Economics Bulletin

Volume 33, Issue 1

An impossibility theorem for secure implementation in discrete public good economies

Katsuhiko Nishizaki
Graduate School of Economics, Osaka University

Abstract

This paper studies the possibility of secure implementation (Saijo, T., T. Sjostrom, and T. Yamato (2007) "Secure implementation," Theoretical Economics 2, pp.203-229) in discrete public good economies with quasi-linear preferences. We find that only constant social choice functions are securely implementable over the domains that satisfy partial dominance introduced in this paper. Partial dominance is a reasonable condition because the set of all strictly increasing and strictly concave valuation functions satisfies this condition.

The author wishes to thank the editor, John P. Conley, the associate editor, Nizar Allouch, and an anonymous referee for their thoughtful comments and efforts. This paper is based on the author's M.A. thesis submitted to the Graduate School of Economics, Osaka University. The author wishes to thank Tatsuyoshi Saijo, Masaki Aoyagi, Shigehiro Serizawa, Yuji Fujinaka, Takuma Wakayama, Shinji Ohseto, Kazuhiko Hashimoto, Shuhei Morimoto, and seminar participants at the 2008 Japanese Economic Association Spring Meeting, Tohoku University, for their helpful comments on the thesis. The author is especially grateful to Tatsuyoshi Saijo, Yuji Fujinaka, and Takuma Wakayama for their valuable advices. Any errors in this paper are entirely the responsibility of the author.
Citation: Katsuhiko Nishizaki, (2013) "An impossibility theorem for secure implementation in discrete public good economies", *Economics Bulletin*, Vol. 33 No. 1 pp. 300-308.
Contact: Katsuhiko Nishizaki - gge008nk@mail2.econ.osaka-u.ac.jp.
Submitted: November 09, 2012. **Published:** February 07, 2013.

1 Introduction

In social choice theory, **strategy-proofness** is a standard for non-manipulability: it requires that truthful revelation is a weakly dominant strategy for each agent. Although strategy-proofness is desirable, it allows the existence of Nash equilibria that induce non-optimal outcomes. This shortcoming might cause some problems for the performance of strategy-proof mechanisms. [1] Saijo, Sjöström, and Yamato (2007) introduced **secure implementation** to solve the problems. [2] This notion is identical with double implementation in dominant strategy equilibria and Nash equilibria. This paper studies securely implementable social choice functions.

Previous studies illustrated how difficult it is to find securely implementable social choice functions with desirable properties: voting environments (Saijo, Sjöström, and Yamato, 2007; Berga and Moreno, 2009), public good economies (Saijo, Sjöström, and Yamato, 2007; Nishizaki, 2011), pure exchange economies (Mizukami and Wakayama, 2005; Nishizaki, 2012b), the problems of providing a divisible and private good with monetary transfers (Saijo, Sjöström, and Yamato, 2007; Kumar, 2011), the problems of allocating indivisible and private goods with monetary transfers (Fujinaka and Wakayama, 2008), queueing problems (Nishizaki, 2012a), Shapley-Scarf housing markets (Fujinaka and Wakayama, 2011), and allotment economies (Bochet and Sakai, 2010). This paper considers discrete public good economies. [3] Examples of the economies are the provision of public facilities (e.g. schools, hospitals, welfare facilities for children and the elderly) and public services (e.g. train, bus, plane services). The provision of information goods (e.g. software, audio-visual contents) and intellectual properties (e.g. patented technologies, copyrighted pieces) are also included in the examples. Note that this paper also considers the provision of an excludable public good with cost shares. In the economies similar to those of this paper, Nishizaki (2011) showed an example of domains over which only constant social choice functions are securely implementable. This paper introduces a domain-richness condition called **partial dominance** including Nishizaki (2011)'s domains and shows a constancy result on secure implementation over partially dominant domains. In non-excludable public good economies, Saijo, Sjöström, and Yamato (2007) showed that secure implementation is more difficult in discrete public economies than divisible ones. This paper strengthens their result by characterizing securely implementable social choice functions in excludable public good economies.

Partial dominance is closely related to **minimal richness** (Fujinaka and Wakayama, 2008) and **weak indifference** (Nishizaki, 2012a). For secure implementation, Fujinaka and Wakayama (2008) showed a constancy result over minimally rich domains in the problem of allocating in-

[1] See Chen (2008) for experimental studies on strategy-proofness in public good economies.

[2] See Cason, Saijo, Sjöström, and Yamato (2006) for experimental studies on secure implementation in public good economies.

[3] See Deb and Razzolini (1999), Ohseto (2000, 2005), Deb, Razzolini, and Seo (2003), and Yu (2007) for strategy-proofness in discrete public good economies.

divisible private goods with monetary transfers and Nishizaki (2012a) also showed over weakly indifferent domains in queueing problems. In the model of this paper, partial dominance is weaker than minimal richness and stronger than weak indifference. Note that weak indifference does not imply a constancy result on secure implementation in the model. Partial dominance is also related to **dual dominance** (Saijo, 1987) which is a condition of social choice functions, not a domain-richness condition. Note that our result is not established by his result straightforwardly because partial dominance is weaker than dual dominance in term of certain dominance relationship.

The reminder of this paper is organized as follows. Section 2 introduces notation and definitions and Section 3 presents the result. Section 4 concludes this paper.

2 Notation and Definitions

Let $N \equiv \{1,\dots,n\}$ ($n \geq 2$) be the set of **agents**. Let $Y \subseteq \mathbb{Z}_+$ be the set of **production levels of the public good** and $c\colon Y \to \mathbb{R}_+$ be the **cost function**. For each $i \in N$, let $(y_i, x_i) \in Y \times \mathbb{R}_+$ be **agent *i*'s consumption bundle**, where y_i is **agent *i*'s consumption of the public good** and x_i is **agent *i*'s cost share**. The **non-excludability of the public good** requires that $y_i = y_j$ for each $i, j \in I$. The **excludability of the public good** allows that $y_i \neq y_j$ for some $i, j \in I$. Note that our model includes the both cases. A profile of consumption of the public good is $\mathbf{y} \equiv (y_i)_{i\in N} \in Y^n$ and a profile of cost shares is $\mathbf{x} \equiv (x_i)_{i\in N} \in \mathbb{R}^n_+$. Let $(\mathbf{y},\mathbf{x}) \in Y^n \times \mathbb{R}^n_+$ be an **allocation** and $Z \equiv \{(\mathbf{y},\mathbf{x}) \in Y^n \times \mathbb{R}^n_+ \mid c\,(\max_{i\in N} y_i) \leq \sum_{i\in N} x_i\}$ be the set of **feasible allocations**.

For each $i \in N$, **agent *i*'s preferences** defined over $Y \times \mathbb{R}_+$ are quasi-linear: for each $(y_i, x_i) \in Y \times \mathbb{R}_+$, $u_i(y_i, x_i) = v_i(y_i) - x_i$, where $v_i\colon Y \to \mathbb{R}_+$ is **agent *i*'s valuation function** that is strictly increasing. For each $i \in N$, let V_i be the set of agent i's valuation functions. A profile of valuation functions is $v \equiv (v_i)_{i\in N} \in V \equiv \prod_{i\in N} V_i$ and a profile of valuation functions other than agent $i \in N$ is $v_{-i} \equiv (v_j)_{j\in N\setminus\{i\}} \in V_{-i} \equiv \prod_{j\in N\setminus\{i\}} V_j$. The set V is called the **domain**.

A **social choice function** $f\colon V \to Z$ associates an allocation $(\mathbf{y},\mathbf{x}) \in Z$ with a profile of valuation functions $v \in V$. For each $v \in V$, let $(\mathbf{y}(v),\mathbf{x}(v)) \in Z$ be the allocation associated with the social choice function f at the profile of valuation functions of the public good v and $(y_i(v), x_i(v))$ be the consumption bundle for agent $i \in N$ at the allocation $(\mathbf{y}(v),\mathbf{x}(v))$.

Saijo, Sjöström, and Yamato (2007) characterized securely implementable social choice functions by the following two conditions. Strategy-proofness requires that truthful revelation is a weakly dominant strategy for each agent. The rectangular property requires that if each agent cannot change the utility by the revelation, then the outcome does not change by all the agents' revelations.

Definition 1. The social choice function f satisfies **strategy-proofness** if and only if for each $v, v' \in V$ and each $i \in N$, $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) \geq v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$.

Definition 2 (Saijo, Sjöström, and Yamato, 2007). The social choice function f satisfies the **rectangular property** if and only if for each $v, v' \in V$, if $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$ for each $i \in N$, then $(\mathbf{y}(v), \mathbf{x}(v)) = (\mathbf{y}(v'), \mathbf{x}(v'))$.

This paper considers securely implementable social choice functions over the domains that satisfy the following domain-richness condition, called partial dominance. The set of all strictly increasing and strictly concave valuation functions is an example of partially dominant domains. [4]

Definition 3. The domain V satisfies **partial dominance** if and only if for each $i \in N$, each $v'_i, v''_i \in V_i$, each $y'_i, y''_i \in Y$ with $y'_i \leq y''_i$, and each $X \in \mathbb{R}$, if $v'_i(y''_i) - v'_i(y'_i) < X < v''_i(y''_i) - v''_i(y'_i)$, then there exists $v_i \in V_i$ such that

(i) $v_i(y''_i) - v_i(y'_i) = X$;

(ii) $v_i(y_i) - v_i(y'_i) \leq v'_i(y_i) - v'_i(y'_i)$ for each $y_i \leq y'_i$; and

(iii) $v_i(y_i) - v_i(y''_i) \leq v''_i(y_i) - v''_i(y''_i)$ for each $y_i \geq y''_i$.

Partial dominance is weaker than minimal richness (Fujinaka and Wakayama, 2008). The domain V satisfies **minimal richness** if and only if for each $i \in N$, each $v'_i, v''_i \in V_i$, each $y'_i, y''_i \in Y$, and each $X \in \mathbb{R}$, if $v'_i(y''_i) - v'_i(y'_i) < X < v''_i(y''_i) - v''_i(y'_i)$, then there exists $v_i \in V_i$ such that (i) $v_i(y''_i) - v_i(y'_i) = X$ and (ii) $v_i(y_i) - v_i(y''_i) \leq v'_i(y_i) - v'_i(y''_i)$ for each $y_i \in Y \setminus \{y'_i, y''_i\}$. In the problems of allocating indivisible private goods with monetary transfers, Fujinaka and Wakayama (2008) show a constancy result on secure implementation over minimally rich domains. Note that the result in this paper is independent of those of Fujinaka and Wakayama (2008) because their models are different. [5]

Partial dominance is stronger than weak indifference (Nishizaki, 2012a). The domain V satisfies **weak indifference** if and only if for each $i \in N$, each $v'_i, v''_i \in V_i$, each $y'_i, y''_i \in Y$, and each $X \in \mathbb{R}$, if $v'_i(y''_i) - v'_i(y'_i) < X < v''_i(y''_i) - v''_i(y'_i)$, then there exists $v_i \in V_i$ such that $v_i(y''_i) - v_i(y'_i) = X$. In queueing problems, Nishizaki (2012a) showed a constancy result on secure implementation over weakly indifferent domains. Note that weak indifference does not imply a constancy result similar to those of this paper because a key point of the result is the condition (iii) in Definition 3. In the following proof, this condition makes the upper limit of consumption of the public good that the agent can induce.

Partial dominance is close to dual dominance (Saijo, 1987) which is a condition of social choice functions. Dual dominance requires that the weak lower contour set for agent i with v_i at (y'_i, x'_i) (resp. (y''_i, x''_i)) includes the set for agent i with v'_i (resp. v''_i) at (y'_i, x'_i) (resp. (y''_i, x''_i))

[4]Nishizaki (2011) considered this domain. The supplementary material illustrates some examples of partially dominant domains available at the website of the Economics Bulletin.

[5]For the relationship between partial dominance and minimal richness, see the supplementary material available at the website of the Economics Bulletin.

over $Y \times \mathbb{R}_+$. On the other hand, partial dominance requires such domination over the "part" of $Y \times \mathbb{R}_+$. [6] Saijo (1987) showed a constancy result on Nash implementation by dual dominance. Similarly, dual dominance implies a constancy result on secure implementation. [7]

Partial dominance is important in the situations where there is little difference between the two extreme consumption of the public good because of certain problems other than the budget constraint. An example of the situations is the construction of schools in urban areas for the adjustment to the growth of population. In the areas, there are limited sites for constructing schools. In addition, we need to select the sites carefully because schools play an important role as evacuation shelters. In the situations, it might be reasonable to assume that each agent's preference is linear, that is, each agent's marginal utility of the public good is constant because we might be able to construct several schools at most. Partial dominance covers the situations but not minimal-richness because the set of all strictly increasing and linear valuation functions satisfies partial dominance but not minimal-richness.

3 Result

The following theorem shows a domain-richness condition that causes the difficulty of secure implementation in discrete public good economies. The proof techniques are similar to those of Fujinaka and Wakayama (2008) and slightly different from those of Nishizaki (2012a).

The social choice function f is **constant** if and only if for each $v, v' \in V$, $(\mathbf{y}(v), \mathbf{x}(v)) = (\mathbf{y}(v'), \mathbf{x}(v'))$. For each $i \in N$ and each $v'_{-i} \in V_{-i}$, let $O_i(v'_{-i}) \equiv \{y_i \in Y \mid \text{there exists } v_i \in V_i \text{ such that } y_i(v_i, v'_{-i}) = y_i\}$ be the **option set for agent i given v'_{-i}**.

Theorem. *The social choice function f satisfies **strategy-proofness** and the **rectangular property** if and only if it is **constant** when the domain V satisfies **partially dominance**.* [8]

Proof. Because the "if" part is obvious, only the "only if" part is demonstrated. We will prove the following four claims. Because the proof of Claim 1 is similar to Fujinaka and Wakayama (2008) and Nishizaki (2012a), it is omitted.

Claim 1. *For each $v, v' \in V$ and each $i \in N$, if $y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i})$, then $x_i(v_i, v'_{-i}) = x_i(v'_i, v'_{-i})$.*

Claim 2. *For each $v, v' \in V$ and each $i \in N$, if $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$, then $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$.*

[6] For the relationship between partial dominance and dual dominance, see the supplementary material available at the website of the Economics Bulletin.

[7] Securely implementable social choice functions satisfying non-dominance (Fujinaka and Wakayama, 2008) which is weaker than dual dominance are also constant. For non-dominance, see the supplementary note provided by Fujinaka and Wakayama (2008) available at: `http://www.iser.osaka-u.ac.jp/library/dp/2007/DP0699N.pdf`

[8] For the tightness of this theorem, see the supplementary material available at the website of the Economics Bulletin.

Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in N$ such that $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$ and $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) \leq v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$. Together with **strategy-proofness**, this implies that $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$. Together with the **rectangular property**, this implies that $y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i})$. This is a contradiction.

Claim 3. *For each $v, v' \in V$ and each $i \in N$, $y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i})$.*

Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in N$ such that $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$. Together with Claim 2, this implies that $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$ and $v'_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}) > v'_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i})$. These imply that

$$v_i(y_i(v'_i, v'_{-i})) - v_i(y_i(v_i, v'_{-i})) < X < v'_i(y_i(v'_i, v'_{-i})) - v'_i(y_i(v_i, v'_{-i})), \tag{1}$$

where $X = x_i(v'_i, v'_{-i}) - x_i(v_i, v'_{-i})$. We consider the case in which

$$y_i(v_i, v'_{-i}) < y_i(v'_i, v'_{-i}). \tag{2}$$

By (1), we can take $v''_i \in V_i$ such that

$$v''_i(y_i(v'_i, v'_{-i})) - v''_i(y_i(v_i, v'_{-i})) = x_i(v'_i, v'_{-i}) - x_i(v_i, v'_{-i}), \tag{3}$$

$$v''_i(y_i) - v''_i(y_i(v_i, v'_{-i})) \leq v_i(y_i) - v_i(y_i(v_i, v'_{-i})) \text{ for each } y_i \leq y_i(v_i, v'_{-i}), \tag{4}$$

$$v''_i(y_i) - v''_i(y_i(v'_i, v'_{-i})) \leq v'_i(y_i) - v'_i(y_i(v'_i, v'_{-i})) \text{ for each } y_i \geq y_i(v'_i, v'_{-i}), \tag{5}$$

because V satisfies **partial dominance**. Let $y_i^*, y_i^{**} \in O_i(v'_{-i})$ be such that $y_i^* \leq y_i(v_i, v'_{-i})$ and $y_i^{**} \geq y_i(v'_i, v'_{-i})$. On the basis of Claim 1, let x_i^* (resp. x_i^{**}) be the cost share of the public good for agent i at y_i^* (resp. y_i^{**}). By **strategy-proofness**, we know that $v_i(y_i^*) - v_i(y_i(v_i, v'_{-i})) \leq x_i^* - x_i(v_i, v'_{-i})$. Together with (4), this implies that

$$v''_i(y_i^*) - x_i^* \leq v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}). \tag{6}$$

Similarly, we find that $v''_i(y_i^{**}) - x_i^{**} \leq v''_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$ by (5) and **strategy-proofness**. According to the position of $y_i(v''_i, v'_{-i})$, we have the following two cases: (i) $y_i(v''_i, v'_{-i}) \leq y_i(v_i, v'_{-i})$ or $y_i(v''_i, v'_{-i}) \geq y_i(v'_i, v'_{-i})$ and (ii) $y_i(v_i, v'_{-i}) < y_i(v''_i, v'_{-i}) < y_i(v'_i, v'_{-i})$.

We consider the case of (i). By **strategy-proofness**, we know that

$$v''_i(y_i(v''_i, v'_{-i})) - x_i(v''_i, v'_{-i}) \geq v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}). \tag{7}$$

If $y_i(v''_i, v'_{-i}) \leq y_i(v_i, v'_{-i})$, then we know that

$$v''_i(y_i(v''_i, v'_{-i})) - x_i(v''_i, v'_{-i}) \leq v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) \tag{8}$$

by (6). By (7) and (8), we find that $v''_i(y_i(v''_i, v'_{-i})) - x_i(v''_i, v'_{-i}) = v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i})$. Together with the **rectangular property**, this implies that

$$(\mathbf{y}(v_i, v'_{-i}), \mathbf{x}(v_i, v'_{-i})) = (\mathbf{y}(v''_i, v'_{-i}), \mathbf{x}(v''_i, v'_{-i})). \tag{9}$$

By (3), (9), and the **rectangular property**, we find that

$$(\mathbf{y}(v_i'', v_{-i}'), \mathbf{x}(v_i'', v_{-i}')) = (\mathbf{y}(v_i', v_{-i}'), \mathbf{x}(v_i', v_{-i}')). \tag{10}$$

By (9) and (10), we find that $(\mathbf{y}(v_i, v_{-i}'), \mathbf{x}(v_i, v_{-i}')) = (\mathbf{y}(v_i', v_{-i}'), \mathbf{x}(v_i', v_{-i}'))$. This contradicts (2). In addition, we have a contradiction to (2) in the above manner if $y_i(v_i'', v_{-i}') \geq y_i(v_i', v_{-i}')$.

We consider the case of (ii). This case is divided into two subcases. In one subcase, at least one of the following conditions is not satisfied:

$$\begin{aligned} v_i(y_i(v_i, v_{-i}')) - x_i(v_i, v_{-i}') &> v_i(y_i(v_i'', v_{-i}')) - x_i(v_i'', v_{-i}'), \\ v_i'(y_i(v_i', v_{-i}')) - x_i(v_i', v_{-i}') &> v_i'(y_i(v_i'', v_{-i}')) - x_i(v_i'', v_{-i}'), \end{aligned} \tag{11}$$

$$v_i''(y_i(v_i, v_{-i}')) - x_i(v_i, v_{-i}') = v_i''(y_i(v_i', v_{-i}')) - x_i(v_i', v_{-i}') < v_i''(y_i(v_i'', v_{-i}')) - x_i(v_i'', v_{-i}'). \tag{12}$$

Note that the equality of (12) is satisfied by (3). In another subcase, all the above conditions are satisfied. In the former subcase, we find that $(\mathbf{y}(v_i, v_{-i}'), \mathbf{x}(v_i, v_{-i}')) = (\mathbf{y}(v_i'', v_{-i}'), \mathbf{x}(v_i'', v_{-i}'))$, $(\mathbf{y}(v_i', v_{-i}'), \mathbf{x}(v_i', v_{-i}')) = (\mathbf{y}(v_i'', v_{-i}'), \mathbf{x}(v_i'', v_{-i}'))$, or the both by **strategy-proofness** and the **rectangular property**. This is a contradiction in this case. In the latter subcase, we find that

$$v_i(y_i(v_i'', v_{-i}')) - v_i(y_i(v_i, v_{-i}')) < X'' < v_i''(y_i(v_i'', v_{-i}')) - v_i''(y_i(v_i, v_{-i}'))$$

by (11) and (12), where $X'' = x_i(v_i'', v_{-i}') - x_i(v_i, v_{-i}')$. By applying an argument similar to the case of v_i'' repeatedly, we only have the case similar to the case of (i) for the last time. This implies a contradiction.

By an argument similar to the case in which $y_i(v_i, v_{-i}') < y_i(v_i', v_{-i}')$, we have a contradiction in the case in which $y_i(v_i, v_{-i}') > y_i(v_i', v_{-i}')$.

Claim 4. *For each* $v, v' \in V$, $(\mathbf{y}(v), \mathbf{x}(v)) = (\mathbf{y}(v'), \mathbf{x}(v'))$.

Let $v, v' \in V$. By Claims 1 and 3, we find that $(y_i(v_i, v_{-i}'), x_i(v_i, v_{-i}')) = (y_i(v_i', v_{-i}'), x_i(v_i', v_{-i}'))$ for each $i \in N$. This implies that $v_i(y_i(v_i, v_{-i}')) - x_i(v_i, v_{-i}') = v_i(y_i(v_i', v_{-i}')) - x_i(v_i', v_{-i}')$ for each $i \in N$. Together with **rectangular property**, this implies that $(\mathbf{y}(v), \mathbf{x}(v)) = (\mathbf{y}(v'), \mathbf{x}(v'))$. □

Together with the result of Saijo, Sjöström, and Yamato (2007), the above theorem implies the following constancy result on secure implementation in discrete public good economies.

Corollary. *The social choice function is **securely implementable** if and only if it is **constant** when the domain satisfies **partially dominance**.*

4 Conclusion

In divisible public good economies, Saijo, Sjöström, and Yamato (2007) showed that Groves mechanisms (Groves, 1973) are securely implementable over certain domains. It is open to characterize the domains and securely implementable social choice functions in the economies including excludable public good economies. On the other hand, Nishizaki (2012b) showed a possibility of constructing desirable social choice functions that are securely implementable in pure exchange economies with Leontief preferences. It is open to characterize securely implementable social choice functions in the economies. These interesting topics remain for our future research.

References

Berga, D. and B. Moreno (2009) "Strategic requirements with indifference: Single-peaked versus single-plateaued preferences," *Social Choice and Welfare* 32, pp.275-298.

Bochet, O. and T. Sakai (2010) "Secure implementation in allotment economies," *Games and Economic Behavior* 68, pp.35-49.

Cason, T., T. Saijo, T. Sjöström, and T. Yamato (2006) "Secure implementation experiments: Do strategy-proof mechanisms really work?" *Games and Economic Behavior* 57, pp.206-235.

Chen, Y. (2008) "Incentive-compatible mechanisms for pure public goods: A survey of experimental literature," in *Handbook of Experimental Economics Results*, edited by C. R. Plott and V. L. Smith, North-Holland.

Deb, R. and L. Razzolini (1999) "Voluntary cost sharing for an excludable public project," *Mathematical Social Sciences* 37, pp.123-138.

Deb, R., L. Razzolini, and T. K. Seo (2003) "Strategy-proof cost sharing, ability to pay and free provision of an indivisible public good," *Mathematical Social Sciences* 45, pp.205-227.

Fujinaka, Y. and T. Wakayama (2008) "Secure implementation in economies with indivisible objects and money," *Economics Letters* 100, pp.91-95.

Fujinaka, Y. and T. Wakayama (2011) "Secure implementation in Shapley-Scarf housing markets," *Economic Theory* 48, pp.147-169.

Groves, T. (1973) "Incentives in teams," *Econometrica* 41, pp.617-631.

Kumar, R. (2011) "Secure implementation in production economies," Department of Economics Working Paper Series No. 2011-02, Louisiana State University.

Mizukami, H. and T. Wakayama (2005) "Bossiness and implementability in pure exchange economies," *RIMS Kokyuroku* 1461, pp.126-140.

Nishizaki, K. (2011) “Secure implementation in discrete and excludable public good economies,” *Osaka Economic Papers* 61, pp.48-56.

Nishizaki, K. (2012a) “Secure implementation in queueing problems,” *Theoretical Economics Letters* 2, pp.561-565.

Nishizaki, K. (2012b) “An equivalence of secure implementability and full implementability in truthful strategies in pure exchange economies with Leontief utility functions,” GCOE Discussion Paper Series No. 279, Osaka University.

Saijo, T. (1987) “On constant Maskin monotonic social choice functions,” *Journal of Economic Theory* 42, pp.382-386.

Ohseto, S. (2000) “Characterizations of strategy-proof mechanisms for excludable versus nonexcludable public projects,” *Games and Economic Behavior* 32, pp.51-66.

Ohseto, S. (2005) “Augmented serial rules for an excludable public good,” *Economic Theory* 26, pp.589-606.

Saijo, T., T. Sjöström, and T.Yamato (2007) “Secure implementation,” *Theoretical Economics* 2, pp.203-229.

Yu, Y. (2007) “Serial cost sharing of an excludable public good available in nultiple units,” *Social Choice and Welfare* 29, pp.539-555.