Economics Bulletin

Volume 33, Issue 2

Sustainability is compatible with decreasing social welfare

Tanguy Isaac
Université catholique de Louvain

Abstract

This note studies the determination of optimal path - i.e. maximizing the intertemporal social welfare - under a sustainability constraint defined as a non-decreasing intertemporal social welfare accross time. We show that this definition suffers from an important drawback : the path obtained is not Pareto optimal. It is a drawback because nothing in our intuition of sustainable allocation justifies to give up a Pareto improvement. This implies also the possibility of false-negative result in the empirical studies aiming to determine if a country is on a sustainable path.

The author thanks Raouf Boucekkine and Arastou Khatibi for their useful help.
Citation: Tanguy Isaac, (2013) "Sustainability is compatible with decreasing social welfare", *Economics Bulletin*, Vol. 33 No. 2 pp. 1116-1125.
Contact: Tanguy Isaac - tanguy.isaac@uclouvain.be.
Submitted: November 12, 2012. **Published:** April 30, 2013.

1 Introduction

It is useless to emphasize the topicality of the intergenerational justice issue. The global warming is maybe the most exemplary case. Other examples are related to limits of natural resources. And many other cases can be cited.

Of course, for a long time, economists are interested in this issue. Huge achievements have been reached. Nevertheless the literature is also characterized by strong impossibility theorems. One of the first impossibility theorems was provided by Diamond (1965). The impossibility that we mention is usually an incompatibility between different axioms which are considered desirable.

Of course, economists are not alone to think to what could be intergenerational fairness. The temptation is large to try to import, in economics, concepts of justice conceived in others fields. For economics, it could be fruitful for two reasons. First, it could provide some new and fresh ways to think the problematic. The hope would be that a renewed approach could help to circumvent the usual problems. The second reason is that it may help resolving the usual incompatibility between desirable axioms. If some axioms are incompatible, an arbitrary choice has to be made between them. If a concept exterior to economics is largely accepted, it may help with the choice of the axioms which have to be favored. Hopefully, the dialog can benefit to the other fields. Economics can help to identify the qualities and the internal incoherences of a concept. In any case, it seems obvious that there exists a space for a profitable dialog between the economics and the alternative approaches.

Sustainable development is certainly one of those successful concepts related to intergenerational justice. The term was used by the Brundtland Commission which coined what has become its most often-quoted definition as development that *meets the needs of the present without compromising the ability of future generations to meet their own needs.*

Sustainable development has already been translated in economics. Several interpretations of sustainability are compatible with the definition quoted above [Pezzey (1992), Solow (1992), Heal (1998), Asheim (2003)]. We will follow the approach adopted by Arrow & al. (2004). One stream of consumption is considered sustainable if the intertemporal social welfare is non-decreasing with time. The intertemporal social welfare is defined as the present discounted value of the flow of utility from consumption from the present to infinity, discounted using a constant rate strictly positive. The goal of Arrow & al. (2004) is to assess empirically if *we are consuming too much*. But nothing was done in a policy making perspective. From this point of view, the criterion used by Arrow & al. (2004) is far from being able to be applied immediately. At least because the criterion is usually satisfied by numerous streams. Bonneuil and Boucekkine (2009) provide an analysis of the set of sustainable trajectories in a Ramsey model.

One possibility among others to manage this multiplicity is to select the path which maximizes the intertemporal social welfare among all the sustainable streams. Without the sustainability criterion, maximizing the intertemporal social welfare is a very common approach. It seems natural to maintain this objective once a sustainability constraint is added. This choice is in line with the proposition of Stavins & al. (2003) to define sustainable development. One appealing property of this objective is that the selected stream is Pareto optimal compared to all the others sustainable paths. Pareto optimality is certainly one property cherished by economists and there is no reason to reject it in the intergenerational justice context.

This article is a first step in the determination of optimal path - i.e. maximizing the intertemporal social welfare - under a sustainability constraint defined as a non-decreasing intertemporal social welfare accross time. This work is done in a Ramsey model as simple as possible.

The positive result is that the determination of this optimal path is not an heroic mathematical task. Of course, it is a desirable characteristic if we want to apply largely this approach to determine policies. Unfortunately, one finds that it happens that the optimal trajectory is not Pareto optimal when compared to all the possible paths. Nothing, in the Brundtland definition or in our intuition, could condemn a better situation for current generations without affecting negatively the future. So, one should maybe reconsider the definition of sustainable development as a path characterized by non-decreasing intertemporal social welfare.

The framework is presented in the next section. The third one provides the optimal path without the sustainability criterion. Section four is devoted to optimal path under sustainability constraint defined as in Arrow & al. (2004). Finally, the conclusion constitutes the fifth section.

2 Framework

The framework is the standard Ramsey model. A planner maximizes, over an infinite time horizon and continuous time, the present value of future utility gains $u(c(t))e^{-\rho^t}$ as a positive function of consumption c at time t and depending on a subjective discount rate ρ belonging to $(0, 1)$. The decision variable for the planner is the consumption $c(t)$ which has to be chosen among all the continuous function defined on $[0, +\infty]$. We denote by $V(t)$ the present value of future utility gains which is also called intertemporal social welfare in this article[1]. More formally,

$$V(t) \equiv \int_t^{+\infty} u(c(\tau))e^{-\rho(\tau-t)}d\tau \tag{1}$$

The novelty comes from the fact that a sustainability constraint is imposed. The sustainability constraint forbids a decrease of intertemporal social welfare, i.e. it requires $\dot{V}(t) \geq 0$. The program can be written

$$\begin{aligned}
&\max_{c(t)} V(0) && (2)\\
&\text{subject to} \\
&\dot{k}(t) = f(k(t)) - \delta k(t) - c(t) \\
&k(0) = k_0 \\
&k(t) \geq 0 \quad c(t) \geq 0 \\
&\dot{V}(t) \geq 0 && (3)
\end{aligned}$$

where δ is the depreciation rate of capital, $k(t)$ the capital per worker, $k_0 > 0$ is given. u is the utility function and f the production function. Both are twice continuously differentiable, strictly increasing, strictly concave and satisfy Inada conditions :

$$\lim_{k \to 0} f'(k) = \infty, \ \lim_{k \to \infty} f'(k) = 0.$$

[1]This notation can surprise readers accustomed with the dynamic programming literature where, for instance, V would also depend on $k(t)$. The choice between the different notations is a matter of taste. Here the notation chosen is the one used in Arrow & al. (2004) and common in the related literature.

Constraint (3) is the sustainability constraint. For the sake of simplicity, there is no population growth.

Thanks to capital depreciation and concavity of production function, we preclude the possibility to have an infinite capital accumulation. Hence, loosely speaking, $c(t)$ is also bounded. By consequence and due to the exponential term, $V(t)$ cannot take an infinite value. Finally, it implies that our program is well defined without needing additional technicalities due to the infinite horizon.

3 Optimal path without sustainability criterion

This section is a reminder about optimal trajectories of this model without any kind of sustainability criteria. More details can be found in Barro and Sala-i-Martin (2004, Chapter 2).

The Pontryagin method gives the velocities $\dot{k}(t)$ and $\dot{c}(t)$ on the optimal paths. In this problem, there is a unique saddle point, where $\dot{k}(t) = 0$, $\dot{c}(t) = 0$, which is (k^*, c^*) with k^* solution of $f'(k^*) - \delta = \rho$ and $c^* \equiv f(k^*) - \delta k^*$, independent of the initial condition $k_0 > 0$.

For any initial condition $k_0 > 0$, the level of consumption is chosen such that the system jumps on the saddle path and moves monotonically towards the stationary point (k^*, c^*). In other words, optimal trajectories remain on a stable branch and converge to the steady state equilibrium. The solution can be represented on a graph

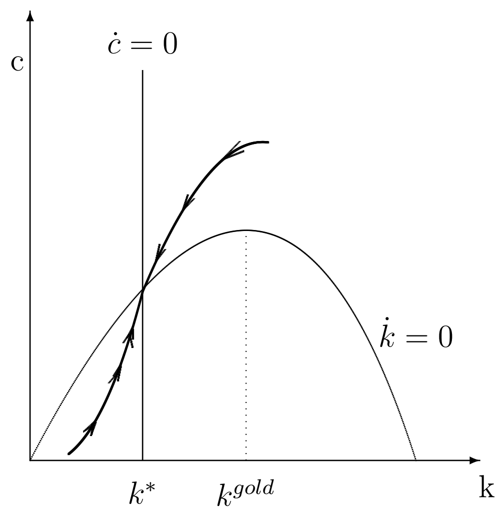

Figure 1: Optimal path without sustainability criterion

k^{gold} is the golden rule steady state level of $k(t)$. It is the solution of $f'(k^{gold}) - \delta = 0$.

Let us remark that, for $k > k^{gold}$, the marginal capital productivity, net of depreciation, is negative. Similarly, when k belongs to (k^*, k^{gold}), the marginal capital productivity, net of depreciation and discount rate, is negative. In other words, for $k \in (k^*, k^{gold})$, a sacrifice in present consumption, in order to increase capital stock, will never improve the intertemporal social welfare. Indeed, the increase in future consumption, allowed by the rise in capital, will not be sufficient to overcome the discount rate effect.

4 Optimal path under sustainability constraint

Theorem 1. *The optimal path under the sustainability constraint, defined as $\dot{V}(t) \geq 0$, is*

- $c(t) = \tilde{c}(t)$ *and* $k(t) = \tilde{k}(t)$ *for all* $k_0 \leq k^*$.
- $c(t) = f(k_0) - \delta k_0$ *for all* $k_0 \in [k^*, k^{gold}]$.
- $c(t) = c^{gold} = f(k^{gold}) - \delta k^{gold}$ *and* $k(t) = \hat{k}(t)$ *for all* $k_0 \geq k^{gold}$.

where $\tilde{c}(t)$ and $\tilde{k}(t)$ represent the optimal path without any kind of sustainability constraint while $\hat{k}(t)$ satisfies

$$\hat{k}(0) = k_0 \text{ and } \dot{\hat{k}}(t) = f(\hat{k}(t)) - \delta\hat{k}(t) - c^{gold} \tag{4}$$

Without an explicit expression for the production function, one cannot give a more precise formulation of $\hat{k}(t)$. Nevertheless, this path is obviously characterized by a decreasing capital level. It is in opposition with the relatively common idea that intertemporal welfare $V(t)$ is nondecreasing in t if and only if genuine investment is non-negative in t. We discuss this important feature and its consequences in the conclusion.

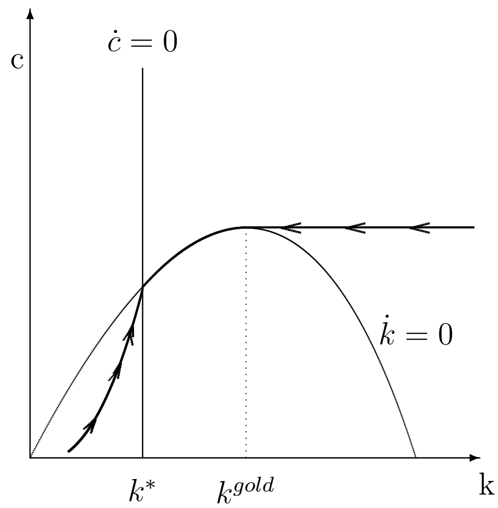

Figure 2: Optimal path under sustainability constraint

The part of the theorem concerning $k_0 \leq k^*$ is trivial. Indeed, if the solution of the problem without sustainability constraint satisfies $\dot{V}(t) \geq 0$ then it is of course a solution of the constrained problem.

The cases where $k_0 > k^*$ are a little bit more complicate. The proof will be in three steps. First, we present necessary conditions for the optimality. Then, we observe that the proposed solution fulfills those conditions. Finally, we show that this path is optimal by comparing it to other admissible paths. This final step will be divided in two : we first consider the paths remaining in the same regime (binding or non-binding with respect to the sustainability constraint), and then, devote our attention to the trajectories switching from one regime to the other.

4.1 Necessary conditions

To write those conditions, we will treat $V(t)$ as a state variable. From the definition, one can compute:

$$\dot{V}(t) = -u(c(t)) + \rho V(t) \tag{5}$$

We know neither the initial value of V nor its final value. Nonetheless, $V(t)$ must satisfy the sustainability constraint. One can then write an equivalent optimization program

$$\max_{c(t)} \int_0^\infty u(c(t))e^{-\rho t}dt \tag{6}$$

subject to

$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t)$$
$$\dot{V}(t) = -u(c(t)) + \rho V(t)$$
$$\rho V(t) - u(c(t)) \geq 0$$
$$c(t) \geq 0 \qquad k(t) \geq 0 \qquad k(0) = k_0 \tag{7}$$

The Hamiltonian is given by

$$\begin{aligned} J &= u(c(t))e^{-\rho t} + \lambda(t)[f(k(t)) - \delta k(t) - c(t)] \\ &+ \gamma(t)[-u(c(t)) + \rho V(t)] + \theta(t)[\rho V(t) - u(c(t))] \end{aligned} \tag{8}$$

Not to mention the transversality conditions, the maximum-principle conditions[2] are

$$\begin{aligned} \frac{\partial J}{\partial c} &= 0 \\ \dot{k} &= \frac{\partial J}{\partial \lambda} \\ \dot{V} &= \frac{\partial J}{\partial \gamma} \\ \dot{\lambda} &= -\frac{\partial J}{\partial k} \\ \dot{\gamma} &= -\frac{\partial J}{\partial V} \\ \theta\frac{\partial J}{\partial \theta} &= 0 \qquad \theta \geq 0 \quad \frac{\partial J}{\partial \theta} \geq 0 \end{aligned}$$

Or after few computations

$$u'(c(t))[e^{-\rho t} - \gamma(t) - \theta(t)] - \lambda(t) = 0 \tag{9}$$
$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t) \tag{10}$$
$$\dot{V}(t) = -u(c(t)) + \rho V(t) \tag{11}$$
$$\dot{\lambda}(t) = -\lambda(t)[f'(k(t)) - \delta] \tag{12}$$
$$\dot{\gamma}(t) = -\rho[\gamma(t) + \theta(t)] \tag{13}$$
$$\theta(t)[\rho V(t) - u(c(t))] = 0 \qquad \theta(t) \geq 0 \quad [\rho V(t) - u(c(t))] \geq 0$$

[2] An introduction to the maximum principle can be found in Barro and Sala-i-Martin (2004) or, in a more formal way, in Chiang (1999).

The transversality conditions are

$$\lim_{t \to \infty} \gamma(t) = 0 \tag{14}$$

$$\lim_{t \to \infty} \lambda(t)k(t) = 0 \tag{15}$$

Basic computations are sufficient to check that the solution proposed in the theorem fulfills all those conditions (with $\lambda(t) = \theta(t) = 0$ and $\gamma(t) = e^{-\rho t}$). Let us remark that those conditions are necessary only for an interior solution - i.e. with $c(t) > 0$ and $k(t) > 0$ - which is clearly our case.

4.2 Paths in permanent regime

A permanently non-binding solution would be the one which does not take into account the sustainability constraint. This solution is valid only for $k_0 \leq k^*$. Indeed, $c(t)$ is always decreasing for $k_0 > k^*$ and thus $V(t)$ also. By consequence, the constraint $\dot{V}(t) \geq 0$ is not satisfied.

In the case of a permanently binding regime - i.e. $\rho V(t) - u(c(t)) = 0$ and $\theta(t) \geq 0$ - one observes that $\dot{V}(t) = 0$ and $V(t) = \frac{u(c(t))}{\rho}$. Thus, $V(t)$ and $c(t)$ are constant. Let us imagine that consumption takes a level $\tilde{c}$ different from the one defined in the theorem. If $\tilde{c}$ is larger then the capital decreases infinitely (see equation (10)) and the condition $k(t) \geq 0$ cannot be satisfied for ever. Now, if $\tilde{c}$ is lower then $V(t)$ cannot be maximized since it is smaller than the level reached with the solution proposed in the theorem.

4.3 Paths with regime switch

Now, we turn to paths which are not permanently in the same regime. We begin with the possibility to start in a binding regime and then to switch to a permanently non-binding one. Our interest is still limited to cases characterized by $k_0 > k^*$. The solution described in the theorem implies $V(t) = \frac{u(f(k_0)-\delta k_0)}{\rho} \equiv V_0$ for $k_0 \leq k^{gold}$ and $V(t) = \frac{u(f(k^{gold})-\delta k^{gold})}{\rho} \equiv V^{gold}$ for $k_0 \geq k^{gold}$. Due to the discussion in previous section, we know that an optimal trajectory ending in a permanently non-binding regime is characterized by $\lim_{t\to\infty} V(t) = \frac{u(f(k^*)-\delta k^*)}{\rho} \equiv V^*$. Since $V(t)$ cannot decrease we know that, for a trajectory switching from a binding regime to a permanently non-binding one, $V(t) \leq V^*$. Obviously, $V^* < V_0$ and $V^* < V^{gold}$. So, a switch from a binding regime to a permanently non-binding one cannot be optimal.

Let us now consider the possibility of a trajectory switching from a non-binding regime to a permanently binding one. Among the permanently binding trajectory, $V(t)$ cannot take an higher value than $V^{gold} = \frac{u(c^{gold})}{\rho}$. The constraint $\dot{V}(t) \geq 0$ requires $V(t) \leq V^{gold} \forall t$. Now, notice that for $k_0 \geq k^{gold}$, the path characterized by $c(t) = c^{gold}$ is such that $\forall t\ V(t) = V^{gold}$. Hence, one cannot do better. By consequence, when $k_0 > k^{gold}$, a path which switches from a non-binding regime to a permanently binding one cannot be better than $c(t) = c^{gold}$.

It remains to consider situations with $k_0 \in (k^*, k^{gold})$. Compared to the solution proposed in the theorem, an alternative optimal path is characterized either by a lower or an higher level of today consumption. In the first case, present consumption is sacrificed to the benefit of future consumption. Nevertheless, as noted in section 3, it cannot be optimal since the marginal capital productivity net of depreciation rate and discount rate is negative. In the second case, due to the higher current consumption, the stock of capital decreases and there exists a T such that $\forall t \geq T$, consumption is lower than $f(k_0) - \delta k_0$. Which means that, at time T, the intertemporal social welfare is lower than $V_0 \equiv \frac{u(f(k_0)-\delta k_0)}{\rho}$. Due to the sustainability constraint, the intertemporal social welfare is also lower than V_0 at time zero. So, it cannot be optimal that consumption at time zero is larger than the one proposed in the theorem.

5 Conclusion

The main contribution of this work is to provide the solution to the maximization of intertemporal social welfare $V(t)$ under the sustainability constraint defined as $\dot{V}(t) \geq 0$.

Clearly, one gets an optimal path under sustainability criterion sometimes characterized by a decreasing capital level. It is in opposition with what is usually assumed. Indeed, according to Arrow & al. (2004), *Clearly, intertemporal welfare $V(t)$ is nondecreasing at t if and only if genuine investment is non-negative at t.* This kind of equivalence is proven by Dasgupta and Mäler (2000). What is wrong? Actually, in Dasgupta and Mäler (2000) there is no capital depreciation. If it was the case in our Ramsey model - i.e. $\delta = 0$ - then k^{gold} would take ∞ as value. There would be no room for a sustainable path with decreasing capital level. The absence of capital depreciation precludes the possibility of an (initial) excessive capital level.

It raises two questions. First, what is the depreciation of the genuine capital? Is it obvious that such a depreciation cannot exist? The second issue is to know if it is reasonable to believe that a country (or the world) could have a stock of capital greater than k^*. A first intuition would be that no optimal behaviour can lead to a capital overaccumulation. This idea seems quite reasonable but does not take into account unexpected shocks. Let us consider a country which possesses large amount of natural resources currently considered as useless. Clearly, those natural resources will not be taken into account by the country to determine its optimal behaviour. Now, let us imagine that due to a discovery, those natural resources become very valuable. Suddenly, this country has a stock of genuine capital quite larger than what was previously thought. Is it really foolish to believe that this country can own a too high capital level, i.e. greater than k^{gold} ?

So, our contribution invite to a reconsideration of the use of genuine capital as a perfect proxy of intertemporal social welfare. Unfortunately, it is not the only bad new revealed by our work. For $k_0 > k^{gold}$, there are numerous feasible paths which Pareto dominate the solution found. Let us consider the path drawn here.

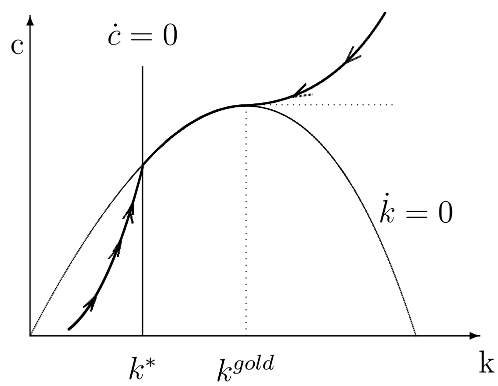

Figure 3: Pareto improvement

We are easily convinced that the path is feasible. Moreover, this path is characterized by a level of consumption larger for all t to the one determined by our theorem when $k_0 > k^{gold}$. Clearly, nothing in the Brundtland's definition or in our intuition condemns a better situation for current generations without negatively affecting the future. There is thus no reason to not exploit the possible Pareto improvements. Hence, the constraint $\dot{V}(t) \geq 0$ appears excessive to express the sustainability constraint.

The intuition which explains why $\dot{V}(t) \geq 0$ is not a good translation of the Brundlandt's definition has two components. First, if it is possible to increase the consumption of current generations without affecting the future ones, then it cannot be argued that it compromises the ability of future generations to meet their own needs. Hence, this kind of Pareto improvement cannot be considered as opposed to the sustainable development. Second, in the case where the optimality requires a reduction of the capital level, it is better to *over*consume rather than waiting that capital depreciates. Indeed, *over*consumption benefits to first generations while no one enjoys the capital depreciation.

Notice that this kind of drawback would also affect the criteria $\dot{c} \geq 0$ and $\dot{k} \geq 0$ which are sometimes used as sustainability criterion. The key feature is the presence of capital depreciation which opens the door to the possibility of an excessive capital level.

The criterion $\dot{V}(t) \geq 0$ has to be modified or abandoned. In the line of the approach adopted in this article, it is natural to take the optimal trajectory found in the previous section as a fall-back position. So, to avoid the excessiveness of $\dot{V}(t) \geq 0$, it can be proposed to operate a second step of maximization under the constraint $V(t) \geq \tilde{V}(t)$ with $\tilde{V}(t)$ which represents the intertemporal social welfare associated to the the optimal path with the constraint $\dot{V}(t) \geq 0$.

From an empirical point of view, the appealing of $\dot{V}(t) \geq 0$ is seriously damaged. We have discovered that this criterion is too demanding, so one can get false-negative result. Moreover, the modification proposed of the criterion loses the simplicity of the original formulation which could create additional difficulties for the empiric applications.

References

Arrow, K., Dasgupta, P., Goulder, L., Daily, G. & al. (2004) "Are we consuming too much?" *Journal of Economic Perspectives* **18**, 147-172.

Asheim, G. (2003) "Green National Accounts for Welfare and Sustainability: A Taxonomy of Assumptions and Results" *Scottish Journal of Political Economy* **50**, 113-30.

Barro, R. and X. Sala-i-Martin (2004) *Economic Growth*, MIT Press: Cambridge.

Bonneuil, N. and R. Boucekkine (2009) "Sustainability, optimality, and viability in the Ramsey model" CORE discussion paper number 2009/74.

Chiang, A. (1999) *Elements of Dynamic Optimization*, Waveland Press: Prospect Heights.

Dasgupta, P. and K.-G. Maler (2000) "Net National Product, Wealth, and Social Well-Being" *Environment and Development Economics* **5**, 69-93.

Diamond, P. (1965) "The Evaluation of Infinite Utility Streams" *Econometrica* **33**, 170-177.

Heal, G. M. (1998) *Valuing the Future: Economic Theory and Sustainability*, Columbia University Press: New York.

Pezzey, J. (1992) "Sustainable Development Concepts: An Economic Analysis" World Bank Environment Paper number 2.

Solow, R. M. (1992) "An Almost Practical Step toward Sustainability" in *Resources for the Future 40th Anniversary Lecture*, Resources for the Future: Washington, D.C.

Stavins, R.N., Wagner, A.F. and G. Wagner (2003) "Interpreting sustainability in economic terms: dynamic efficiency plus intergenerational equity" *Economics Letters* **79**, 339-343.