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Application of Jones' Inequality to the n-country, m-good Ricardo–Graham Model

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Abstract

This paper discusses the results of the application of Jones' Inequality to the n-country, m-good Ricardo–Graham model. In the Jones' Inequality model, the number of countries is the same as the number of goods produced. On one hand, if a country is divided into regions for the purposes of this model, Jones' Inequality can be applied to the case where the number of goods is larger than the number of countries. On the other hand, when two or more countries specialize in the same good, the model distinguishes the same good as being a different commodity for the other countries.

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1. Introduction

The many-country, many-good Ricardian model was first examined by Graham (1923, 1932, 1948). Graham (1948) noted the importance of the model in a key example of his four-country, three-good model. In this case, generally, the number of countries may be different from the number of goods produced. This generality is maintained by McKenzie (1954a, 1954b, 1955), who developed the general framework of the many-country, many-good Ricardo–Graham model. However, when Jones (1961) provided the solution for the model, he used the same number of countries and goods produced. Jones (1961) considered the production assignment model, which is the key concept of the model, and he showed an inequality as the solution to the model. Jones' Inequality was widely accepted as the solution to the Ricardo–Graham model, so the various cases where there was a difference between the number of countries and the number of goods has not been studied. Ikema (1993) (written in Japanese) showed illustration of the many-country, three-good model, but his illustration is essentially limited only to the three-good case. Shiozawa (2007) focused on a case where the number of goods produced is larger than the number of countries in the model, the goods, in this case, being intermediate goods. However, his focus is not the condition stipulated by Jones' Inequality, but rather the existence of the solution that points to a "strongly-shared pattern of specialization," which extends the concept of patterns of specialization to the case where the number of goods produced is larger than the number of countries.[1]

It is interesting to note that the way to apply Jones' Inequality to the model with a different number of countries than the goods produced is not obvious, especially in the case where the number of countries is larger than the number of goods produced. Therefore, this paper discusses both cases, i.e., where the number of countries is larger and smaller than the number of goods produced. Consider the n-country, m-good Ricardo–Graham model. To complete the analysis of the many-country, many-good Ricardo–Graham model, it is essential to use the application discussed in this paper.

This paper shows how to determine the strongly-shared pattern of specialization with Jones' Inequality, when the number of goods produced is larger than that of countries. The method is to virtually divide a country, which makes two or more goods that are not produced by other countries, into regions. When the country is divided into regions for the purpose of the model, the technology's parameters, i.e., labor input coefficients, are the same for each region of a country as they would be for the country before it was divided into regions. Thus, the number of regions can be the same as that of goods, i.e., m. To interchange countries with regions, Jones' Inequality can be applied to the m-region, m-good model. The regions are equated to countries, so it is understood that each country produces the same goods in which regions from that country specialize.

This paper also shows how to determine efficient patterns of specialization using Jones' Inequality when the number of countries is larger than the number of goods produced. In the efficient patterns of specialization, there is a good in which two or more countries specialize. The way to find the pattern, for the purposes of the model, is to distinguish the good as a different commodity if the country producing the good is also different, even if the goods produced are the same in each country. Therefore, the number of commodities can be the same as that of countries, i.e., n. By changing countries to regions, Jones' Inequality can be applied to the n-country, n-commodity model. It is understood that both countries specialize in the same good.

In the next section, we consider the n-country, m-good Ricardo–Graham model. In this third section, we discuss the case where the number of goods produced is larger than the number of countries. The purpose, essentially, is to determine the strongly-shared pattern of specialization in Jones' Inequality. In the fourth section, we discuss the case where the number of countries is larger than the number of goods produced. The purpose of the section is to find efficient patterns of specialization with Jones' Inequality. The final section is the conclusion.

[1] See Shiozawa (2007).

2. The model and Jones' Inequality

The analysis is based on the Ricardo–Graham model with n countries and m goods. The production function is linear with one factor, labor. Let l_j^i be the amount of labor needed to produce one unit of the j th good in the i th country $(i=1,2,\cdots,n;\ j=1,2,\cdots,m)$, let x_j^i be the quantity of the j th good in the i th country, and L^i be the positive labor endowment in the i th country. Then the resource constraint is $\sum_j l_j^i x_j^i \le L^i$ for any $i=1,2,\cdots,n$. The world production set W is as follows:

$$W \equiv \left\{ \left(\sum_i x_1^i, \sum_i x_2^i, \cdots, \sum_i x_m^i \right) \ge 0 \;\middle|\; \sum_j l_j^i x_j^i \le L^i, \quad x_j^i \ge 0 \right\}.^{2}$$

If $(p_1, p_2, \cdots, p_m) >> 0$ is assumed as the world price and $X = (X_1, X_2, \cdots, X_m)$ as the world production point, then a point $X = \left(\sum_i x_1^i, \sum_i x_2^i, \cdots, \sum_i x_m^i \right)$ is on the frontier iff there is a vector $(p_1, p_2, \cdots, p_m) >> 0$ such that X maximizes the world output maximization problem:

$$\max_{x_j^i \ge 0} \sum_{i,j} p_j x_j^i \quad \text{s.t.} \quad \sum_j l_j^i x_j^i \le L^i \quad (i=1,2,\cdots,n). \tag{1}$$

To show the result of Jones (1961), the following definitions are useful.[3]

Definition

1. $X^* \in W$ is efficient iff there is no production point $X \in W$ satisfying $X > X^*$.
2. $X^* \in W$ is an extreme point iff there is no bundle $(A,B,t) \in W \times W \times (0,1)$ satisfying $tA + (1-t)B = X^*$.
3. When $n = m$, the $i-i$ assignment is the world economy satisfying that the i th country puts all of its labor into the production of the i th good.

The following theorem is essentially pointed out by Jones (1961).[4]

Jones' Theorem (1961) Consider $X^* (\in W)$ is produced with the $i-i$ assignment when $n = m$.

1. The following three are equivalent in the meaning of a strong, efficient world economy.

(1) X^* is the only one extreme point under the condition that every good is produced.

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[2] Inequality signs are $\ge, >, >>$.

[3] In this area, the word "an extreme point" is firstly used not in Jones(1961) but in Kuhn(1968). However, for explanation, the word "an extreme point" is useful to distinguish the uniqueness (strong) argument from the universal (weak) argument, so this paper uses the terminology of Kuhn(1968).

[4] See Jones (1961) and Kuhn (1968) for example.

(2) X^* can be produced with only the $i-i$ assignment and there is a world price $(p_1, p_2, \cdots, p_n) >> 0$ such that under the price, X^* uniquely maximizes world output.

(3) The following strict Jones' Inequality holds: for any permutation σ (except the identity),

$$\prod_i l_i^i < \prod_i l_{\sigma(i)}^i .$$

2. The following three are equivalent in the meaning of a weak, efficient world economy.

(1) X^* is efficient.

(2) There is a price $(p_1, p_2, \cdots, p_n) >> 0$ such that under the price, X^* maximizes the world output.

(3) The following weak Jones' Inequality holds: for any permutation σ,

$$\prod_i l_i^i \le \prod_i l_{\sigma(i)}^i .$$

From the next section, we apply the theorem to the case where the number of countries is different than the number of goods.

3. The case when the number of goods is larger than the number of countries

In this section, we consider the case where the number of goods is larger than the number of countries. In the n-country, m-good model, we suppose $n < m$. When every good is produced in the world, some countries choose incomplete specialization. Thus, the concept of patterns of specialization cannot be applied directly in the model. First, we define the extended concept of patterns of specialization.[5]

Definition A shared pattern of specialization is a world economy where every good is produced, but the number of countries producing each good is only one.

The following lemma is shown by Shiozawa (2007) for intermediate goods, but the essence is the same for this model, so we omit the proof.

Lemma 1 There is an efficient, shared pattern of specialization in the model.

Moreover, under the definition, the following lemma can be proved straightforwardly, nevertheless it is necessary for starting the analysis.

Lemma 2 Consider sets of shared patterns of specialization in which the number of goods each country produces is the same. The number of sets is as follows:

$$\frac{(m-1)(m-2)\cdots(m-n+1)}{(n-1)!} .$$

[5] This definition is essentially same as Shiozawa (2007), but some way for writing is different.

For example, in the case of a three-country, four-good model, the number is 3. (If one country produces two kinds of goods, the set is determined, so the number of sets is the same as the number of countries.) If $n = m$, the set is only one, which is suited for the analysis by Jones (1961). The role for Lemma 2 is as follows: For the $m > n$ case, one can consider a set of specializations in which country i produces m_i kinds of good, where $\sum_{i=1}^{n} m_i = m$. The method below identifies the efficient specialization for a given set. Lemma 2 identifies the number of such distinct sets of specializations.

To complete the analysis, we consider each set of the shared-patterns of specialization. However, the method is similar. Moreover, the essence of the analysis in the n-country, m-good model can be shown only for the three-country, four-good model. Therefore, we will firstly focus only on the three-country, four-good model here.[6] Suppose the indexes of the countries are 0, 1, and 2, while the indexes of goods are 1, 2, 3, and 4.

Let L^i be labor endowment in the i^{th} country $(i = 0,1,2)$, and l_j^i be the amount of labor needed to produce one unit of the j^{th} good in the i^{th} country $(i = 0,1,2;\ j = 1,2,3,4)$. Consider the case where the 0$^{\text{th}}$ country produces the third and fourth goods, while the first country specializes in the first good and the second country specializes in the second good. To check whether the shared pattern of specialization is efficient, it is essential to divide the 0$^{\text{th}}$ country into regions.

First, we define the regions as follows: Let the first region be the same as the first country. Similarly, let the second region be the same as the second country. The third and fourth regions' technologies, i.e., labor input coefficients, are defined as that of the 0$^{\text{th}}$ country. The sum of labor for the third and fourth country is defined as that of the 0$^{\text{th}}$ country. Therefore,

$$\hat{l}_j^i \equiv l_j^i, \quad \hat{L}^i \equiv L^i \ (i = 1,2), \quad \hat{l}_j^3 = \hat{l}_j^4 \equiv l_j^0, \quad \hat{L}^3 + \hat{L}^4 = L^0,$$

are satisfied, where hats mean regions. We can consider this model as the four-region, four-good model. We can apply Jones' Inequality. We would like to claim that the world economy, where the i^{th} region specializes in the i^{th} good (that is, the $i-i$ assignment) for any $i = 1,2,3,4$ is efficient, in the strong meaning of the term, iff

$$\prod_i \hat{l}_i^i < \prod_i \hat{l}_{\sigma(i)}^i$$

is satisfied for any permutation σ. However, the third and fourth regions have the same technologies. Therefore, we must except not only the identity but also the permutation σ satisfying that $\sigma(1) = 1, \sigma(2) = 2, \sigma(4) = 3,$ $\sigma(3) = 4$. The new permutation excepted changes the pair within the same country (the 0$^{\text{th}}$ country) divided into regions from the $i-i$ assignment. Similarly, the world economy in the above model is efficient, in the weak meaning of the term, iff

$$\prod_i \hat{l}_i^i \le \prod_i \hat{l}_{\sigma(i)}^i$$

is satisfied for any permutation σ. Consider the shared pattern of specialization satisfying that each country

[6] The easiest model in this section is a two-country, three-good model, but in the two-country case, the answer is already known by Haberler (1936).

produces the same good that regions divided from each country specialize in. These inequalities can be the condition that the shared pattern of specialization is efficient in a strong or a weak meaning of the term. This method can be applied in the general, efficient, shared patterns of specialization in the n-country, m-good model ($n < m$). We can conclude the following claim.

Claim 1 To characterize any efficient shared patterns of specialization in the n-country, m-good model ($n < m$), Jones' Inequality can be applied by dividing some countries into regions.

Similarly to the case of three-country, four-good model, the way whether one shared patterns of specialization is efficient or not is as follows: In the n-country, m-good model ($n < m$), each good is produced by only one country, but each country produces m_i kinds of good, where $\sum_{i=1}^{n} m_i = m$. Firstly, divide each country to m_i regions virtually. Each region has the same technology of the country divided and the sum of each region's labor endowment is equal to that in the country divided. Secondly, apply Jones' inequality to the m-region, m-good model. Therefore, the generalization to Jones' inequality to the n-country, m-good case ($n < m$) was completed.

As Shiozawa (2007) focused on the case where the number of goods is larger than the number of countries, the case is real if one minutely subdivided each good. In the section the case is treated.

Next, we consider the case when the number of countries is larger than the number of goods produced.

4. The case when the number of countries is larger than the number of goods produced

In this section, we consider the case when the number of countries is larger than the number of goods produced. In the n-country, m-good model, we suppose $n > m$. In the model, the following concept of classes defined by Jones (1961) is useful.

Definition

1. A pattern of specialization is a world economy where each country specializes in one good, i.e., each country puts its labor into the production of that one good.
2. A class is a set of patterns of specialization in which each good is specialized in by the same number of countries.

Under the definition, the following lemma can be proved directly, but it is necessary for beginning the analysis.

Lemma 3 The number of classes producing every good is

$$\frac{(n-1)(n-2)\cdots(n-m+1)}{(m-1)!}.$$

For example, in the four-country, three-good model, which is the focus of Graham's (1948) example, the number is 3. Moreover, the following lemma is essentially proven by Jones (1961).

Lemma 4 The efficient pattern of specialization in each class is unique without the case of a tie.

In Lemma 4, the word "tie" is discussed by Jones (1961). Moreover, from Lemma 1 by Shiozawa (2007), the following lemma is also satisfied.

Lemma 5 In the case of $n = m$, there is an efficient pattern of specialization.

From now, we show the way to apply Jones' Inequality in the case where the number of countries is larger than the number of goods produced. In essence, only the four-country, three-good model, which is Graham's (1948) focus, can be shown, so we only consider that model firstly.

The method used is similar to those used in the other cases, so we only explain that, in this case, the first (second) country specializes in the first (second) good, and the third and fourth countries specialize in the 0^{th} good. (shown by *) The concept is to treat the commodities as different, if the country making the good is different, even if the goods are the same. The concept is shown on Table 1 and Table 2 as follows:

(Table 1) $\Longrightarrow$ (Table 2)

First, we define the commodity as follows. On one hand, each good specialized in by only one country is treated as the same commodity. In the case above, the first (second) commodity is defined as the first (second) good. The labor input coefficients of the commodities (shown by the hat) are defined as those for the same goods. In the example above, the input coefficients of the first and second commodities are the same as those for the first and second goods.

Second, each good specialized in by two or more countries is treated as a different commodity. In the case above, the third commodity is defined as the 0^{th} good produced by the third country. Similarly, the fourth commodity is defined as the 0^{th} good produced by the fourth country. The labor input coefficients (shown by the hat) are also defined as follows: In the case of the commodity specialized in by the country, the labor input coefficients are the same as the good. In the example above, the labor input coefficient of the third (fourth) commodity in the third (fourth) country is the same as the third (fourth) good in the third (fourth) country.

In the case of commodity specialized by another country, if the good specialized in is the same as the commodity specialized by the country in the model, the labor input coefficients (shown by the hat) are defined as infinity. In the example above, the labor input coefficient of the third (fourth) commodity in the fourth (third) country is infinity.

If the good specialized is different from the commodity the country specializes in, the labor input coefficients for the commodity (shown by the hat) are the same as that of the labor coefficients for the good. In the example above, the labor input coefficients for the third and fourth commodities in the first and second country are the same as those for the third and fourth goods in the same countries. Therefore, we can consider the fourth-country, fourth-commodity model in which Jones' Inequality can be applied. This method can be applied for any pattern of specialization in the general n-country, m-good model in which the number of countries is larger than the number of goods. We can conclude the following claim.

Claim 2 To characterize any efficient patterns of specialization in the n-country, m-good model ($n > m$), Jones' Inequality can be applied to treat the same good as a different commodity in the country where it is made.

Similarly to the case of four-country, three-good model, the way whether one patterns of specialization is efficient or not is as follows: In the n-country, m-good model ($n > m$), each country produces only one good, but each good is produced by n_i countries, where $\sum_{i=1}^{m} n_i = n$. Firstly, rename each good as n_i commodities virtually. If two or more countries specialize the same good, the commodities are treated as different by countries making the good. If country A and country B make the same good, consider that country A cannot make the country B's commodity virtually, and vice versa. In the case impossible, the labor input coefficient can be defined infinity. However, country C, which doesn't make the good, can make both country A's commodity and country B's commodity with the same technology of the good in country C. Secondly, apply Jones' inequality to the n-country, n-commodity model. Therefore, the generalization to Jones' inequality to the n-country, m-good case ($n < m$) was completed.

The importance of this method can be understood by reviewing Graham's (1948) example. Table 3 shows the example. The pattern of specialization shown by the * is actually efficient, but the pattern of specialization shown by the circle is a lower product of labor input coefficient than the pattern of specialization shown by the *.[7] In the case of *, see Table 4, the four-country, four-commodity model, the pattern of specialization shown by * is the lowest product of labor input coefficients, which implies that from a strict application of Jones' Inequality, the pattern of specialization shown by the * is efficient. However, in the case of the circle, see Table 5, the lowest product of labor input coefficients is not the pattern of specialization shown by the circle, but the pattern of specialization shown by the *, which implies that from a weak Jones' Inequality, the pattern of specialization shown by the circle is not efficient.

(Table 3) $\Longrightarrow$ (Tables 4 and 5)

5. Conclusion

In this paper, we applied Jones' Inequality to the model in which the number of countries is different from the number of goods produced. Jones' Inequality is the central result of the Ricardo–Graham model, but the inequality can be used only in the n-country, n-good model. In the case where the number of goods is larger than the number of countries, Jones' Inequality can be applied to the model when some countries are divided into regions. Conversely, in the case where the number of countries is larger than the number of goods produced, the method is to treat the same goods as different commodities, if the countries producing the goods are different even if the goods are actually the same. These cases are actually important because, generally, the number of goods produced may be different from the number of countries. This paper complements Jones' general application of the model.

As Jones (1961), Deardorff (2005a, 2005b), and Shiozawa (2007) argued, Ricardian comparative advantage with intermediate inputs has been analyzed for a long time, but the theory is not completed.[8] The paper's method has a potential in the case with intermediate inputs. At least in the case where the number of goods is larger than the number of countries as Shiozawa (2007) focused on, the method of this paper can be generalized in the model with intermediate goods straightforwardly. Therefore, because of researches like this paper, a future research of Ricardian comparative advantage with tradable intermediate goods can be focused only on the case with the same number of countries and goods.

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[7] Many previous works has been made in this area because the example is a little paradoxical. However, compared to Jones' (1961) inequality, the example is still paradoxical. Therefore, this paper is using Graham's example to illustrate the application of the method.

[8] For example, as Deardorff (2005a, 2005b), the correct definition of Ricardian comparative advantage has not been determined.

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good\country	1st	2nd	3rd	4th
1st	$*l^1_1$	l^2_1	l^3_1	l^4_1
2nd	l^1_2	$*l^2_2$	l^3_2	l^4_2
0th	l^1_0	l^2_0	$*l^3_0$	$*l^4_0$

Table 1

commodity\country	1st	2nd	3rd	4th
1st	$\hat{l}^1_1 \equiv l^1_1$	$\hat{l}^2_1 \equiv l^2_1$	$\hat{l}^3_1 \equiv l^3_1$	$\hat{l}^4_1 \equiv l^4_1$
2nd	$\hat{l}^1_2 \equiv l^1_2$	$\hat{l}^2_2 \equiv l^2_2$	$\hat{l}^3_2 \equiv l^3_2$	$\hat{l}^4_2 \equiv l^4_2$
3rd	$\hat{l}^1_3 \equiv l^1_0$	$\hat{l}^2_3 \equiv l^2_0$	$\hat{l}^3_3 \equiv l^3_0$	$\hat{l}^4_3 \equiv \infty$
4th	$\hat{l}^1_4 \equiv l^1_0$	$\hat{l}^2_4 \equiv l^2_0$	$\hat{l}^3_1 \equiv \infty$	$\hat{l}^4_4 \equiv l^4_0$

Table 2

good\country	1st	2nd	3rd	4th
1st	1/10	1/10*	1/10○	1/10
2nd	1/19	1/20○	1/15	1/28*
3rd	1/42*○	1/24	1/30*	1/40○

Table 3

commodity\country	1st	2nd	3rd	4th
1st	1/10	1/10	1/10*	1/10
2nd	1/19	1/20*	1/15	∞
4th	1/19	∞	1/15	1/28*
3rd	1/42*	1/24	1/30	1/40

Table 4

commodity\country	1st	2nd	3rd	4th
1st	1/10	1/10*	1/10○	1/10
2nd	1/19	1/20○	1/15	1/28*
3rd	1/42*○	1/24	∞	1/40
4th	∞	1/24	1/30*	1/40○

Table 5