Economics Bulletin

Volume 33, Issue 1

Debt and growth: Is there a non-monotonic relation?

Alfred Greiner
Bielefeld University

Abstract

In this note we theoretically investigate the question of whether the relationship between public debt and economic growth is characterized by an inverse U-shaped functional form. Starting point of our analysis is the paper by Checherita-Westphal et al. (2012) who present an endogenous growth model with public capital and public debt that displays a hump-shaped relation between debt and economic growth. We highlight the mechanism that generates this outcome and we generalize their model by allowing for a more general debt policy. We demonstrate that this non-monotonic relation only holds if public deficits are exogenously fixed and exactly equal to public investment at each point in time. With a more general debt policy, one realizes that smaller public deficits and lower public debt always lead to a higher balanced growth rate. Thus, starting from a situation where the public deficit equals public investment, governments can raise the long-run growth rate by reducing their deficits.

Citation: Alfred Greiner, (2013) "Debt and growth: Is there a non-monotonic relation?", *Economics Bulletin*, Vol. 33 No. 1 pp. 340-347.
Contact: Alfred Greiner - agreiner@wiwi.uni-bielefeld.de.
Submitted: December 10, 2012. **Published:** February 08, 2013.

1. Introduction

The current public debt crisis in some member countries of the euro area raises the question of how public debt and economic growth are correlated. From an empirical point of view there is evidence that this relation is described by an inverted U-shaped pattern. First, higher public debt to GDP ratios go along with higher GDP growth rates before the relation becomes negative, implying that there is a growth maximizing public debt to GDP ratio. For example, Checherita and Rother (2010) as well as Reinhart und Rogoff (2010) find empirical evidence for such a functional form. However, that result is far from being robust since one can also find studies that detect a strictly negative correlation between the debt to GDP ratio and economic growth (see for example Ferreira 2009 or Kumar and Woo 2011).

From a theoretical perspective, the relation between the public debt to GDP ratio and economic growth seems to be rather characterized by a negative correlation. There exist studies resorting to standard endogenous growth models with an infinitely lived representative household that come to this result. For example, Futagami et al. (2008) present an endogenous growth model with productive public spending and public debt where the government is not allowed to raise the debt to GDP ratio beyond a certain critical value. For that model, it turns out that the balanced growth rate is the higher the smaller the public debt to GDP ratio is, although those authors do not mention this outcome explicitly since they are primarily interested in the dynamics of their model. Greiner (2008) presents an endogenous growth model with public capital and public debt where the government sets the primary surplus such that the inter-temporal budget constraint of the government is fulfilled. In that model, it turns out that a zero debt to GDP ratio implies a higher balanced growth rate compared to a situation with a positive debt ratio. An asymptotically zero debt to GDP ratio can be obtained either through a balanced government budget or through public deficits that imply a positive growth rate of public debt that, however, is smaller than the growth rate of GDP. Minea and Villieu (2009) also present an endogenous growth model with public capital and public debt and posit that the government is allowed to run deficits only in order to finance public investment, that is the government obeys the 'golden rule of public finance'. They find that a balanced budget rule yields a higher growth rate than an economy where the government runs permanent deficits and, erroneously, conclude that the 'golden rule of public finance' implies a smaller long-run growth rate than the balanced budget rule. Their conclusion is not correct since it is not the 'golden rule of public finance' that generates the result, but rather the fact public debt grows at the same rate as GDP. Thus, it can be shown that the 'golden rule of public finance' and the balanced budget rule give identical long-run growth rates when public debt grows but less than GDP (see Greiner 2010 for details).

The fact that public debt and economic growth are negatively correlated can also be observed for endogenous growth models without productive public spending. For example, Greiner (2011) presents an endogenous growth model with elastic labour supply where ongoing growth results from positive externalities of private capital. In that model public debt and economic growth are negatively correlated, too. It must also be pointed out that it is not the standard crowding-out mechanism that generates this outcome because, as the debt ratio rises, the primary surplus rises, too, to guarantee sustainabilty. Consequently, for a fixed tax rate public spending declines, thus, preventing a crowding-out of private investment. Rather, the decline in the growth rate, as a consequence of higher public debt, is due to the fact that higher public debt leads to a lower shadow price of savings which reduces labour supply and investment. However, once incomplete markets are assumed, things change. For example, if the labour market is characterized by wage rigidities, leading to unemployment, public debt is neutral in the sense that it does not affect the allocation of resources ceteris paribus (see Greiner 2012). Then, a higher debt ratio can lead to higher growth and less unemployment if the deficit is used for productive public investment, as shown in Greiner and Flaschel (2010) with the help of numerical examples. However, an inverted U-shaped relation between debt and growth does not exist, but the growth rate rises until the economy reaches the full employment state.

In a recent contribution, Checherita-Westphal et al. (2012) present an endogenous growth model with public capital and public debt and find an inverted U-shaped relation between debt and growth and they estimate the growth maximizing debt to GDP ratios for OECD, EU and euro area countries. We demonstrate that their model is structurally the same as the model without public debt that is characterized by an inverted U-shaped relation between the tax rate and growth. Since they assume that the deficit is exogenously fixed and equal to public investment at each point in time, the debt ratio equals the ratio of public capital to GDP that is a monotonic function of the tax rate in equilibrium. But, once the model is generalized to allow for different deficit policies, we find that the growth rate is the higher the smaller public deficits and public debt are.

In the rest of the paper, we proceed as follows. In the next section we present the model which is analyzed and discussed in section 3. Section 4, finally, concludes the paper.

2. The growth model with public capital and debt

We begin our presentation with a description of the structure of the growth model with public capital and debt that extends the model by Checherita-Westphal et al. (2012).

2.1 The private sector

There exists a continuum of rational and identical households that maximize utility over an infinite time horizon arising from per-capita consumption, $C(t)$, subject to the budget constraint. Population is constant and set equal to one. Neglecting the time argument t when no ambiguity arises, the maximization problem of the representative household can be written as,

$$\max_{C} \int_0^\infty e^{-\rho t} \ln C \, dt, \tag{1}$$

subject to

$$\dot{K} + \dot{B} = rB + (1 - \tau)Y - C. \tag{2}$$

The coefficient ρ is the household's rate of time preference, r is the interest rate and we assume a logarithmic utility function without loss of generality.[1] The variable Y gives GDP which is equal to output in the economy, K denotes private capital where we neglect depreciation and B denotes public debt. Finally, $\tau \in (0, 1)$ is the constant tax rate on output and the dot over a variable stands for the time derivative.

Output Y is given by

$$Y = AK^{1-\alpha}G^{\alpha}, \tag{3}$$

with $\alpha \in (0, 1)$ the elasticity of output with respect to private capital and $1 - \alpha$ is the elasticity of output with respect to public capital, G, and A is a technology parameter. In equilibrium, the interest rate equals the net marginal product of private capital and is given by,

$$r = (1 - \tau)(1 - \alpha)AK^{-\alpha}G^{\alpha}. \tag{4}$$

2.2 The government

The period budget constraint of the government in our economy is as follows,

$$\dot{B} = rB - \tau Y + I_p, \tag{5}$$

with I_p denoting public investment. Neglecting depreciation, public capital, G, evolves according to,

$$\dot{G} = I_p. \tag{6}$$

The government obeys the inter-temporal budget constraint such that $\lim_{t\to\infty} e^{-rt}B(t) = 0$ holds and it follows the 'golden rule of public finance' implying that public deficits are smaller or equal to public investment. The latter gives,

$$\dot{B} = \psi I_p,\ \psi \in [0, 1]. \tag{7}$$

3. Analysis of the model

Solving the optimization problem of the household and using (4), the growth rate of consumption, g_C, is obtained as,

$$g_C := \frac{\dot{C}}{C} = -\rho + (1 - \tau)(1 - \alpha)AK^{-\alpha}G^{\alpha}. \tag{8}$$

[1]Allowing for a more general CRRA utility function does not change the outcome.

Further, combining the budget constraint of the household, (2), with that of the government, (5), yields the economy-wide resource constraint that determines the growth rate of private capital as,

$$g_K := \frac{\dot{K}}{K} = A\left(\frac{G}{K}\right)^{\alpha} - \frac{C}{K} - \left(\frac{I_p}{G}\right)\left(\frac{G}{K}\right). \tag{9}$$

The growth rates of public debt and of public capital are obtained from (5)-(7) as,

$$g_B := \frac{\dot{B}}{B} = \psi\left(\frac{G}{K}\right)\left(\frac{K}{B}\right)\left(\frac{\dot{G}}{G}\right), \tag{10}$$

$$g_G := \frac{\dot{G}}{G} = (1-\psi)^{-1}\left(\tau A\left(\frac{G}{K}\right)^{\alpha-1} - (1-\tau)(1-\alpha)A\left(\frac{G}{K}\right)^{\alpha-1}\frac{B}{K}\right). \tag{11}$$

To analyze our system further, we define $z := G/K$, $c := C/K$ and $b := B/K$. First, we consider the case $\psi = 1$ which gives the model studied by Checherita-Westphal et al. (2012).

3.3 The model for $\psi = 1$

Setting $\psi = 1$ and noting that $g_B = g_G = g$ must hold on the balanced growth path (BGP), with g denoting the balanced growth rate, we immediately see from equation (10) that this implies $z = b$. Further, $\psi = 1$ also implies $\dot{B} = \dot{G}$ and, thus, $\tau Y = rB$ (from (5) to (7)). Using $b = z$ and (4), $\tau Y = rB$ leads to,

$$b = z = \frac{\tau}{(1-\tau)}\frac{1}{(1-\alpha)}. \tag{12}$$

Equation (12) shows that the ratio of public debt to private capital equals the ratio of public to private capital on the BGP and is determined by the tax rate. Inserting (12) in (8) yields the balanced growth rate as,

$$g = -\rho + A(1-\tau)^{1-\alpha}\tau^{\alpha}(1-\alpha)^{1-\alpha}. \tag{13}$$

Equation (13) demonstrates that the balanced growth rate is a hump-shaped function of the tax rate τ and, because of (12), a hump-shaped function of the debt to capital and of the public to private capital ratio. Maximizing the balanced growth rate with respect to τ, shows that the maximum growth rate is achieved for $\tau = \alpha$ which is identical to the result in the model without public debt (see Greiner and Hanusch 1998).

The reason for that outcome is that the assumption of a deficit that equals public investment at each point in time, gives the budget constraint of the government as $\tau = (g+\rho)\cdot(G/Y)$ which is structurally equivalent to that in the model without public debt, given by $\tau = g \cdot (G/Y)$. Therefore, the model with public debt is basically the same as the model without debt, and one obtains the result that the balanced growth rate is a hump-shaped function of the tax rate and of the ratio of public to private capital which is equal to the debt ratio. That result only holds for $\psi = 1$ as we will show next.

3.4 The model for $\psi \in [0, 1)$

For $\psi \in (0, 1)$ a BGP is given for $g_C = g_G = g_K = g_B = g$ and for $\psi = 0$ we have $b = 0$ on the BGP so that in this case the BGP is obtained for $g_C = g_G = g_K = g$ and $g_B = 0$. Note that in this case, $\dot{b} = b(g_B - g_K) = 0$ holds because of $b = 0$. Setting $g = g_B = g_G$ one obtains $z\psi = b$. Using the latter, we get for $q := g_C - g_G$,

$$q = -\rho + (1-\tau)(1-\alpha)Az^{\alpha} - (1-\psi)^{-1}\left(\tau A z^{\alpha-1} - \psi(1-\tau)(1-\alpha)Az^{\alpha}\right) = 0\ . \quad (14)$$

By implicitly differentiating (14) and using $g - r = -\rho$, we obtain:

$$\frac{dz}{d\psi} = \frac{-\rho}{(1-\psi)\,(\partial q/\partial z)} < 0\ . \quad (15)$$

Equation (8) shows that $\partial g/\partial z > 0$ holds. Hence, equation (15) demonstrates that the balanced growth rate is the higher, the smaller public deficits and public debt are. The highest balanced growth rate is obtained for a balanced budget that implies a zero debt to capital ratio and, for a given tax rate, any value of $\psi \in [0, 1)$ generates a higher balanced growth rate than $\psi = 1$, which is also seen from (15).

3.5 An example

In order to illustrate our analytical model we now present a numerical example. The parameter values are set as follows: $\rho = 0.05$, $\alpha = 0.25$, $\tau = 0.15$, $A = 0.15$. Table 1 shows the balanced growth rate and the debt to private capital ratio for different values of the public deficit.

Table 1. Balanced growth rate (g) and the debt to private capital ratio (B/K) for different values of the public deficit (ψ).

ψ	g	B/K
0.99999	1.66%	0.235
0.85	1.86%	0.225
0.7	2.07%	0.209
0.5	2.38%	0.178
0.35	2.65%	0.143
0.2	2.94%	0.095
0	3.36%	0

Table 1 clearly shows that the balanced growth rate is the higher and the debt ratio is the smaller, the lower the public deficit ψ is chosen.

Setting $\psi = 1$ leads to a balanced growth rate of $g = 1.66\%$ and a corresponding debt to capital ratio of $b = 0.235$ for $\tau = 0.15$, which is, of course, identical to the situation with $\psi = 0.99999$ in table 1. With $\psi = 1$, the maximum growth rate is $g = 1.89\%$ which is obtained for $\tau = 0.25$ that implies a debt to capital ratio of $z = 0.44444$.

This example illustrates the outcome of our analytical model. It clearly demonstrates that the balanced growth rate is the higher, the smaller public deficits and public debt are. The highest growth rate is achieved for a balanced budget, i.e. for $\psi = 0$, that implies a zero debt ratio and the lowest growth rate is obtained when the public deficit equals public investment, i.e. for $\psi = 1$, for a given tax rate.

4. Conclusion

In this paper we have shown that the inverted U-shaped relation between economic growth and public debt in the theoretical model by Checherita-Westphal et al. (2012) is the result of their assumption that the public deficit is exogenously fixed and set such that it is equal to public investment at each point in time. That makes the stock of public debt identical to the stock of public capital, relative to private capital or to GDP respectively, and those two variables are monotonically rising with the tax rate. Since there is a hump-shaped relation between the tax rate and economic growth in this class of endogenous growth models with productive public spending, there also exists such a relation between debt and growth.

However, once a more general debt policy is considered, one finds that smaller public deficits and lower public debt always generate a higher growth rate. Consequently, starting from the situation where the public deficit equals public investment, governments can raise the long-run growth rate by reducing their deficits. Such a policy implies a smaller debt ratio leading to higher public and private investment in the long-run, thus, raising the balanced growth rate.[2]

References

Checherita-Westphal, C. and P. Rother (2010) ”The impact of high and growing debt on economic growth. An empirical investigation for the euro area” ECB working paper series number 1237

Checherita-Westphal, C., A.H. Hallett and P. Rother (2012) ”Fiscal sustainability using growth-maximising debt targets” ECB working paper series number 1472

[2]Transition dynamics of such a policy are studied in Greiner (2008), for example.

Ferreira, C. (2009) "Public Debt and Economic Growth: a Granger Causality Panel Data Approach" School of Economics and Management, Technical University of Lisbon, working paper 24/2009

Futagami, K., T. Iwaisako and R. Ohdoi (2008) "Debt policy rule, productive government spending, and multiple growth paths." *Macroeconomic Dynamics* **12,** 445-62.

Greiner, A. (2008) "Does it pay to have a balanced government budget?" *Journal of Institutional and Theoretical Economics (JITE)* **164,** 460-476.

Greiner, A. (2010) "Does the 'Golden Rule of Public Finance' imply a lower long-run growth rate? A clarification." *Economics Bulletin* **30,** 975-982.

Greiner, A. (2011) "Economic growth, public debt and welfare: Comparing three budgetary rules" *German Economic Review* **12,** 205-222.

Greiner, A. (2012) "Sustainable public debt and economic growth under wage rigidity" *Metroeconomica,* first published online: 22 NOV 2012, DOI: 10.1111/meca.12006

Greiner, A. and H. Hanusch (1998) "Growth and welfare effects of fiscal policy in an endogenous growth model with public investment." *International Tax and Public Finance* **5,** 249-261.

Greiner, A. and P. Flaschel (2010) "Public debt and public investment in an endogenous growth model with real wage rigidities" *Scottish Journal of Political Economy* **57,** 68-84.

Kumar, M.S. and J. Woo (2010) "Public debt and growth" IMF working paper number WP/10/174

Minea, A. and P. Villieu (2009) "Borrowing to finance public investment? The 'golden rule of public finance' reconsidered in a growth setting" *Fiscal Studies* **30,** 103-133.

Reinhart, C.M. and K.S. Rogoff (2010) "Growth in a time of debt" NBER working paper number 15639