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Dynamics of price and advertising as quality signals: anything goes

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Abstract

This paper demonstrates that signaling motive may lead to many different time patterns of advertising. Therefore, it is not possible to rule out signaling as an explanation for advertising by looking solely at the time patterns of price and advertising expenditures.

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1 Introduction

Whether advertising can be interpreted as a signal of product quality is a long standing discussion. There have been attempts to test this hypothesis by looking for correlations between quality and advertising expenditures. However, the difficulty of measuring quality renders this approach problematic.[1] A second approach is to compare the observed dynamics of advertising and prices to the patterns implied by the signaling model. This approach is taken by Hortsmann and MacDonald (2003). The authors analyze data from the market for compact disc players and conclude that the standard signaling models fail to explain the observed patterns.[2]

This paper presents an infinite horizon dynamic model of prices and advertising as signals of quality. Even though the model is stylized, it admits a large number of equilibria, even when restricting attention to a refined set. Moreover, various time patterns of advertising are featured within this class of equilibria, implying that equilibrium patterns of advertising are, to a large extent, indeterminate. This demonstrates that it is not possible to rule out signaling as an explanation for advertising by solely looking at the time patterns of advertising and prices as signals of quality.

As is well-understood in the static signaling literature (e.g. Milgrom and Roberts (1986)), credible signaling of quality requires that sufficiently costly actions (in the form of uninformative advertising or distorted prices) are undertaken by the high quality producer to ensure non-mimickery by the lower quality producers. Since in general, advertising and price setting decisions are not one-shot decisions, the costly signaling actions may be allocated among different time periods subject only to certain non-mimickery constraints. As demonstrated in this paper, these non-mimickery constraints need not (and in general do not) uniquely pin down the patterns of advertising.[3] This is why various time patterns of advertising may be consistent with the signaling motive. Even when one refines the set of equilibrium patterns of advertising and prices by focusing on "least costly separating equilibria", the time patterns of advertising consistent with such equilibria are numerous. Among the possible such patterns are intense introductory advertising that later dies down, as well as sustained moderate levels of advertising.

On the methodological side, this paper demonstrates how to solve a repeated signaling game when there is more than one signaling variable: prices and advertising in this case. This is a generalization of methods developed in Kaya (2009), which characterizes the least costly separating equilibria of repeated signaling games with a single signaling variable.

The rest of this paper is organized as follows: Section 2 introduces the model, Section 3 characterizes equilibrium conditions, Section 4 presents a characterization of equilibrium patterns of advertising under an assumption guaranteeing that advertising will be used in equilibrium, and Section 5 concludes.

2 Preliminaries

The unique producer of a commodity is privately informed about the quality θ of its product, where $\theta \in \{H, L\}$ with $H > L \in \mathbb{R}_+$. The quality of the product is determined by the producer's technology and

[1] For a discussion of this literature and related issues see Hortsmann and MacDonald (2003) and the references therein.

[2] They suggest, but not pursue, ways to extend the standard model to account for these facts.

[3] This possibility was mentioned in Bagwell and Riordan (1991), in their discussion of "hindsighted consumers", in a context with only prices as signals of quality. Also, it is explored in detail in Kaya (2009) for general repeated signaling games with a unique signaling variable.

is perfectly persistent over time. The production exhibits constant marginal cost which is 0 for the product of quality L and $c > 0$ for quality H.

I consider an infinite horizon model. Every period there is a unit mass of potential customers, each with a unit demand. Two interpretations are possible: first, the commodity can be a durable good, and a new set of customers arrive every period. Second, the same set of buyers may come to the market every period, purchasing a perishable good the quality of which is not perfectly observable upon consumption. If the quality of the product is known to be L, then the reservation value of each customer for the good is $l > 0$. The reservation value, conditional on quality H, is drawn uniformly over $[l, l+1]$. Each consumer purchases a unit of the good as long as the price does not exceed his willingness to pay. A consumer's willingness to pay is the expected value of his reservation value based on the beliefs he holds about the quality of the product.

An obvious factor affecting the incentives to advertise is the extent of the diffusion of the information about product quality among the consumers. To incorporate this element, I assume that each consumer in the market may be informed or uninformed about the true quality of the product being offered, having, with some probability, been exposed to informative reviews of the product via media outlets or private channels.[4] It is conceivable that the probability of a consumer being informed depends on his reservation value for the H-quality product, since, for instance, a consumer with a higher willingness to pay for the high quality product may be more likely to seek information. Accordingly, I assume that the diffusion process is captured by a sequence of functions

$$\alpha_t : [l, l+1] \to [0, 1],$$

where $\alpha_t(r)$ represents the probability that a consumer who has reservation value r for the H-quality product is informed in period t. I further assume that $\alpha_t(r)$ evolves according to an *exogenous* process for each r which is commonly known by all agents in the model. Finally, I assume that for any r, $\alpha_t(r) \leq \alpha_{t'}(r)$ whenever $t' > t$, and for any t, $\alpha_t(r)$ is non-decreasing. The exogeneity assumption is perhaps less benign in that one can imagine introductory pricing or intense introductory advertising to impact the speed of diffusion of a product into the market. However, not letting the rate of diffusion depend, in particular, on the intensity of advertising isolates the "signaling" aspect of advertising from its "informative" aspect: allowing the speed of diffusion to be influenced by the intensity of advertising introduces distinct motives to the choice of amount and time-pattern of advertising. Moreover, this assumption greatly simplifies the analysis and is unlikely to be a driving assumption of the main result of the paper. The assumption that $\alpha_t(r)$ is increasing in t simply means that the probability of being informed about the quality of the product that has been on the market for a longer period is higher. On the other hand, $\alpha_t(r)$ being non-decreasing in r is meant to capture the intuition that consumers with higher willingness to pay for a high quality product are more likely to be informed, perhaps due to the fact that they might pay more attention to news about this product or may expend more resources researching it.[5]

At the beginning of each period t, the producer picks a price p_t and an amount of advertising expenditure A_t. The history of these choices is observable by the potential customers. The customers form beliefs about the quality of the product based on the history and decide whether to purchase.

The focus is on the pure strategy perfect Bayesian equilibria (PBE). The definition of PBE is standard. The set of equilibria is further refined by considering only the "least costly separating equilibria". A *separating equilibrium* is a PBE in which after every equilibrium path history, the beliefs are degenerate. A *least costly separating equilibrium* (LCSE) is a separating equilibrium which maximizes the expected discounted sum of

[4]This model is similar to Linnemer (2002) and Bagwell and Riordan (1991). However, both papers consider a static model.

[5]On the practical side, this assumption guarantees that the optimization problem presented in the next section is well behaved.

payoffs for the producer of type H, among all separating equilibria.

3 Equilibrium conditions

This section characterizes the LCSE of the model as a solution to an appropriately specified dynamic optimization problem.[6]

In any equilibrium of this repeated signaling game, the buyers are aware of the persistence of the product quality. Therefore, in making inferences based on a given history of prices and advertisement choices, they take this into account: i.e. they understand that the monopolist's choices in earlier periods are indicative of his product quality in the current period. In other words, early advertising or price choices may be used to signal about quality in future periods. Moreover, as is well-understood by now, without the requirement that beliefs should remain fixed once they become degenerate, costly signals after initial separation can be supported in equilibrium via the threat of switching to unfavorable beliefs.[7] Therefore, in a repeated situation as in the current model, it is possible to spread signaling costs over time. These two observations suggest why many different patterns of advertising and prices may be consistent with the equilibrium behavior of a high quality producer in a repeated signaling model.

Even though many different patterns of prices and advertising may be consistent with equilibrium behavior of a high quality producer, naturally there has to be some constraints on these patterns. Such constraints would ensure that the producer, if he were of low-quality, would not mimic this sequence of actions in return for being believed to be the H-type. Instead, he would rather reveal his type by choosing his full information optimal price. In particular, at any time period T, the history thus far must be consistent with this requirement. That is, in order for a sequence $\{p_t, A_t\}_{t=1}^{\infty}$ to be the equilibrium sequence of actions of an H-type producer it is necessary that

$$\forall\, T = 1, 2, 3, ... : \quad \sum_{t=1}^{T} \delta^{t-1}\, \pi_L(p_t, A_t|\alpha_t) \;\leq\; l \times \frac{1-\delta^T}{1-\delta} \tag{1}$$

where $\pi_L(p, A|\alpha)$ is the per-period payoff of the L-type producer from choosing p, A in a period where α proportion of the consumers are informed, and the rest believes that he is of type H. That is,

$$\pi_L(p, A|\alpha) = p \int_p^{l+1} (1 - \alpha_t(r))dr - A.$$

To understand (1) note that the left-hand-side is the accumulated payoff—up to time T—of the L-type producer, from mimicking the sequence $\{p_t, A_t\}$, while the right-hand-side is the corresponding payoff from choosing his full information optimal strategy of $p = l$ and $A = 0$. Note also that (1) is *weaker* than requiring that $\pi_L(p_t, A_t|\alpha_t) \;\leq\; l$ for all t, which would be equivalent to requiring that the non-mimickery constraint holds period-by-period. The intuition that earlier actions can signal future quality allows to relax the period-by-period requirement into (1) which simply necessitates that the signals are *sufficiently front-loaded*.

[6]The methods applied in this section are a generalization of the analysis in Kaya (2009) to the case of more than one signaling variable.

[7]See for instance Madrigal et al. (1987); Noldeke and van Damme (1990). Also see Kaya (2009) for implications of this in a repeated signaling environment.

In fact, it is possible to show that (1) is not only necessary but also sufficient for $\{p_t, A_t\}_{t=1}^{\infty}$ to be the equilibrium sequence of actions of an H-type producer in a separating equilibrium.[8] In this light, letting $\pi_H(p, A) = (p - c)(1 + l - p) - A$ to be the per-period payoff of the H-type producer from choosing (p, A) when the uninformed consumers believe that he is indeed of type H, a least costly separating equilibrium (henceforth, LCSE) maximizes

$$\max_{(p_t, A_t)_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta^{t-1} \pi_H(p_t, A_t) = \sum_{t=1}^{\infty} \delta^{t-1} [(l + 1 - p_t)(p_t - c) - A_t] \tag{2}$$

subject to (1). In sum, any price-advertising sequence $\{p_t, A_t\}_{t=1}^{\infty}$ that satisfies (1) is consistent with the equilibrium behavior of the H-type producer in a separating equilibrium, while any $\{p_t, A_t\}_{t=1}^{\infty}$ that solves (2) subject to (1) is consistent with his behavior in a LCSE.

4 Indeterminacy of time patterns of advertising

This section characterizes the solution of (2) subject to (1) under a simplifying assumption that guarantees that the least costly separating equilibrium path does involve costly advertising in addition to price distortions. We formally state and discuss this assumption below. As a solution to this problem, we demonstrate that many different time-patterns of advertising can be consistent with the signaling motive.

It is convenient to use a dual approach to this problem: first, compute the "cheapest way" to deliver a given level, say q, of within-period "mimicking payoff" to the L-type producer, then using this as an analogue of a cost function, find the cost minimizing sequence $\{q_t\}_{t=1}^{\infty}$ of such payoffs that satisfy (1) (i.e. $\sum_{t=1}^{T} \delta^{t-1} q_t \leq l \times \frac{1-\delta^T}{1-\delta}$). Formally,

Step 1 Find the optimal combination of prices and advertising such that the within-period payoff of the L-type producer from choosing this pair is at most q:

$$\Pi(q|\alpha) = \max_{p,A} \quad \pi_H(p\ ,\ A) \qquad \text{subject to} \qquad \pi_L(p, A \mid \alpha) \leq q. \tag{3}$$

This step defines a static "value function", $\Pi(q|\alpha)$ for holding the low-quality producer to a profit level of q when each consumer is informed with probability α.

Step 2 Choose the optimal sequence of per-period payoffs for the L-type producer that respect the constraints in (1), solving:

$$\max_{\{q_t\}_{t=1}^{\infty}} \sum_{t=1}^{T} \delta^{t-1} \Pi(q_t|\alpha_t) \qquad \text{subject to} \qquad \forall\, T: \sum_{t=1}^{T} \delta^{t-1}(q_t - l) \leq 0. \tag{4}$$

It is easy to see that (4) is equivalent to (2)-(1), using the variable q introduced in (3).

[8] Strictly speaking, another sequence of constraints that guarantees that type H is willing to follow (A_t, p_t) sequence rather than pool with type L, is necessary. However, these are not binding in a LCSE. For a formal proof of this assertion and a discussion of other (more reasonable) off-equilibrium beliefs that would support these sequences of actions in equilibria, see Kaya (2009).

Let's first discuss the characterization of (3). Let $p^*(\alpha)$ and $q^*(\alpha)$ be defined by

$$p^*(\alpha) = \mathbf{argmax}_{p \le l+1} \quad \Pi_H(p, A) - \Pi_L(p, A|\alpha) = p \int_p^{l+1} \alpha(r)dr - c(l + 1 - p),$$

and

$$q^*(\alpha) = \pi_L(p^*(\alpha), 0|\alpha).$$

First, it is easy to show using second order conditions that $p^*(\alpha)$ and therefore $q^*(\alpha)$ are uniquely determined. Next, to understand the meaning of $p^*(\alpha)$ and $q^*(\alpha)$, it is useful to consider the problem of finding the cheapest (for the high-quality monopolist) way to reduce the mimicking profit of the low-quality monopolist by a small amount, say $\varepsilon > 0$. There are two tools available to the high-quality monopolist to achieve this goal: (1) distorting prices, (2) using advertising. To use the first tool, it is necessary to distort (increase) the price by $\frac{\varepsilon}{\partial \pi_L(p|\alpha)/\partial p}$, which reduces the profit of the high-quality producer by approximately $\varepsilon \frac{\partial \pi_H(p,A)/\partial p}{\partial \pi_L(p,A|\alpha)/\partial p}$. To use the second tool it is necessary to increase advertising by ε, the cost of which is exactly ε. Then, $p^*(\alpha)$ is exactly the price at which these two costs are equalized, with $\varepsilon \frac{\partial \pi_H(p,A)/\partial p}{\partial \pi_L(p,A|\alpha)/\partial p} \le \varepsilon$ if and only if $p \le p^*(\alpha)$.[9] This implies, in particular, that for the solution of (3), if $q \ge q^*(\alpha)$, then it is optimal to solely use prices. Otherwise, it is optimal to set $p = p^*(\alpha)$ and choose $A = q^*(\alpha) - q$.

Intuitively, if the price sequence $q^*(\alpha_t)$ satisfies (1), then the least costly separating equilibrium does not involve any advertising. In the opposite case, the optimal path necessarily involves some advertising. Yet, that (1) is not satisfied does not imply that along the optimal path the price sequence is pinned down as $\{p^*(\alpha_t)\}$. Since the purpose of this paper is to demonstrate the various time patterns of advertising that are consistent with the signaling motive, in order to abstract from further complications and keep the exposition short, I make the following stronger assumption that guarantees that in any least costly separating equilibrium $\{p^*(\alpha_t)\}$ is the sequence of prices.

Assumption 1 *For all t, $q^*(\alpha_t) > l$.*

Proposition 1 *Under Assumption 1, the price sequence on the path of any LCSE satisfies $p_t = p^*(\alpha_t)$. The advertising expenditure sequence A_t on the path of any LCSE satisfies*

$$\sum_{t=1}^{\infty} \delta^{t-1} A_t = \sum_{t=1}^{\infty} \delta^{t-1}(q^*(\alpha_t) - l). \tag{5}$$

The timing of advertising is indeterminate as long as (1) is satisfied.

Proof: See Appendix.

Even though Proposition 1 pins down the price sequence and the discounted sum of advertising expenditures, the timing of these expenditures along the optimal path are largely indeterminate. Reducing advertising in one period and increasing it in a different period while maintaining the sum of the present discounted values of advertising does not affect the objective function. Moreover, (1) continues to be satisfied as long as the resulting sequence is sufficiently front-loaded. This allows, in particular, for optimal paths that have constant, declining, or first increasing and then declining advertising patterns.

[9] This is because the problem is concave. The concavity is guaranteed by the assumption that $\alpha(r)$ is non-decreasing.

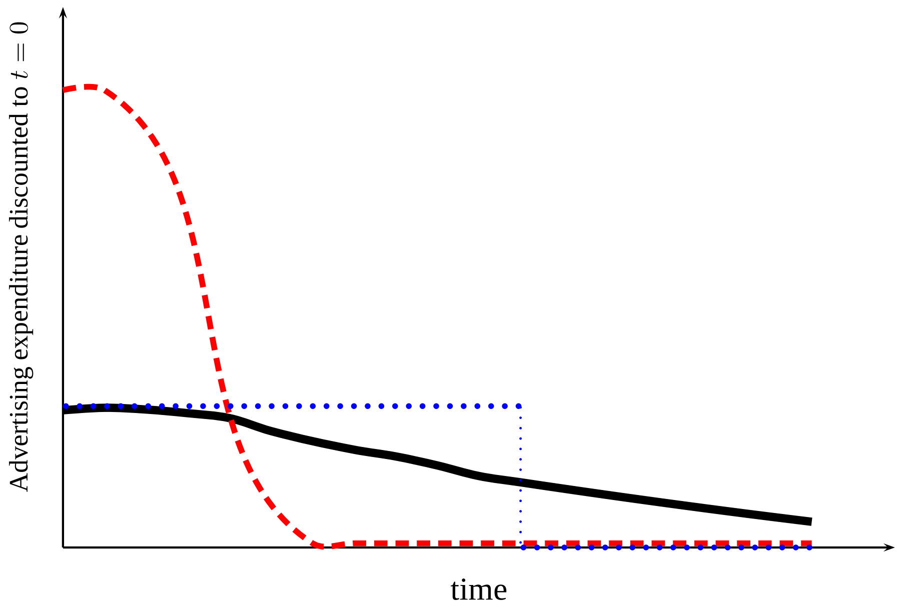

Figure 1: Possible time patterns of advertising

Figure 1 visualizes three possible sequences of advertising expenditures. The horizontal axis is time. The curves represent various sequences of $\delta^{t-1}A_t$, (that is, the advertising expenditures as a function of t discounted to time 0) that can be an outcome of a LCSE.[10] The solid black line represents the "least front-loaded" equilibrium sequence of advertising. More precisely, it plots the values

$$A_t = q^*(\alpha_t) - l.$$

Therefore, the area under this curve up to any time T approximates $l \times \frac{1-\delta^T}{1-\delta}$; i.e. the right hand side of the constraint (1). Then, by Proposition 1, any modification of this curve such that (i) the total area under it is the same as that under the solid black curve; *and* (ii) the area under it up to any T is larger than that under the solid black line, represents an equilibrium sequence of advertising expenditures. The requirement (i) guarantees that (5) is satisfied while the requirement (ii) guarantees that the constraint (1) is satisfied. The dashed and dotted lines demonstrate two patterns that satisfy these two requirements: one in which advertising expenditure is first increasing and then declining, and the second in which it is first constant and then abruptly stops. It is easy to see that there are a continuum of different patterns that satisfy the requirements (i)-(ii) and therefore can be explained by a signaling motive.

[10]The values are calculated for an example where $\alpha_t(r) \equiv \alpha_t$. That is, each consumer is equally likely to be informed at a given time period t. Also, in this example, α_t is increasing *linearly* over time, varying between .2 and .6, and becoming constant thereafter.

5 Conclusion

This paper has demonstrated that the signaling motive is consistent with many different time patterns of advertising. This suggests that, it is not possible to rule out signaling motive for advertising solely by observing the time patterns of advertising and prices, and the actual observed patterns of advertising are likely to be pinned down by other motives such as providing information about the product.[11]

Our results are obtained under several simplifying assumptions about the production and the distribution of the buyer valuations in order to facilitate exposition. Yet, it is easy to see that the analysis would extend to more general environments under certain regularity conditions guaranteeing that the optimization problems discussed are well-behaved. One assumption which may appear less innocuous is that the advertising expenditure is linear in the amount of advertising, as linearity is typically associated with indeterminacy of solutions to optimization problems. However, it is easy to see that our main conclusion that advertising time patterns are indeterminate would continue to hold regardless of the functional form of the cost of advertising as long as it is additively separable from the rest of the profits.

Finally, looking at a model of advertising in a monopolistic, rather than an oligopolistic environment, is admittedly restrictive. The issue of competitive signaling has recently been taken up in static settings by several authors (see for instance Daughety and Reinganum (2004); Fluet and Garella (2002); Hertzendorf and Overgaard (2001)). Introducing this aspect in the dynamic environment may change the predictions about the time patterns of price and advertising. This remains an open question.

6 Appendix

Proof of Proposition 1: Suppose for some T, $p_T < p^*(\alpha_T)$. If there is a period $T' > T$ with $A_{T'} > 0$, then increasing p_T and reducing $A_{T'}$ by an appropriate amount so that $\sum_{t=1}^{T'} \delta^{t-1} \pi_L(p_t, A_t|\alpha_t)$ remains the same relaxes all constraints (1) corresponding to $T \leq t < T'$ and leaves those corresponding to $t \geq T'$ unaffected. Moreover, it is easy to see that such a variation increases the objective function. If there is no period $T' > T$ with $A_{T'} > 0$, let $T'' \leq T$ such that $A_{T''} > 0$ and constraint (1) does not bind at T''. Such T'' exists by Assumption 1. Then, reducing $A_{T''}$ by $\epsilon > 0$ and increasing p_T sufficiently so that the quantity $\sum_{t=1}^{T} \delta^{t-1} \pi_L(p_t, A_t|\alpha_t)$ remains the same strictly increases the objective for small enough ϵ. Moreover, the constraint (1) is unaffected at each $T \neq T''$ and continues to be satisfied at T''.

Suppose $p_T > p^*(\alpha_T)$. Then, reducing p^T by $\epsilon > 0$ and increasing A_T so that q_T remains the same increases the objective and does not affect any of the constraints. Finally, if $\sum_{t=1}^{\infty} \delta^{t-1} A_t < \sum_{t=1}^{\infty} \delta^{t-1}(q^*(\alpha_t) - l)$, then the sequence (p_t, A_t) is not feasible and if $\sum_{t=1}^{\infty} \delta^{t-1} A_t > \sum_{t=1}^{\infty} \delta^{t-1}(q^*(\alpha_t) - l)$, the sequence (p_t, A_t) is clearly not optimal. □

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[11]See, for instance, Saak (2012) for an analysis of how time patterns of advertising are determined when the advertising is informative.

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