The explanatory power of signed jumps for the risk-return tradeoff

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Abstract

Patton and Sheppard (2011) develop the concept of signed jumps as the difference between positive and negative realized positive semivariances. This quantity is well-suited for gauging the risk-return trade-off at high-frequency as it is well-defined each day and, contrary to the squared jump contribution following Barndorff-Nielsen and Shephard (2004, 2006) which is dedicated to rare jumps, it is signed. We show that signed jumps only occasionally help in explaining future returns, at least when the horizon of interest is one-day ahead as in Bali and Peng (2006).

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1 Introduction

The risk-return trade-off is a central question in financial economics since the theoretical contribution from Merton (1973). While the principle of a linear relationship between expected risk and expected return is a well-accepted assumption among researchers, it lacks empirical support. Indeed, since French et al. (1987), a large number of papers have found mixed evidence of a positive relationship between risk and return and some articles even found a significant negative relation (see Lettau and Ludvigson (2009) for a survey of this issue).

Three main hypothesis emerged in the literature to explain the weak empirical support to the existence of a risk-return trade-off. First, as indicated in the original contribution by Merton (1973), the relation may be recovered only when the hedging component materializing alternative investment opportunities is correctly modeled. In words, the literature so far may have missed the risk-return trade-off by ignoring the hedging component in predictive regressions (see Guo and Whitelaw (2006) for a paper dealing mainly with this issue). We will not specifically consider this issue here but will use several proxies, in line with the existing literature, to empirically take into account the hedging component.

The second explanation, which will be of interest to us in the present work, is on the expected risk that we should consider when estimating the predictive regression. Indeed, while returns are observable, risk is not and should then be estimated. The recent literature suggested a number of alternatives to deal with the issue of modelling the expected risk. Ghysels et al. (2005) propose to use MIDAS regressions along with monthly realized measures of risk. Other papers have emphasized the existence of two risk components that should be considered to recover the risk-return trade-off (Engle and Lee (1999), Maheu and McCurdy (2007) or Adrian and Rosenberg (2008)). One is a short-term risk component while the other is a longer-term risk component.\(^1\) Lastly, alternative measures of risk have been considered to figure out downside risk (Bali et al. (2009), Ghysels et al. (2011), Conrad et al. (2012) among many others) or the variance risk premium.

\(^{1}\)And both may contribute differently to the relationship between risk and return.
Finally, the third explanation is that the risk-return trade-off only exists at specific horizons. In particular, Ghysels et al. (2005) show that the one-month ahead horizon considered in French et al. (1987) is not sufficient and that a positive relationship can be recovered using the same technique but considering horizons up to six months. Most of the literature now uses such horizons to empirically estimate a risk-return relation. An striking exception is Bali and Peng (2006) who only focus on the one-day ahead risk-return trade-off. Interestingly, their empirical results are very significant and their contribution unambiguously sheds light on a short-term risk-return trade-off for the S&P 500.

In the present paper, we consider the same issue as in Bali and Peng (2006) but use an alternative measure of risk that has been recently suggested in Patton and Sheppard (2011): signed jumps. While Bali and Peng (2006) mainly use realized variance (and realized volatility) computed using intraday data, we complement their analysis with the signed jump component defined as the difference between positive and negative realized semivariances. The jump component in asset returns has received much attention last years and we do believe that their contribution to the risk-return is worth an investigation. Two notable exceptions already examined the possible role of jumps in the ICAPM framework. In his Section 6.2, Evans (2011) investigates the predictability of returns using the measures developed in Tauchen and Zhou (2011). The author finds evidence of return predictability but his work does consider much larger horizons (several months) than we do. S´evi and Baena (2012) study the contribution of rare jumps defined following Barndorff-Nielsen and Shephard (2004, 2006) for the one-day ahead risk-return trade-off and only find mixed evidence for the contributing role of jumps.

The particular advantage of signed jumps is that, contrary to the quadratic jump con-

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2 Realized measures are much less noisy estimates of conditional risk than more standard measures used in the literature. This might partly explain the strong empirical findings in Bali and Peng (2006).

3 The next Section present how signed jumps are formally defined.

4 The particular role of jumps has been emphasized for volatility forecasting (Giot and Laurent (2007), Andersen et al. (2007), Patton and Sheppard (2011) and Corsi et al. (2011)), in the volatility volume debate (Giot et al. (2011)), in modeling excess bond premia (Wright and Zhou (2009)) or the credit spread at the aggregate level (Tauchen and Zhou (2005)) or for individual firms (Zhang et al. (2009)).
tribution used in the aforementioned papers, we can observe the direction of the jump component during a given day which may be of central importance for the issue considered here.  

Our results point to the limited contribution of signed jumps to the risk-return trade-off for the S&P 500 front-month futures. Empirical evidence from rolling window regressions show that signed jumps only marginally contribute to explain future returns thereby highlighting the important role of the continuous risk component in shaping the risk-return relation.

The rest of the note is as follows. The next section briefly presents the concept of signed jumps recently developed in Patton and Sheppard (2011). Section 3 provides our empirical estimates of the role of signed jumps in a standard one-day ahead predictive regression framework. Section 4 provides concluding remarks.

2 Signed jumps

Patton and Sheppard (2011) study the forecasting accuracy of different realized measures of volatility. In particular, they emphasize the essential role of negative realized semivariance in predicting the future realized volatility. Interestingly, the authors show that positive realized volatility has very limited forecasting power with respect to the future realized volatility. Let $RV_{t,N}$ be the realized variance (the square of realized volatility) formally defined as follows:

$$RV_{t,N} = \sum_{i=1}^{N} r_{t,i}^2,$$ (1)

The realized variance is computed as the sum of the $r_{t,i}$ which are intraday returns ($r_{t,i} = p_{t,i} - p_{t,i-1}$ for $i = 1, ..., N$). $p_{t,i}$ are intraday observations allowing to compute $N$ continuously compounded intraday returns each day so the realized variance is the sum

Identifying jumps in a stochastic process is important because it has implications for risk management, option pricing, portfolio selection and also has consequences for optimal hedging strategies. The impact of jumps in returns and volatility is studied in Andersen et al. (2002), Eraker et al. (2003), Chernov et al. (2003), Eraker (2004), Broadie et al. (2007). A risk premium (jump risk premium) can be raised in reference to jumps (Pan (2000) or Li (2011)). Our work is based on nonparametric estimates of jumps which is a different approach than in all these papers.
of the $N$ equally spaced squared intraday returns.

Following Barndorff-Nielsen et al. (2010), it is possible to define the negative realized semivariance:

$$RSV_{t,N}^- = \sum_{i=1}^{N} r_{t,i}^2 I[r_{t,i} < 0],$$

(2)

which may be a proxy for downside risk. Positive realized semivariance can be defined in a similar manner when only positive intraday returns are considered:

$$RSV_{t,N}^+ = \sum_{i=1}^{N} r_{t,i}^2 I[r_{t,i} > 0].$$

(3)

By subtracting the negative realized semivariance from the positive one, Patton and Sheppard (2011) define signed jumps:

$$\Delta J_t^2 = \sum_{i=1}^{N} r_{t,i}^2 I[r_{t,i} > 0] - \sum_{i=1}^{N} r_{t,i}^2 I[r_{t,i} < 0].$$

(4)

Interestingly, signed jump variation in day $t$ as computed in Eq. (4) is a signed measure of the squared jump component following the asymptotic limit of realized semivariance presented in Barndorff-Nielsen et al. (2010). As such, this is an alternative measure to the well-known squared jump variation extracted using multi-power variations theory in Barndorff-Nielsen and Shephard (2004, 2006) which is not signed. Indeed, the sign may be of utmost importance when looking at a relation between risk and return.

3 Empirical findings

3.1 Data

We use transaction (tick) data from January 2, 1996 to July 31, 2008. A continuous time-series is built with the front-month contract S&P 500 futures and we check the length of the trading period each day so that reliable estimates for realized quantities

Note, in addition, that the measure is nonparametric as realized semivariances are themselves nonparametric. As such, as mentioned in the Introduction, our analysis departs from numerous studies (Pan (2000), Eraker et al. (2003), Chernov et al. (2003), Eraker (2004), Broadie et al. (2007), Li (2011)) where a the existence of a risk premium for the jump risk is investigated in a parametric framework.
can be computed.\footnote{Days with shortened trading period are removed from the sample.} We end with 3166 days where all these requirements are met. The average number of trades for these days is 3,090 and this variable is quite stable during the period under consideration. We only consider the S&P 500 futures contract because both the cash index and the CRSP Value-Weighted Index that have been considered in Bali and Peng (2006) have some notable drawbacks. In particular, these are not tradable assets and thus no available transaction data exist. Martens (2002) further indicates that using futures data when considering high-frequency measures helps to deal with the well-known asynchronicity issue (in a large index, many stocks are not traded in some intraday intervals) present in cash-index.

Figure 1
Annualized volatility computed from realized variance

Figure 1 plots the annualized volatility calculated using realized variance, itself computed using 5-minute returns. The signed jumps over the same period are plotted in Figure 2. The most notable feature for these two series is the time-varying behavior of the realized volatility and signed jumps with some clustering for both series.

3.2 Empirical methodology

As for most of the contributions aiming at testing the existence of a risk-return trade-off, we run regressions of the following general form:
\[
rt - rf,t = \mu + \gamma E_{t-1}[RISK_t] + \Pi X_{t-1} + \varepsilon_t,
\]
where the \(E_{t-1}[RISK_t]\) stands for the expectation up to \(t - 1\) of a conditional measure of risk, \(X_{t-1}\) represents a vector for the hedging component and \(\gamma\) is an estimate of the relative risk aversion coefficient. In fact, Eq. (5) is a testable form of the theoretical relation in Merton (1973): \(E_{t-1}[r_t - rf,t] = \gamma E_{t-1}[\sigma_t^2]\), where the conditional expected excess return of the stock market index should be a linear function of the expectation of the conditional variance plus a hedging component.

In our empirical work, we will allow for several measures of risk: lagged values of realized variance, signed jumps and implied variance as well as their square-root and log transformations. We will thus be able to compare the contribution of these three measures for the existence of an empirical trade-off between risk and return at the daily horizon. As for modelling the hedging component, we will thoroughly follow Bali and Peng (2006) in using three financial proxies: \(FED\), \(DEF\) and \(TERM\). \(FED\) is the federal funds rate, \(DEF\) is the default spread calculated as the difference between the yields on BAA- and AAA-rated corporate bonds and \(TERM\) is the term spread calculated as the difference between the yields on the 10-year Treasury bond and the three-month Treasury bill.
An important characteristic of our estimation of Merton’s (1973) result is that we use lagged values as optimal predictors for risk measures. Bali and Peng (2006) also use lagged values in their main analysis due to the high persistence of realized volatility which exhibits long-memory. An alternative is to use fitted values from ARMA models but the analysis is then prone to the look-ahead bias. \(^8\)

### 3.3 Full sample evidence

While full sample estimates may be of little interest for a phenomenon such as the risk-return trade-off, as discussed in Chou et al. (1992), we first consider the general Eq. (5) to gauge the overall significance of our risk measures for predicting returns. Table 1 reports results from different specifications of Eq. (5). The first row estimates the most standard specification in Bali and Peng (2006) where the endogenous variable is the daily excess return for the S&P 500 front-month futures contract\(^9\) for day \(t\) and the single exogenous variable is the realized variance computed from intraday returns for day \(t - 1\).\(^10\) The slope is then an estimate of the relative risk aversion coefficient as developed in Merton (1973). Here, for the period 1996-2008, we obtain an estimate of 6.16, significant at the 1% threshold using the Newey-West correction for the \(t\)-stat. This estimate is in line with existing estimates where the relative risk aversion coefficient is generally found to lie between 1 and 8. The regression has a limited explanatory power, very similar to the one in Bali and Peng (2006), because daily data are noisy variables. In summary, we confirm the findings in Bali and Peng (2006) who analyzed the 1986-2002 period.

\(^8\)Bali and Peng (2006) provide such results in their robustness check Section, in line with classical contribution (see French et al. (1987)). We also experimented with the HAR model developed in Corsi (2009) which has good properties for mimicking the long-memory behavior of variance series and delivering good risk forecasts. Our results are better with lagged values so we do not report results for the HAR model here.

\(^9\)Bali and Peng (2006) also consider the S&P 500 cash index and the CRSP Value-Weighted Index but find qualitatively and quantitatively similar results for the three series.

\(^10\)For a discussion of the microstructure noise which is likely to bias empirical estimates of realized variance, see Hansen and Lunde (2006). A very large number of alternative estimators have been proposed in the literature to deal with this problem. Because we use S&P 500 futures data, which is a very liquid asset, and has been used previously in the rest of the literature with a 5-min sampling interval, giving satisfactory results, we adopt the same sampling frequency here. We also computed the TSRV estimator of Zhang et al. (2005) and found similar results in all our regressions. Results are available upon request from the authors.
Using realized variance in a regression framework gives rise to the generated regressor problem as advocated in Pagan and Ullah (1988) for the particular case of risk terms. The variance in the estimation of realized variance is low when the sampling frequency converges to infinity but it could well be significant with a 5-minute sampling interval. To deal with the generated regressor problem, Bali and Peng (2006) provide robustness check results using an instrumental variables (IV) approach with GMM where instruments are chosen to be lagged values of realized variance with lags going from 2 to 5. We estimate the same regression and found a slope of 5.59 with a Newey-West adjusted t-statistic of 2.99. This result allows to conclude that the generated regressor problem has no impact on our empirical findings.

The results about the contribution of signed jumps in the risk-return trade-off are reported in the second row of Table 1. We observe that the estimated coefficient for the slope, while negative, is not significant at any standard level. This conclusion does not strictly imply that signed jumps do not participate in shaping the risk-return trade-off at the daily horizon. Indeed, they may have a high explanatory power during some periods and a low contribution in other periods and their overall impact might then be hidden in a

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>RV&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>ΔJ&lt;sup&gt;2&lt;/sup&gt;</th>
<th>VIX&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>FED&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>DEF&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>TERM&lt;sub&gt;t-1&lt;/sub&gt;</th>
<th>Adj. R&lt;sup&gt;2&lt;/sup&gt;</th>
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<tbody>
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<td>Constant</td>
<td>-0.0004*</td>
<td>6.1615***</td>
<td>-2.8336</td>
<td>-0.0011</td>
<td>6.3738***</td>
<td>-1.9817</td>
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<tr>
<td></td>
<td>(-1.6616)</td>
<td>(2.9517)</td>
<td>(-1.1434)</td>
<td>(-1.3726)</td>
<td>(3.6571)</td>
<td>(-1.5911)</td>
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<tr>
<td></td>
<td>1.0489e-04</td>
<td>-0.0007*</td>
<td>5.1644**</td>
<td>0.0009</td>
<td>6.5224***</td>
<td>-7.4461e-06</td>
<td>0.11%</td>
</tr>
<tr>
<td></td>
<td>(1.3076)</td>
<td>(-1.9394)</td>
<td>(2.2742)</td>
<td>(0.4707)</td>
<td>(3.0335)</td>
<td>(-0.0301)</td>
<td></td>
</tr>
</tbody>
</table>

Relation between daily excess market return and daily estimated risk (realized variance using 5-minute returns in this table). The dependent variable is the one-day-ahead excess return on the S&P 500 index futures and the risk free rate is the equivalent one-day rate computed from the three month Treasury bill. All regressions are estimated using ordinary least square. Standard deviations are computed using Newey-West (1987) HAC. Estimated coefficients are those of Eq. (5). Asterisks indicate statistical significance at the 1% (***) or 5% (**) level.
full sample regression. However, it appears that the contribution of signed jumps for the risk-return trade-off is likely to be rather limited.

The row three of Table 1 shows the contribution of signed jumps when realized variance is also considered. This idea is in line with Patton and Sheppard’s investigation of the contribution of signed jumps in forecasting realized variance. The authors consider signed jumps along with various forms of realized variance (at various horizons). Our results indicate that signed jumps are near to be significant at the 10% threshold when considered with lagged realized variance. This indicates some complementarities between signed jumps and the standard realized variance while also indicating the rather low contribution of signed jumps in shaping the risk-return trade-off.

The penultimate row in Table 1 provides similar evidence to Bali and Peng (2006) that lagged VIX operates as a good measure risk to recover the risk-return trade-off at the daily horizon. The estimated relative risk aversion coefficient is comparable with the one using realized variance but is less significant. This may be due to the calibration of the VIX which is computed using derivatives with an 22-day horizon. However, the VIX remains an interesting measure of risk to recover a relation between expected risk and return. Kanas (2012) includes the VIX measure in the variance equation of a GARCH model and provides evidence of significant positive risk-return relation for the S&P 100.

Finally, the last row in Table 1 reports results for a regression including the realized variance as well as control variables. The aim of this specification is to control for the hedging component described in the original contribution by Merton (1973) and further studied in Scruggs (1998) and Guo and Whitelaw (2006) among others. When significant alternative investment opportunities are available, the risk-return relation may be altered. To recover the original relation, it is necessary to empirically consider a set of proxies so that the hedging component is included in the risk-return regression. Here, we use the same set of proxies as in Bali and Peng (2006) thereby allowing for a possible comparison. The estimates for the FED, DEF and TERM variables are not significant.

\[\text{To obtain comparable results with the realized variance regression, we use the daily implied variance which is obtained from the VIX as follows: } \left[\frac{\text{VIX}}{(100 \times \sqrt{252})}\right]^2.\]
thus confirming the original results in Bali and Peng (2006).\footnote{Of course, insignificant control variables for the hedging component raise the issue of the choice of these variables. We do not pursue here a systematic search of more adequate variables but use similar variables as in Bali and Peng (2006) to allow for comparison as already mentioned.}

### 3.4 Time-varying empirical estimates

Lettau and Ludvigson (2009) provide theoretical arguments for such time-variability and discuss existing contribution on this issue. We thus now present empirical evidence based on rolling windows to gauge the time-variability of the risk-return trade-off. Bali and Peng (2006) also rely on this methodology which provides simple time-varying estimates while avoiding the computational difficulties of Kalman filter (or more generally time-varying)
estimation for the coefficients of the regression.

The length of the rolling windows will be either 1000 or 2000 days to ensure that our results do not depend on the sample size.\textsuperscript{13} Figures 3 and 4 plot the estimates of the slope in regression (5) for the lagged realized variance and lagged signed jump, respectively, using 1000 days for estimation. While the relative risk aversion coefficient estimate is positive in a realistic range and significant half of the time, the estimate for signed jumps is negative and only significant for a short period of time (about one year) at the end of the sample.\textsuperscript{14}

Empirical conclusions are quite different for the rolling windows of 2000 days (around eight years of daily data). Indeed, the estimate of the relative risk aversion is even more realistic as it is between 2 and 4 and, more importantly, it is significant at the 1\% threshold over the full period. This first result highlights the importance of the time horizon for the estimation of the risk-return trade-off and confirms the main empirical finding in Bali and Peng (2006) of a significant relation between risk and return at the daily horizon using intraday data for the estimation of the variance (risk). As for signed jumps, the empirical evidence is unambiguous about their role in shaping the risk-return relation at the daily horizon. The estimate is never significant thereby pointing the superiority of realized variance, including the continuous and the jump component, as a factor driving the risk-return relation. This confirms the main result in Sévi and Baena (2012) that the jump component estimated in its quadratic form using the testing methodology in Barndorff-Nielsen and Shephard (2004, 2006) does not help in estimating the risk-return trade-off, at least at the daily horizon.

4 Conclusion

This note investigates the role of signed jumps in shaping the so-called risk-return trade-off. The chosen horizon for the analysis is the daily horizon as in Bali and Peng (2006).

\textsuperscript{13}It is 2177 days in Bali and Peng (2006).
\textsuperscript{14}Note that we do not include control variables for the hedging component in the regressions using rolling windows on the basis of the empirical evidence of their insignificance provided in the previous section.
We show that signed jumps, while convenient tools for a daily analysis as they do not cluster at zero most of the time, have only limited explanatory power for future returns.

Our results at the daily horizon contrast with those of Brekenfelder and Tédongap (2012). The authors define the “ARV measure of risk” (Eq. 8) which is exactly similar to the signed jump measure in Patton and Sheppard (2011) but investigate the risk-return trade-
Figure 5
Rolling window estimation of the RRA coefficient in Eq. (5) using 2000 observations and RV

Figure 6
Rolling window estimation of the slope coefficient in Eq. (5) using 2000 observations and signed jumps
off at larger horizons than we do. The ARV measure is aggregated at four or five years while the future return is computed over intervals going from one to six months. Evidence of a significant negative relation is found for all horizons thereby indicating the importance of the time period for recovering a risk-return trade-off.
References


