Economics Bulletin

Volume 33, Issue 2

An axiomatic characterization of the strong constrained egalitarian solution

Francesc Llerena
Universitat Rovira i Virgili

Cori Vilella
Universitat Rovira i Virgili

Abstract

In this note we axiomatize the strong constrained egalitarian solution (Dutta and Ray, 1991) over the class of weak superadditive games using constrained egalitarianism, order-consistency, and converse order-consistency.

We want to thank Antonio Quesada, Carles Rafels, William Thomson, the associate editor Timothy Van Zandt and the anonymous referees for helpful comments and suggestions that improve this work. Financial support from research grants ECO2011-22765 (Ministerio de Ciencia e Innovación) and 2009SGR900 (Generalitat de Catalunya) is gratefully acknowledged.

Citation: Francesc Llerena and Cori Vilella, (2013) "An axiomatic characterization of the strong constrained egalitarian solution", *Economics Bulletin*, Vol. 33 No. 2 pp. 1438-1445.

Contact: Francesc Llerena - francisco.llerena@urv.cat, Cori Vilella - cori.vilella@urv.cat.

Submitted: September 28, 2012. **Published:** June 14, 2013.

1. **Introduction**

On the space of transferable utility cooperative games, several solution concepts have been motivated by the idea of egalitarianism. One of the best known is the **weak constrained egalitarian solution** (WCES for short), introduced by Dutta and Ray (1989). This solution is defined under the assumption that agents believe in egalitarianism as a social value, but their individual preferences dictate selfish behavior. On the domain of convex games, the WCES has many desirable properties and it has been axiomatized by different authors (see Dutta, 1990; Klijn et al., 2000; Hougaard et al., 2001; Arin et al., 2003). Nevertheless, existence is only guaranteed for the class of convex games. In order to widen the potential class of applications, Dutta and Ray (1991) introduced the **strong constrained egalitarian solution** (SCES for short), a parallel concept that exists for a large class of games. However, there is no a proper characterization result for the SCES. Thus, it seems a worthwhile exercise to provide an axiomatic characterization for this solution concept. With this objective in mind, we begin introducing some notation and terminology. In Section 3 we present the axiomatic result.

2. **Notation and terminology**

The set of natural numbers $\mathbb{N}$ denotes the universe of potential players. By $N \subset \mathbb{N}$ we denote a finite set of players, in general $N = \{1, \ldots, n\}$. A **transferable utility coalitional game (a game)** is a pair (N, v) where $v : 2^N \longrightarrow \mathbb{R}$ is the characteristic function with $v(\emptyset) = 0$ and 2^N denotes the set of all subsets (coalitions) of N. Here we only consider games with $|N| \geq 2$. Let Γ denote the set of all games. We use $S \subset T$ to indicate strict inclusion, that is $S \subseteq T$ but $S \neq T$. By $|S|$ we denote the cardinality of the coalition $S \subseteq N$. A **subgame** of (N, v) is a game (T, v^T) where $\emptyset \neq T \subset N$ and $v^T(S) = v(S)$ for all $S \subseteq T$. The subgame (T, v^T) will also be denoted by (T, v).

The set of **feasible payoff vectors** of a game (N, v) is defined by $X^*(N, v) := \{x \in \mathbb{R}^N \,|\, x(N) \leq v(N)\}$. A **solution** on a class of games $\Gamma' \subseteq \Gamma$, is a mapping σ which associates with each game $(N, v) \in \Gamma'$ a subset $\sigma(N, v)$ of $X^*(N, v)$. Notice that $\sigma(N, v)$ is allowed to be empty. The **pre-imputation set** of a game (N, v) is defined by $X(N, v) := \{x \in \mathbb{R}^N \,|\, x(N) = v(N)\}$, and the set of **imputations** by $I(N, v) := \{x \in X(N, v) \,|\, x(i) \geq v(\{i\}),$ for all $i \in N\}$. The **core** of (N, v) is defined by $C(N, v) = \{x \in X(N, v) \mid x(S) \geq v(S)$ for all $S \subseteq N\}$. A game (N, v) is **convex** (Shapley, 1971) if, for every $S, T \subseteq N$, $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$. We denote by Γ_{vex} the class of all convex games. A game (N, v) is **weakly superadditive** if for all partition, $\{S_1, \ldots, S_m\}$ of N, $v(S_1) + \cdots + v(S_m) \leq v(N)$. We denote by Γ_{ws} the class of all weak superadditive games. Notice that $\Gamma_{vex} \subseteq \Gamma_{ws}$.

Let $\mathbb{R}^N$ stand for the space of real-valued vectors indexed by N, $x = (x_i)_{i \in N}$, and for all $S \subseteq N$, $x(S) = \sum_{i \in S} x_i$, with the convention $x(\emptyset) = 0$. For each $x \in \mathbb{R}^N$ and $T \subseteq N$, x_T denotes the restriction of x to T: $x_T = (x_i)_{i \in T} \in \mathbb{R}^T$. Given two vectors $x, y \in \mathbb{R}^N$, $x \geq y$ if $x_i \geq y_i$ for all $i \in N$. We say that $x > y$ if $x \geq y$ and for some $j \in N$, $x_j > y_j$. Moreover, $x \gg y$ if $x_i > y_i$ for all $i \in N$. For any $x \in \mathbb{R}^N$, denote by $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$

the vector obtained from x by rearranging its coordinates in a non-decreasing order, that is, $\hat{x}_1 \leq \hat{x}_2 \leq \ldots \leq \hat{x}_n$. For any two vectors $y, x \in \mathbb{R}^N$ with $y(N) = x(N)$, we say that y **Lorenz dominates** x, denoted by $y \succ_L x$, if $\sum_{j=1}^{k} \hat{y}_j \geq \sum_{j=1}^{k} \hat{x}_j$, for all $k \in \{1, \ldots, |N|\}$, with at least one strict inequality. Given a coalition $\emptyset \neq S \subseteq N$ and a set $A \subseteq \mathbb{R}^S$, EA denotes the set of allocations that are Lorenz undominated within A. That is, $EA := \{x \in A \mid \text{there is no } y \in A \text{ such that } y \succ_L x\}$. Given a game (N, v), the **strong Lorenz core** (Dutta and Ray, 1989) is defined in a recursive way as follows: the strong Lorenz core of a singleton coalition is $L^*(\{i\}, v) = \{v(\{i\})\}$. Now suppose that the strong Lorenz core for all coalitions of cardinality k or less have been defined, where $1 < k < |N|$. The strong Lorenz core of a coalition $S \subset \mathbb{N}$ of size $(k+1)$ is defined by $L^*(S, v) = \{x \in \mathbb{R}^S \mid x(S) = v(S), \text{ and there is no } T \subset S \text{ and } y \in EL^*(T, v) \text{ such that } y \gg x_T\}$. The **weak Lorenz core** is defined similarly, but replacing $\gg$ by $>$. The **strong constrained egalitarian** solution (Dutta and Ray, 1991), $EL^*(N, v)$, selects the Lorenz undominated allocations within the strong Lorenz core. The **weak constrained egalitarian** solution (Dutta and Ray, 1989), denoted by **DR**, selects the Lorenz undominated allocations within the weak Lorenz core. For $(N, v) \in \Gamma$, $|DR(N, v)| \leq 1$.

3. An axiomatic characterization of the strong constrained egalitarian solution

In this section, we provide an axiomatization of the SCES based on the **consistency** principle and its converse, together with **constrained egalitarianism** (Dutta, 1990), a prescriptive property which fix the solution for two person games.

Before introducing consistency we need to define the concept of a reduced game. The terminology is taken from Thomson (2006a).

Definition 1. Let $(N, v) \in \Gamma$, $y \in \mathbb{R}^N$ and $\emptyset \neq T \subset N$. The **max reduced game** (Davis and Maschler, 1965) **relative to** T **at** y is the game $\left(T, r^T_{y,DM}(v)\right)$ defined by

$$r^T_{y,DM}(v)(S) := \begin{cases} 0 & \text{if } S = \emptyset, \\ \max\limits_{Q \subseteq N \setminus T} \{v(S \cup Q) - y(Q)\} & \text{if } S \subset T, \\ v(N) - y(N \setminus T) & \text{if } S = T. \end{cases}$$

For an interpretation of the max reduced game see Peleg (1986).

Roughly speaking, consistency says that there is no difference in what the players of the reduced game will get in both the original game and in the reduced game. The dual of consistency is named **converse consistency**. This property states that if an efficient payoff vector is accepted for every pair of players, then it is accepted for the set of all players.[1] Let us now introduce formally these properties.

Let $(\{i, j\}, v)$ be a 2-person weak-superadditive game. Without loss of generality, assume $v(\{i\}) \leq v(\{j\})$. The **constrained egalitarian solution** of the game $(\{i, j\}, v)$,

[1]See Thomson (2006a) for a survey on consistency and its converse

denoted by $CE(\{i,j\},v)$, is defined as follows: $CE_j(\{i,j\},v) := \max\left\{\frac{v(\{i,j\})}{2}, v(\{j\})\right\}$, and $CE_i(\{i,j\},v) := v(\{i,j\}) - CE_j(\{i,j\},v)$.

A solution σ on Γ_{ws} satisfies

- **constrained egalitarianism** if it coincides with the constrained egalitarian solution for all 2-person weak-superadditive games.

A solution σ on Γ' satisfies

- **max consistency** if for each $(N,v) \in \Gamma'$, each $\emptyset \neq T \subset N$, and each $y \in \sigma(N,v)$, then $\left(T, r^T_{y,DM}(v)\right) \in \Gamma'$ and $y_T \in \sigma\left(T, r^T_{y,DM}(v)\right)$.
- **converse max consistency** if for each $(N,v) \in \Gamma'$, $y \in X(N,v)$, and for each $T = \{i,j\} \subseteq N$, $\left(T, r^T_{y,DM}(v)\right) \in \Gamma'$ and $y_T \in \sigma\left(T, r^T_{y,DM}(v)\right)$, then $y \in \sigma(N,v)$.

These properties have been widely studied and appear in axiomatizations of several solution concepts. For instance, they appear among other properties in Peleg's (1986) characterization of the core. On the domain of convex games, Dutta (1990) characterizes the WCES by means of constrained egalitarianism and max consistency. The next example shows that the SCES is not max consistent.

Example 1. Dutta and Ray (1991)
Let $(N,v) \in \Gamma$ where $N = \{1,2,3\}$, $v(S) = v(N) = 1$ if $|S| = 2$, and $v(S) = 0$ if $|S| = 1$. Here $EL^*(N,v) = \{(0.5,0.5,0),(0.5,0,0.5),(0,0.5,0.5)\}$ (Dutta and Ray, 1991). Consider the max reduced game $\left(\{1,2\}, r^{\{1,2\}}_{x,DM}(v)\right)$, where $x = (0.5,0.5,0)$. As the reader can easy check, $I\left(\{1,2\}, r^{\{1,2\}}_{x,DM}(v)\right) = \emptyset$ thus $EL^*\left(\{1,2\}, r^{\{1,2\}}_{x,DM}(v)\right) = \emptyset$.

In fact, the above example shows that there are no max consistent solutions if we assume that solutions only assign imputations to games, and are non-empty valued. [2] Indeed, let $y = (y_1,y_2,y_3) \in I(N,v)$. Then, $y_i \geq 0$ for all $i \in N$ and $y(N) = 1$. Take $T = \{1,2\}$ and the max reduced game $\left(T, r^T_{y,DM}(v)\right)$. Write $r = r^T_{y,DM}(v)$. A simple computation, taking into account that $0 \leq y_3 \leq 1$, shows that $r(\{1\}) = r(\{2\}) = r(\{1,2\}) = 1 - y_3$. Therefore, $\left(T, r^T_{y,DM}(v)\right)$ is weakly superadditive precisely when $2 - 2y_3 \leq 1 - y_3$, which implies $y_3 \geq 1$ and thus $y_3 = 1$. In the same way we get $y_1 = y_2 = 1$, which is a contradiction.

The conclusion is that for the game (N,v), being itself a weakly superadditive game, there does not exist any imputation y such that all max reduced games with respect to y are also weakly superadditive. Therefore, no solution will be max consistent simply because of the resulting max reduced games are not always in the class of weakly superadditive games. We may try to overcome this drawback with the following slight modification of the max consistency notion.

A solution σ on Γ' is **conditional max consistent** if for each $(N,v) \in \Gamma'$, each $\emptyset \neq T \subset N$ and each $y \in \sigma(N,v)$, if $\left(T, r^T_{y,DM}(v)\right) \in \Gamma'$ then $y_T \in \sigma\left(T, r^T_{y,DM}(v)\right)$.

[2] We thank an antonymous referee for enlighten us with this fact.

The advantage of this approach is that not all games of the form $\left(T, r^T_{y,\,DM}(v)\right)$ need to be in $\Gamma^{'}$. However, as shown in the following example, the SCES is not conditional max consistent.

Example 2. Dutta and Ray (1991)
Let $(N, v) \in \Gamma$ *where* $N = \{1, 2, 3, 4\}$, $v(\{1\}) = 23$, $v(\{2\}) = 40$, $v(\{3\}) = 39$, $v(\{1, 2\}) = 100$, $v(\{1, 3\}) = 82$ $v(\{2, 3\}) = 79$, $v(\{1, 2, 3\}) = 159$, $v(N) = 167$ *and for all* $S \neq \{1, 2, 3\}$, $v(S \cup \{4\}) = v(S)$. *Take* $y = (53, 40, 39, 35) \in EL^*(N, v)$ *and consider the max reduced game* $\left(\{1, 3, 4\},\ r^{\{1,3,4\}}_{y,DM}(v)\right)$. *If we write* $r = r^{\{1,3,4\}}_{y,DM}(v)$, *it is easy to check that* $r(\{1\}) = 60$, $r(\{3\}) = 39$, $r(\{4\}) = 0$, $r(\{1, 3\}) = 119$, $r(\{1, 4\}) = 60$, $r(\{3, 4\}) = 39$, $r(\{1, 3, 4\}) = 127$ *and it is a weakly superadditive game. The unique SCES of this max reduced game is* $(60, 39, 28)$, *which is different from* y_T.

Hence, to characterize the SCES by means of consistency we introduce a different notion of reduced game. To this end, we need additional definitions.

An ordering $\theta = (i_1, i_2, \ldots, i_n)$ of N where $|N| = n$, is a bijection from $\{1, 2, \ldots, n\}$ to N. We denote by Θ_N the set of all orderings of N.

Definition 2. Given $(N, v) \in \Gamma$ and $\theta = (i_1, \ldots, i_n) \in \Theta_N$, let $x^\theta \in \mathbb{R}^N$ be defined as follows:

$$x^\theta_{i_k} := \max_{S \in P_{i_k}} \left\{ \frac{v(S)}{|S|} \right\}, \text{ for } k = 1, \ldots, n, \tag{1}$$

where $P_{i_1} := \{S \subseteq N \mid i_1 \in S\}$ and $P_{i_k} := \{S \subseteq N \mid i_1, \ldots, i_{k-1} \notin S,\ i_k \in S\}$, for $k = 2, \ldots, n$.

Observe that in the construction of x^θ underlines the principle of equal division. As Selten (1972) showed by a great number of experimental games, this principle is a strong distributive norm which influences the behavior of players. Thus, we can consider x^θ as a vector of "natural" claims that players can require in a sequential way.

Given $(N, v) \in \Gamma$ and $\theta \in \Theta_N$, let

$$\Delta^\theta(v) := \{y \in X(N, v) \,\text{such that}\, y \geq x^\theta\},$$

and

$$\Theta^{\preceq}_N(v) := \{\theta^{'} \in \Theta_N \,|\, \text{there is not}\, \theta \in \Theta_N \,\text{such that}\, \theta \succ_L \theta^{'}\},$$

where $\theta \succ_L \theta^{'}$ means that there is $x \in \Delta^\theta(v)$ such that $x \succ_L y$, for all $y \in \Delta^{\theta^{'}}(v)$.

Definition 3. Let $(N, v) \in \Gamma$, $\emptyset \neq T \subset N$, $\theta = (i_1, i_2, \ldots, i_n) \in \Theta^{\preceq}_N(v)$ and $y \in \mathbb{R}^N$. The **order-reduced game on T at** y is the game $\left(T, r^T_{y,\theta}(v)\right)$, where

$$r^T_{y,\theta}(v)(S) := \begin{cases} 0 & \text{if } S = \emptyset \\ \max\limits_{Q \subseteq N \setminus T} \left\{x^\theta(S \cup Q) - y(Q)\right\} & \text{if } S \subset T, \\ v(N) - y(N \setminus T) & \text{if } S = T. \end{cases}$$

In the definition of the order-reduced game, we suppose that the worth of a non-empty coalition $S \subset T$ is revaluated under the assumption that the members of S can choose the best partners in $N \setminus T$ in order to maximize the "natural" requirements given by x^θ, provided that it pays them their components of y. This is as in the Davis and Maschler (1965) definition. Assuming that all the members of N agree that the members of $N \setminus T$ will get $y_{N \setminus T}$, the members of T may get $v(N) - y(N \setminus T)$.

A solution σ on Γ_{ws} satisfies

- **order-consistency** if for each $(N, v) \in \Gamma_{ws}$ and each $y \in \sigma(N, v)$, there is $\theta \in \Theta_N^{\preceq}(v)$ such that, for all $\emptyset \neq T \subset N$, $\left(T, r_{y,\theta}^T(v)\right) \in \Gamma_{ws}$ and $y_T \in \sigma\left(T, r_{y,\theta}^T(v)\right)$.
- **converse order-consistency** if for each $(N, v) \in \Gamma_{ws}$ and $y \in X(N, v)$, each $\theta \in \Theta_N^{\preceq}(v)$ and for each $T = \{i, j\} \subseteq N$, $\left(T, r_{y,\theta}^T(v)\right) \in \Gamma_{ws}$ and $y_T \in \sigma\left(T, r_{y,\theta}^T(v)\right)$, then $y \in \sigma(N, v)$.

Now we characterize the SCES.

Theorem 3.1. *On the domain of weak superadditive games, the SCES is the only solution satisfying constrained egalitarianism, order-consistency, and converse order-consistency.*

Proof: Constrained egalitarianism follows directly from the fact that for 2-person games both the strong and the weak constrained egalitarian solution coincide. To prove the remaining axioms, we need a geometrical decomposition of the strong Lorenz core. Let $(N, v) \in \Gamma$, we claim that

$$L^*(N, v) = \bigcup_{\theta \in \Theta_N} \Delta^\theta(v). \tag{2}$$

Indeed, let $y \in L^*(N, v)$ and $S_1 \in \arg\max_{S \subseteq N} \left\{ \frac{v(S)}{|S|} \right\}$. Since $y \in L^*(N, v)$, there is $i_1 \in S_1$ such that $y_{i_1} \geq \frac{v(S_1)}{|S_1|}$. Now let $S_2 \in \arg\max_{S \subseteq N \setminus \{i_1\}} \left\{ \frac{v(S)}{|S|} \right\}$ and $i_2 \in S_2$ such that $y_{i_2} \geq \frac{v(S_2)}{|S_2|}$. Following this process step by step, we can generate an order $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N$. Let $x^\theta \in \mathbb{R}^N$ as given in Definition 2. Since $y \geq x^\theta$ and $y(N) = v(N)$, we have that $y \in \Delta^\theta(v)$. To show the reverse inclusion, let $\theta \in \Theta_N$ and $y \in \Delta^\theta(v)$. Let $S \subset N$ be a non-empty coalition and $i_k \in S$ the first player in S with respect to θ. Then, $y_{i_k} \geq x_{i_k}^\theta \geq \frac{v(S)}{|S|}$ and so $y \in L^*(N, v)$.

Next we prove *order-consistency*. Let $(N, v) \in \Gamma_{ws}$ and $y \in EL^*(N, v)$. From expression (2) we know that there is $\theta = (i_1, i_2, \ldots, i_n) \in \Theta_N$ such that $y \in \Delta^\theta(v)$. Suppose that $\theta \notin \Theta_N^{\preceq}(v)$. Then, there are $\theta' \in \Theta_N$ and $y' \in \Delta^{\theta'}(v)$ such that $y' \succ_L y$, a contradiction because, from (2), $\Delta^{\theta'}(v) \subseteq L^*(N, v)$. Hence, $\theta \in \Theta_N^{\preceq}(v)$. Let $\emptyset \neq T \subset N$, and consider the order-reduced game $\left(T, r_{y,\theta}^T(v)\right)$. Now define the game (N, v_θ) as follows:

$$v_\theta(R) := \begin{cases} x^\theta(R) & \text{if } R \neq N, \\ v(N) & \text{if } R = N. \end{cases} \tag{3}$$

Since $x^\theta(N) \leq v(N)$, we have $(N, v_\theta) \in \Gamma_{vex}$. Consider the max reduced game $\left(T, r^T_{y,\,DM}(v_\theta)\right)$. Notice that

$$\left(T, r^T_{y,\theta}(v)\right) = \left(T, r^T_{y,\,DM}(v_\theta)\right). \tag{4}$$

Clearly $C(N, v_\theta) = \Delta^\theta(v)$, and so $y = DR(N, v_\theta)$ (Dutta-Ray, 1989). Since the WCES is *max consistent* on the domain of convex games (Dutta, 1990), $\left(T, r^T_{y,\,DM}(v_\theta)\right) \in \Gamma_{ex} \subseteq \Gamma_{ws}$ and $y_T = DR\left(T, r^T_{y,\,DM}(v_\theta)\right)$. Observe that $C\left(T, r^T_{y,\,DM}(v_\theta)\right) = L^*\left(T, r^T_{y,\,DM}(v_\theta)\right)$. From Dutta-Ray (1989) we know that $y_T \in C\left(T, r^T_{y,\,DM}(v_\theta)\right)$. Thus, from (4) we conclude that $\left(T, r^T_{y,\theta}(v)\right) \in \Gamma_{ws}$ and $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$.

Next we prove *converse order-consistency*. Let $y \in X(N, v)$ and $\theta = (i_1, i_2, \ldots, i_n) \in \Theta^{\preceq}_N(v)$ such that, for all $T = \{i_k, i_l\} \subseteq N$, $\left(T, r^T_{y,\theta}(v)\right) \in \Gamma_{ws}$ and $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$. Thus, for all $i \in N$, $y_i \geq r^T_{y,\theta}(v)(\{i\}) \geq x^\theta_i$. Since $y(N) = v(N)$, we have that $y \in \Delta^\theta(v)$. Consider the game (N, v_θ) as defined in (3). Let $T = \{i, j\} \subseteq N$. As we have seen before $\left(T, r^T_{y,\theta}(v)\right) = \left(T, r^T_{y,\,DM}(v_\theta)\right)$. For 2-person games, the SCES and WCES coincide, so $y_T = DR\left(T, r^T_{y,\,DM}(v_\theta)\right)$. Since the WCES is *converse max consistent* on the domain of convex games (Dutta, 1990), $y = DR\left(N, v_\theta\right)$. Moreover, $y \in C\left(N, v_\theta\right) = \Delta^\theta(v) \subseteq L^*(N, v)$ (expression (2)). Suppose that $y \notin EL^*(N, v)$. Then, there is $y' \in L^*(N, v)$ such that $y' \succ_L y$. From (2) there is $\theta' \in \Theta_N$ such that $y' \in \Delta^{\theta'}(v)$. Since $y' \succ_L y \succ_L x$, for all $x \in \Delta^\theta(v)$, we get $\theta' \succ_L \theta$, a contradiction. Thus, $y \in EL^*(N, v)$.

Finally, to show uniqueness, suppose there is a another solution σ on Γ_{ws} satisfying the above three axioms. For 2-person games, constrained egalitarianism implies $\sigma = EL^*$. Let $(N, v) \in \Gamma_{ws}$ with $|N| \geq 3$. First note that *constrained egalitarianism* and *order-consistency* imply *efficiency*. Let $y \in \sigma(N, v)$. By *order-consistency* there is $\theta \in \Theta^{\preceq}_N(v)$ such that, for each $T \subset N$, $|T| = 2$, $y_T \in \sigma\left(T, r^T_{y,\theta}(v)\right)$. By *constrained egalitarianism*, $y_T \in EL^*\left(T, r^T_{y,\theta}(v)\right)$. Now applying *converse order-consistency* we get $y \in EL^*(N, v)$. Following a symmetric argument we obtain the reverse inclusion, $EL^*(N, v) \subseteq \sigma(N, v)$. [3] $\square$

The following examples show that in Theorem 3.1 the axioms are independent:

- Let $\sigma^1(N, v)$ be defined as follows: $\sigma^1(N, v) := \emptyset$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^1 satisfies *order-consistency*, *converse order-consistency*, but not *constrained egalitarianism*.
- Let $\sigma^2(N, v)$ be defined as follows: $\sigma^2(N, v) := CE(N, v)$ if $|N| = 2$ and $\sigma^2(N, v) := X(N, v)$ if $|N| > 2$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^2 satisfies *constrained egalitarianism*, *converse order-consistency*, but not *order-consistency*.

[3]Notice that this is a standard application of the Elevator Lemma (for more details see Thomson (2006b)).

- Let $\sigma^3(N, v)$ be defined as follows: $\sigma^3(N, v) := CE(N, v)$ if $|N| = 2$, and $\sigma^3(N, v) := \emptyset$ if $|N| > 2$, for each $(N, v) \in \Gamma_{ws}$. Then, σ^3 satisfies *constrained egalitarianism, order-consistency, but not converse order-consistency.*

References

Arin, J., Kuipers, J., Vermeulen, D.(2003) "Some characterizations of egalitarian solutions on classes of TU-games" *Mathematical Social Sciences* **46**, 327–345.

Davis, M., Maschler, M. (1965) "The kernel of a cooperative game" *Naval Research Logistics Quarterly* **12**, 223–259.

Dutta, B.(1990) "The egalitarian solution and reduced game properties in convex games" *International Journal of Game Theory* **19**, 153–169.

Dutta, B., Ray, D.(1989) "A concept of egalitarianism under participation constraints" *Econometrica* **57**, 615–635.

Dutta, B., Ray, D.(1991) "Constrained Egalitarian Allocations" *Games and Economic Behavior* **3**, 403–422.

Hougaard, J.L., Peleg, B., Thorlund-Petersen, L.(2001) "On the set of Lorenz-maximal imputations in the core of a balanced game" *International Journal of Game Theory* **30**, 147–165.

Klijn, F., Sliker, M., Tijs, S., Zarzuelo, J. (2000) "The egalitarian solution on convex games: some characterizations" *Mathematical Social Sciences* **40**, 111–121.

Peleg, B.(1986) "On the reduced game property and its converse" *International Journal of Game Theory* **15**, 187–200.

Selten, R.(1972) "Equal share analysis of characteristic function experiments. Contributions to Experimental Economics III" *J.C.B. Mohr*, 130–165, In: Sauermann H (eds), Tubingen.

Shapley, L.S.(1971) "Cores of convex games" *International Journal of Game Theory* **1**, 11–16.

Thomson, W.(2006a) "Consistent allocation rules" *Discussion Paper. Department of Economics, University of Rochester, USA.*

Thomson, W.(2006b) "How to divide when there isn't enough. From the Talmud to game theory" *Mimeo.*