339	0.0347

* [**] indicates that the test statistic is significant at the 10% [5%] significance level.

3.3 Simulation results on the power of the rank tests

To understand the differences in the test results between the methods of Saikkonen and Choi (2004) and Breitung (2001) applied to the same data series, we decided to run some simulations. Specifically, we employ the estimation results in Christopoulos and León-Ledesma (2007) for logistic smooth transition regression [$LSTR$] and exponential smooth transition regression [$ESTR$] models as the nonlinear data generating process [DGP], and assess the performance of the Breitung (2001) tests applied to the models.

The following relation is considered:

$$i_t = \alpha + \beta_1 \pi_t + \beta_2 F(\pi_t; \gamma; c) + \varepsilon_t, \tag{8}$$

where $\varepsilon_t \sim N(0,\, 1)$. The most popular choices for F are the logistic and exponential smoothing transition functions:

$$F(\pi_t; \gamma; c) = \frac{1}{1 + e^{-\gamma(\pi_t - c)}} \tag{9}$$

and

$$F(\pi_t; \gamma; c) = 1 - e^{-\gamma(\pi_t - c)^2} \tag{10}$$

where $\gamma > 0$ and c is the threshold. The nonlinear model (8) with the transition function (9) or (10) will be called $LSTR$ or $ESTR$, respectively.

We conduct three different sets of simulations. In first simulations, either the *LSTR* or *ESTR* model is assumed to be the DGP for the whole 180 observations. In the second set of simulations, we assume that for the initial sample period ending at 1978:Q4, the *ESTR* model is the DGP, whereas for the remaining sample period starting at 1979:Q1, the *LSTR* model is the DGP. Therefore, the DGP is a mixture of the two *STR* models. The two sub-samples contain 76 and 104 observations, respectively. The division of the sample period into two sub-samples is intended to reflect the changes in the monetary policies during the Volcker era. According to Christopoulos and León-Ledesma (2007), the mixture model is the most plausible one for the U.S. data. In the final set of simulations, we also employ a mixture model with the *LSTR* model for the first sub-sample and the *ESTR* model for the second sub-sample.

The parameter values employed in simulating the nonlinear *STR* models are listed in Table II. They are the two-step Gauss-Newton estimates from Saikkonen and Choi (2004). We also assume that π_t is a random walk:

$$\pi_t = \pi_{t-1} + \eta_t,$$

where $\eta_t \sim N(0,\ 1)$. The total number of simulations is $10,000$.

Table II: Parameter values employed in simulations

model	T	α	β_1	β_2	γ	c
LSTR	180	1.430	1.182	-0.402	2.855	5.567
ESTR	180	2.113	1.202	-0.461	1.564	4.062
ESTR	76	2.735	0.582	-0.147	0.037	5.857
LSTR	104	2.701	0.350	0.762	5.070	3.121
LSTR	76	2.081	0.768	-0.376	0.434	7.310
ESTR	104	1.765	1.482	-0.574	1.331	4.168

First, we discuss the simulation results for the whole sample period with either the *LSTR* or *ESTR* model, as reported in the first two rows of Table III. All of the rank tests strongly reject the null hypothesis of no cointegration. Second, similar results are also found from the mixture of *STR* models, as reported in the last two rows of Table III. Even though the power of the tests is lowered slightly, the rank tests still possess high power for the mixture *STR* models. Finally, we conduct additional simulations with more observations.

As expected, the power of the tests increases, indicating the consistency of the tests. Because the simulation results are rather intuitive, we do not report them here to save space.

Table III: Power of the rank tests against nonlinear models

model	T	κ_T	ξ_T	κ_T^*	ξ_T^*	κ_T^{**}	ξ_T^{**}	$\Xi_T^*[1]$
LSTR	180	0.925	0.975	1.000	1.000	1.000	1.000	1.000
ESTR	180	0.947	0.987	1.000	1.000	1.000	1.000	1.000
ESTR+LSTR	76+104	0.821	0.839	0.926	0.940	0.907	0.894	0.949
LSTR+ESTR	76+104	0.776	0.871	0.971	0.994	0.964	0.986	0.995

The rejection rates of the null hypothesis of no cointegration are reported.

Overall, the Breitung's (2001) rank tests have good power against the *LSTR* and *ESTR* models for the U.S. data. The test results in Table I cannot be attributed to the possibly low power of the rank tests against the nonlinear models. Our test results are very different from the current consensus that the U.S. interest rate and inflation are nonlinearly cointegrated, if not linearly. We find herein no evidence for nonlinear cointegration.

4 Concluding remarks

Recent advances in nonlinear cointegration analysis find evidence for a nonlinear long-run relation between the U.S. interest rate and inflation. In this study, we reach a conclusion that conflicts with the consensus view with the Breitung's (2001) rank tests. We also demonstrate, via simulations, that the tests possess excellent power for the plausible nonlinear models for the U.S. data. The evidence for nonlinearity in the long-run relation between the U.S. interest rate and inflation is not overwhelming and more study in this area will be required.

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Figure 1: U.S. interest rate and inflation, 1960:Q1∼2004:Q4