Economics Bulletin

Volume 33, Issue 2

Consumption Decisions in an Economy with Heterogeneous Preferences Defined by a Bivariate Distribution

Alfredo Omar Palafox-Roca
Instituto Politécnico Nacional, Escuela Superior de Economía

Francisco Venegas-Martínez
Instituto Politécnico Nacional, Escuela Superior de Economía

Abstract

This paper considers an economy populated by heterogeneous individuals in two respects: both parameters representing the subjective discount and the risk aversion rates are supposed to have a joint distribution. That is, consumers differ in their level of anxiety for present consumption and their risk aversion rate. The utility index is of the negative exponential type. This research provides a closed-form optimal consumption path of the average infinite-lived agent. Finally, some comparative statics experiments are carried out.

Citation: Alfredo Omar Palafox-Roca and Francisco Venegas-Martinez, (2013) "Consumption Decisions in an Economy with Heterogeneous Preferences Defined by a Bivariate Distribution", *Economics Bulletin*, Vol. 33 No. 2 pp. 993-1000.
Contact: Alfredo Omar Palafox-Roca - aocontreras@gmail.com, Francisco Venegas-Martinez - fvenegas1111@yahoo.com.mx.
Submitted: May 30, 2012. **Published:** April 16, 2013.

1. Introduction

Heterogeneity has been studied from various points of view. For instance, Constantinides (1982) considers heterogeneous consumers by introducing a distribution of wealth among population. Constantinides and Duffie (1996) assume heterogeneity in labor income. Basak (2003) establishes heterogeneity through arbitrary utility functions. Chan and Kogan (2001) suppose that agents differ in their utility function curvature. Luttmer and Marioti (2003) suppose arbitrary subjective discount factors. Jouini and Napp (2007) introduce heterogeneity of beliefs as a source of risk. Follmer *et al.* (2005) study a framework with switches in beliefs when individuals move from one period to another. Finally, Lucas' model of asset prices in an exchange economy is modified by Li (2007) by allowing investors with different subjective discount rates.

This paper develops a model of an economy populated by heterogeneous agents. Heterogeneity refers to individuals with different tastes in two respects: the parameters representing the subjective discount and the risk aversion rates are supposed to have an exponential distribution. Therefore, consumers in the economy differ in their level of anxiety for present consumption and their risk aversion. Utility is assumed to be of the negative exponential type. This proposal differs from the above literature in the following respects: 1) heterogeneity comes from risk aversion rates, 2) there is a joint distribution of the subjective discount and the risk aversion rates, 3) the dynamics of capital and the welfare function are determined, and 4) the simplicity of the obtained results.

This paper is organized as follows: section 2 provides the set up of the economy and establishes the problem faced by a central planner; section 3 defines the firms' behavior in the economy; section 4 deals with resource allocation, which depends on the identity of national income; section 5 provides the optimal consumption path of the average infinite-lived agent; section 6 deals with the welfare function of the average individual, and, finally, section 7 presents conclusions and acknowledges limitations.

2. Assumptions of the economy

The economy consumes and produces a single perishable good and is populated by heterogeneous agents in preferences. The heterogeneity in tastes of agents is represented by two distribution functions. The first distribution, $F = F(\rho),\quad \rho > 0$, considers the subjective discount rate. Similarly, the second one, $G = G(\alpha),\quad \alpha > 0$, takes into account the risk aversion rate, α, of a negative exponential utility function. The modeling allows assigning different agents to the same values of the two above parameters. It is assumed that ρ and α are stochastically independent. This seems to be a reasonable assumption since anxiety about present consumption is not related to risk aversion; it also simplifies the algebraic computations.

2.1 Central planner's problem

It is assumed that a central planner wishes to maximize the satisfaction of the average agent. Specifically, the central planner wishes to solve

$$\text{Maximize}\quad \int_0^\infty \left(\int_0^\infty \left(\int_0^\infty -e^{-\alpha c_t} e^{-\rho t}\,\mathrm{d}t \right) \mu e^{-\mu\alpha}\,\mathrm{d}\alpha \right) \lambda e^{-\lambda\rho}\,\mathrm{d}\rho \tag{1}$$

subject to an economy constraint. It is also assumed that the functional form of the utility function is exponential negative, *i.e.*, $u(c_t;\alpha) = -e^{-\alpha c_t}$, $\alpha > 0$. Moreover, it is assumed that the distribution function of α is $G(\alpha) = 1 - e^{-\alpha\rho}$, $\alpha > 0$. Similarly, the distribution function of ρ is given by $F(\rho) = 1 - e^{-\lambda\rho}$, $\lambda > 0$.

3. Firms' Behavior

It is assumed that the production is carried out by a representative firm using an Ak technology, *i.e.*, $y_t = f(k_t) = Ak_t$. The present value, PV, of the representative firm is given by:

$$PV = \int_{t=0}^{\infty} (Ak_t - rk_t) e^{-rt} \mathrm{d}t$$

Note that the above expression represents the discounted benefits of the firm. Here, r is the real interest rate. The first order condition leads to $r = A$. Thus, the marginal product of capital equals the real interest rate.

4. Resource Allocation

It is assumed that resource allocation is given by the national income identity (for a closed economy without government, *i.e.*, an autarky) and not by a price system, as in López-Herrera *et al.* (2012). Suppose also that the rate of depreciation of capital is zero. Thus,

$$rk_t = c_t + \dot{k}_t.$$

After discounting and integrating both sides of this identity and considering a transversality condition, it follows that

$$0 = \int_0^{\infty} c_s e^{-rs} \mathrm{d}s + \lim_{t\to\infty} k_t e^{-rt} - k_0.$$

Therefore, $k_0 = \int_0^{\infty} c_s e^{-rs} \mathrm{d}s$, where k_0 is given. Notice that $rk_t = c_t + \dot{k}_t$ is the economy constraint and the central planner solves the problem of maximizing utility of the average agent, not the agent himself. Thus, even if all individuals are different, the constraint is the same for everyone. This is not a competitive equilibrium model because it does not determined how much each individual consumes, but the consumption of the average agent (or the population average consumption). The model dos not aggregate consumption and, hence, the average agent approach cannot be supported by a competitive equilibrium.

5. Optimal consumption paths of the average agent

The problem facing the central planner becomes:

$$\text{Maximize} \quad \int_0^{\infty}\left(\int_0^{\infty}\left(\int_0^{\infty} -e^{-\alpha c_t} e^{-\rho t} \mathrm{d}t\right)\mu e^{-\mu\alpha} \mathrm{d}\alpha\right)\lambda e^{-\lambda\rho} \mathrm{d}\rho$$

$$\text{subject to} \quad k_0 = \int_0^{\infty} c_t e^{-rt} \mathrm{d}t.$$

Both μ and λ are known parameters. The objective function, given the above assumptions and Fubini's theorem, can be rewritten as follows:

$$\text{Maximize} \quad \int_0^\infty \frac{-\mu\lambda}{(t+\lambda)(\mu+c_t)}\mathrm{d}t.$$

The Lagrangian for this problem is:

$$\mathfrak{L}(c_t,\lambda)=\frac{-\mu\lambda}{(t+\lambda)(c_t+\mu)}+\beta(y-c_t)e^{-rt}.$$

Differentiating with respect to c_t is found that

$$\frac{\mu\lambda}{(t+\lambda)(c_t+\mu)^2}-\beta e^{-rt}=0,$$

and a solving for c_t is obtained

$$c_t=\sqrt{\frac{\mu\lambda}{\beta}}\sqrt{\frac{e^{rt}}{(t+\lambda)}}-\mu. \tag{2}$$

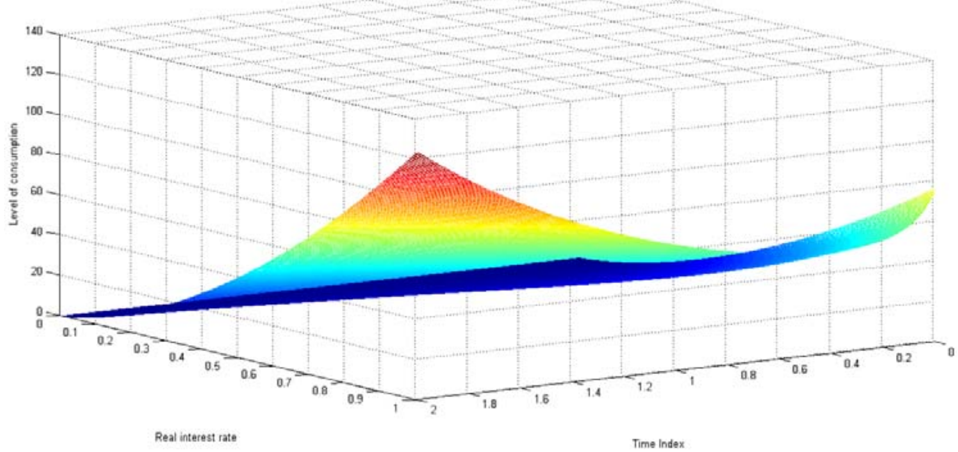
In the above equation the Lagrange multiplier, β, is unknown. In order to find it, equation (2) is substituted in the constraint. The computations are shown in an Appendix. The optimal consumption path satisfies

$$c_t=\frac{e^{\frac{A}{2}(t-\lambda)}}{\sqrt{t+\lambda}}\left[\frac{Ak_0+\mu}{\sqrt{8\pi A\left(1-\Phi\left(\sqrt{A\lambda}\right)\right)}}\right]-\mu. \tag{3}$$

where Φ represents the cumulative distribution function of a standard normal random variable, and, as before, $r = A$. Notice that at time $t = 0$,

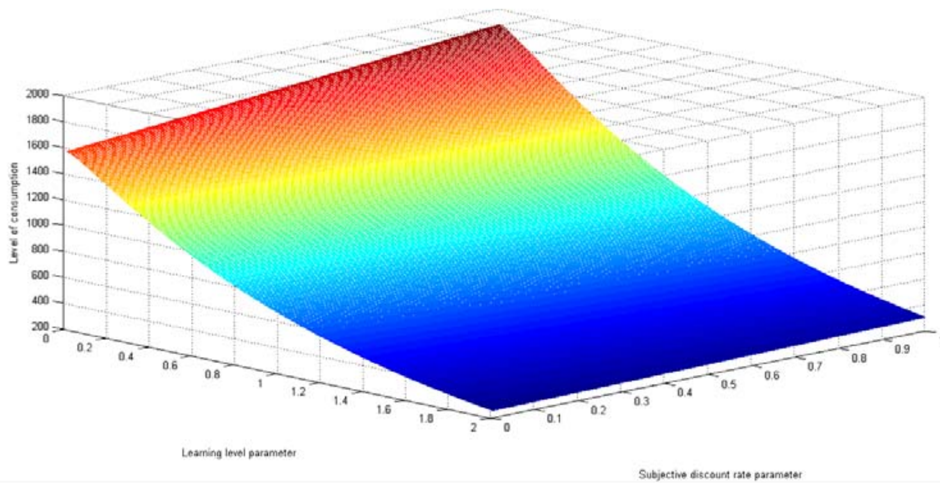
$$\frac{e^{-\frac{r\lambda}{2}}}{\sqrt{\lambda}}\left[\frac{rk_0+\mu}{\sqrt{8\pi r\left(1-\Phi\left(\sqrt{r\lambda}\right)\right)}}\right]>\mu. \tag{4}$$

Though equation (3) was obtained at $t = 0$, this equation guarantees that consumption remains positive for all $t>0$. Graph 1 illustrates the path of consumption as a function of A and t; all other parameters remaining constant. In this case, $k_0=100$, $\lambda = 0.1$, $\alpha = 0.2$, and $r\in(0,1]$.



Graph 1. The level of consumption as a function of *r* and *t* (Source: own elaboration).

From Graph 1 it can be seen that consumption increases with *r* and *t*. Moreover, Graph 2 shows the behavior of the optimal path of consumption when $\lambda \in (0,1]$ and $\mu \in (0,2]$. In this case, that consumption increases when both *r* and *t* rise. These experiments provide a visual way of comparative statics.



Graph 2. The level of consumption as a function of the risk aversion parameter and the subjective discount rate (Source: own elaboration).

Finally, by substituting optimal consumption of the average individual in the national income identity, it follows that (details are available to interested readers)

$$k_t = e^{rt}\left[k_0 + \frac{\mu}{r}\right] - \frac{\mu}{r} - \left[\frac{rk_0 + \mu}{\sqrt{8\pi r\left(1-\Phi\left(\sqrt{r\lambda}\right)\right)}}\right]\left[\left(2\sqrt{t+\lambda}-1\right)e^{\frac{r}{2}(t+\lambda)} - \left(2\sqrt{\lambda}-1\right)e^{-r\left(t-\frac{\lambda}{2}\right)}\right].$$

This is the path followed by state variable. As well as for the optimal consumption path of the average consumer, visual experiments of comparative statics can be carried out for capital.

6. The welfare function of the average consumer

With some calculations it is possible to find the expression for welfare function, W. By substituting $c_t + \mu$ in equation (2.1), it is found that

$$W = \int_0^\infty \frac{-\mu\lambda}{(t+\lambda)\left(c_t^{\lambda,\mu} + \mu\right)} \mathrm{d}t = \frac{-(\mu\lambda) e^{\frac{r\lambda}{2}} \sqrt{8\pi r\left(1-\Phi\left(\sqrt{r\lambda}\right)\right)}}{rk_0 + \mu} \int_0^\infty \frac{e^{-\frac{rt}{2}}}{(t+\lambda)^{\frac{1}{2}}} \mathrm{d}t,$$

$$= \left(2\sqrt{\lambda} - 1\right) \frac{(\mu\lambda) e^{\frac{r\lambda}{2}} \sqrt{8\pi r\left(1-\Phi\left(\sqrt{r\lambda}\right)\right)}}{rk_0 + \mu}.$$

This is a closed formula when all the parameters are known (details are available to interested readers).

7. Conclusions

This paper has focused on a new approach of heterogeneity of individuals regarding their risk aversion rates. One advantage is the simplicity of the setting and the possibility of modeling in a richer and more realistic way intertemporal consumption decisions of the average consumer. Moreover, the capital path and the welfare function of the average consumer are determined Needless to say, the obtained results depend on the utility function and in the functional form of the distributions of the parameters; more research is needed in this sense.

Appendix

Applying twice Fubini's theorem to the objective function (1), of the average individual, and solving the integrals, the problem is reformulated as:

$$\text{Maximize} \quad \int_0^\infty \int_0^\infty \int_0^\infty -\mu\lambda e^{-\alpha(c_t+\mu)} e^{-\rho(t+\lambda)} \mathrm{d}\alpha \, \mathrm{d}\rho \, \mathrm{d}t$$

$$\text{subject to} \quad k_0 = \int_0^\infty c_t e^{-rt} \mathrm{d}t.$$

In solving the first two (inner) integrals of the objective function is obtained

$$\int_0^\infty \int_0^\infty -\mu\lambda e^{-\rho(t+\lambda)} \int_0^\infty e^{-\alpha(c_t+\mu)} \mathrm{d}\alpha \, \mathrm{d}\rho \, \mathrm{d}t = \int_0^\infty \int_0^\infty \frac{-\mu\lambda e^{-\rho(t+\lambda)}}{c_t + \mu} \mathrm{d}\rho \, \mathrm{d}t = \int_0^\infty \frac{-\mu\lambda}{(t+\lambda)(c_t+\mu)} \mathrm{d}t.$$

Now, the maximization problem becomes:

$$\text{Maximize} \quad \int_0^\infty \frac{-\mu\lambda}{(t+\lambda)(c_t+\mu)} \mathrm{d}t$$

$$\text{subject to} \quad k_0 = \int_0^\infty c_t e^{-rt} \mathrm{d}t.$$

From equation (2),

$$c_t = \sqrt{\frac{\mu\lambda}{\beta}} \sqrt{\frac{e^{rt}}{(t+\lambda)}} - \mu.$$

In substituting this expression into the constraint is found that

$$k_0 = \int_0^\infty \sqrt{\frac{\mu\lambda}{\beta} \frac{e^{rt}}{(t+\lambda)}} e^{-rt} \mathrm{d}t - \int_0^\infty \mu e^{-rt} \mathrm{d}t = \sqrt{\frac{\mu\lambda}{\beta}} \int_0^\infty \frac{1}{(t+\lambda)^{1/2}} e^{-\frac{r}{2}t} \mathrm{d}t - \frac{\mu}{r},$$

Notice that $$\int_0^\infty \frac{1}{(t+\lambda)^{1/2}} e^{-\frac{r}{2}t} \mathrm{d}t = 2\int_0^\infty \frac{1}{2(t+\lambda)^{1/2}} e^{-\frac{r}{2}t} \mathrm{d}t.$$

Let $z=(t+\lambda)^{1/2}$, $\mathrm{d}z = 1\big/\left(2(t+\lambda)\right)^{1/2}\mathrm{d}t$, then,

$$2\int_{\sqrt{\lambda}}^\infty e^{-\frac{r}{2}(z^2-\lambda)}\mathrm{d}z = 2e^{\frac{r\lambda}{2}}\int_{\sqrt{\lambda}}^\infty e^{-\frac{1}{2}\left(\frac{z}{(1/\sqrt{r})}\right)^2}\mathrm{d}z = 2e^{\frac{r\lambda}{2}}\frac{1}{\sqrt{r}}\int_{\sqrt{\lambda}}^\infty e^{-\frac{1}{2}\left(\frac{z}{(1/\sqrt{r})}\right)^2}\sqrt{r}\mathrm{d}z.$$

With a new change of variable, $y=\sqrt{r}z$, $\mathrm{d}y=\sqrt{r}\mathrm{d}z$,

$$2e^{\frac{r\lambda}{2}}\frac{1}{\sqrt{r}}\int_{\sqrt{r\lambda}}^\infty e^{-\frac{1}{2}y^2}\mathrm{d}y = 2e^{\frac{r\lambda}{2}}\frac{\sqrt{2\pi}}{\sqrt{r}}\int_{\sqrt{r\lambda}}^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}\mathrm{d}y = 2e^{\frac{r\lambda}{2}}\frac{\sqrt{2\pi}}{\sqrt{r}}\left[1-\Phi\left(\sqrt{r\lambda}\right)\right].$$

Therefore, $$k_0 = \sqrt{\frac{\mu\lambda}{\beta}}\left[2e^{\frac{r\lambda}{2}}\frac{\sqrt{2\pi}}{\sqrt{r}}\left[1-\Phi\left(\sqrt{r\lambda}\right)\right]\right]-\frac{\mu}{r}.$$

It follows that, $$\sqrt{\frac{\mu\lambda}{\beta}} = \frac{rk_0+\mu}{\left[\sqrt{8\pi r}e^{\frac{r\lambda}{2}}\left[1-\Phi\left(\sqrt{r\lambda}\right)\right]\right]}.$$

Hence, $$c_t = \frac{e^{\frac{rt}{2}}}{\sqrt{t+\lambda}}\frac{rk_0+\mu}{\left[\sqrt{8\pi r}e^{\frac{r\lambda}{2}}\left[1-\Phi\left(\sqrt{r\lambda}\right)\right]\right]}-\mu.$$

References

Basak, S. (2000) "A model of dynamic equilibrium asset pricing with heterogeneous beliefs and extraneous risk" *Journal of Economic Dynamics and Control*, 24(1), 63-95.

Chan, Y.L. and L. Kogan (2001) "Catching up with the Joneses Heterogeneous Preferences and the Dynamics of Asset Prices" *National Bureau of Economic Research*, 8607.

Constantinides, G.M. (1982) "Intertemporal Asset Pricing with Heterogeneous Consumers and Without Demand Aggregation" *The Journal of Business*, 55(2), 253-267.

Constantinides, G.M. and D. Duffie (1996) "Asset Pricing with Heterogeneous Consumers" *The Journal of Political Economy*, 104(2), 219-240.

Föllmer, H., U. Horst and A. Kirman (2005) "Equilibria in financial markets with heterogeneous agents: a probabilistic perspective" *The Journal of Mathematical Economics*, 41(1-2), 123-155.

Jouini, E. and C. Napp (2007) "Consensus Consumer and Intertemporal Asset Pricing with Heterogeneous Beliefs" *The Review of Economic Studies*, 74(4), 1149-1174.

Li, T (2007) "Heterogeneous beliefs, asset prices, and volatility in a pure exchange economy" *Journal of Economic Dynamics and Control*, 31(5), 1697-1727.

López-Herrera, F., F. Ortiz-Arango y F. Venegas-Martínez, Crecimiento, desarrollo y crisis financiera: una visión crítica (2012), Edited by Universidad Panamericana. Coordinator F. Ortiz-Arango.

Luttmer, E. G. J. and T. Marioti (2003) "Subjective discounting in an Exchange economy" *Journal of Political Economy*. 111(5), 959-989.