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Comparative Advantage and Skill Premium of Regions

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Abstract

This paper provides one explanation for why a positive correlation is observed between the skill premium and income of regions. In doing so, this paper provides a model of self-organized sorting and skill premium with a continuum of heterogeneous individuals as well as a continuum of industries or tasks within a production process. It is found that the positive correlation emerges through the interaction between the location-occupation choice by individuals and regional comparative advantage. Spatial equilibrium, sorting, and product differentiation play a key role in determining the way in which such an interaction works.

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1 Introduction

This paper provides one explanation for why a positive correlation between the size and skill premium of a region emerges by providing a comparative advantage model with a continuum of mobile heterogeneous individuals as well as a continuum of final goods sectors that are different in terms of their skill intensities of intermediate goods. All individuals choose their occupations depending on their productivity, and any occupation can freely migrate across regions unlike footloose entrepreneur models such as Forslid and Ottaviano (2003). This location-occupation choice then interacts with the regional comparative advantage in final goods sectors which depends on the regional offer prices of two different types of intermediate goods, one of which features monopolistic competition à la Dixit and Stiglitz (1977). Although regions are ex-ante identical, interactions between individuals' location-occupation choices and regional comparative advantage result in a self-organized positive correlation between the skill premium and income of regions. The theory can also accommodate the interpretation that the regional difference in skill premium is caused by specialization in task trade within firms, not industries.

The basic mechanism is simple: Since regions are ex-ante identical in their environment including land, some initial shock or history which reallocates the economy's expenditure across regions unevenly results in a cross-region variation in land rents. Free migration of workers is then associated with a compensating differential, i.e., wage rates in regions with higher land rents must be associated with higher wage rates in order for workers to reside in such regions. Because of cross variation in factor prices, regions with higher prices have no comparative advantage in producing non-differentiated intermediate goods. However, by making the average productivity of high-skilled workers higher through sorting, higher land rents give such regions a comparative advantage in producing skill-intensive intermediate goods. Reflecting this regional comparative advantage, final goods sectors relocate across regions, and such relocation of industries makes the initial reallocation of expenditures sustainable. Thus, a positive correlation between skill premium and the size of regions is observed.

This paper is related to at least two lines of research. The first concerns trade models with Ricardian comparative advantage. The current model is an application of Matsuyama (2013) to the regional context. His model is basically an extension of Dornbusch *et al.* (1977), where the comparative advantage of countries is determined endogenously through firms' entry into a monopolistically competitive sector and the number of countries is increased arbitrarily. Although one of his motivations is to construct a theory of income distribution across a large number of countries, I focus on a two-region case. Unlike the international economy, the regional economy is more complicated in that individuals are mobile across regions, which makes it difficult to derive the distribution of regional income explicitly. In addition to individuals' mobility, the current model differs from Matsuyama's (2013) in that individuals choose their occupations, to be either workers or entrepreneurs; that there are two types of intermediate goods sectors, one of which is characterized by monopolistic competition; and that land, which is one of the usual elements in the urban economics literature, is introduced.

The second line of research involves models of the spatial sorting of individuals. Amongst these, Davis and Dingel (2012) is the most related in the sense that it shares the same motivation and the assumption of identical cities or regions and zero trade costs of some goods. Although both studies feature a self-organized positive correlation between the skill premium and the size of regions, the key mechanism is quite different. In their paper, knowledge exchange works as an agglomeration force, while this force is regional specialization in different industries or tasks in mine.

The remainder of this paper is organized as follows: I first introduce the model in Section 2. I summarize the basic mechanism working through general equilibrium and conduct a numerical exercise in order to verify it in Section 3. In Section 4, I conclude the paper.

2 The Model

The economy consists of two ex-ante identical regions: Region 1 and Region 2. Each region is endowed with one unit of land, which is owned by a competitive landowner outside the economy. Individuals, the mass of which is normalized to unity, are ex-ante heterogeneous in their entrepreneurial productivity, and they choose their occupation, to be a worker or entrepreneur, depending on their productivity as well as their residential choice. There is a $[0, 1]$-continuum of final goods sectors, each of which is different in its share parameters of two types of intermediate goods: labour- and skill- intensive intermediate goods.

2.1 Final Goods Sectors

Competitive final goods sectors exist on a $[0, 1]$ interval. Each sector $s \in [0, 1]$ uses a Cobb-Douglas production technology with constant returns to scale, inputs of which are local differentiated skill-intensive intermediate goods and local homogeneous labour-intensive intermediate goods. $\gamma(s) \in [0, 1]$ is the share parameter of the former, and $\gamma'(s) > 0$.

The location of each sector s is determined through competition, resulting in the price $P(s)$ of sector-s final good which is equal to the lowest unit cost of production. Letting $\chi_j(s)$ and $\mathbb{S}_j \subseteq [0, 1]$ denote the unit cost of production of sector s active in Region j and the set of sectors active in Region j, respectively, it holds that $P(s) = \chi_j(s)$ if $s \in \mathbb{S}_j$.

Formally, a typical firm in sector s residing in Region j, i.e., $s \in \mathbb{S}_j$, solves

$$\max_{\{M_{i,j}(s)\}_{i\in\{E,L\}}\{m_{i,j}(\varphi,s)\}_\varphi} P(s)M_{E,j}(s)^{\gamma(s)}M_{L,j}(s)^{1-\gamma(s)} - \int_0^\infty p_{E,j}(\varphi)m_{E,j}(\varphi,s)N_{E,j}g_j^*(\varphi)d\varphi - P_{L,j}M_{L,j}(s)$$

$$s.t.$$

$$M_{E,j}(s) = \left[\int_0^\infty m_{E,j}(\varphi,s)^{\frac{\sigma-1}{\sigma}}N_{E,j}g_j^*(\varphi)d\varphi\right]^{\frac{\sigma}{\sigma-1}}.$$

$m_{E,j}(\varphi, s)$ is the sector-s demand for a variety of skill-intensive intermediate goods produced locally by a firm with productivity φ (hereafter variety-φ skill-intensive intermediate good).[1] $N_{E,j}$ is the mass of skill-intensive intermediate goods or entrepreneurs in Region j. $g_j^*(\varphi)$ denotes the density function of productivity conditional on location. These differentiated goods are aggregated to $M_{E,j}(s)$ by technology with constant elasticity $\sigma > 1$ of substitution. $M_{L,j}(s)$ is the sector-s demand for homogeneous labor-intensive intermediate goods produced locally. Prices are denoted by

[1]Subscripts E and L are used when making it explicit that variables or parameters with subscripts E and L are specific to the skill- and labor- intensive intermediate goods sectors, respectively. Subscript E is used because the skill-intensive intermediate goods are produced by entrepreneurs.

$p_{E,j}(\varphi)$ and $P_{L,j}$ for variety-φ skill-intensive intermediate good and homogeneous labor-intensive intermediate goods, respectively.

Profit maximization implies the following demand for variety-φ skill-intensive good:

$$m_{E,j}(\varphi, s) = \left[\frac{p_{E,j}(\varphi)}{P_{E,j}}\right]^{-\sigma} M_{E,j}(s), \tag{1}$$

where $P_{E,j}$ is the price index of Region-j skill-intensive intermediate goods defined by

$$P_{E,j} \equiv N_{E,j}^{-\theta} \left[\int_0^\infty p_{E,j}(\varphi)^{-\frac{1}{\theta}} g_j^*(\varphi) d\varphi\right]^{-\theta}, \quad \theta \equiv 1/(\sigma - 1). \tag{2}$$

2.2 Labour-intensive Intermediate Goods Sectors

The local labour-intensive intermediate goods sector in each region is competitive. Firms can access a Cobb-Douglas production technology with constant returns to scale, the inputs of which consist of workers' labour services $L_{L,j}$ and land $T_{L,j}$.

$$\max_{L_{L,j}, T_{L,j}} P_{L,j} B L_{L,j}^{\beta_L} T_{L,j}^{1-\beta_L} - W_{L,j} L_{L,j} - R_j T_{L,j}, \quad B \equiv \beta_L^{-\beta_L}(1-\beta_L)^{-(1-\beta_L)},$$

where $\beta_L \in (0, 1)$ and R_j are the share parameter of labour and land price in Region j.

2.3 Skill-intensive Intermediate Goods Sectors

The local skill-intensive intermediate goods sector is characterized by monopolistic competition à la Dixit and Stiglitz (1977), where each entrepreneur produces one variety of goods using workers' labour services and land as production inputs. Specifically, each entrepreneur must rent f units of land for her office and then use workers' labour services and land as variable inputs.

Therefore, the income $\pi_j(\varphi)$ of an entrepreneur residing in Region j with productivity φ is given by her sales net of input costs:

$$\begin{aligned} \pi_j(\varphi) &= \max_{p_{E,j}(\varphi), q_{E,j}(\varphi)} \left[p_{E,j}(\varphi) - W_j^{\beta_E} R_j^{1-\beta_E} \varphi^{-1}\right] q_{E,j}(\varphi) - R_j f \\ & s.t. \\ & q_{E,j}(\varphi) = \int_{\mathbb{S}_j} m_{E,j}(\varphi, s) ds = \int_{\mathbb{S}_j} \left[\frac{p_{E,j}(\varphi)}{P_{E,j}}\right]^{-\sigma} M_{E,j}(s) ds, \end{aligned}$$

where $q_{E,j}(\varphi)$ is the output of variety-φ skill-intensive intermediate good produced in Region j. Here, it is assumed that the unit cost of production is some amount of the Cobb-Douglas composite of workers' labour services and land, in which β_E governs the labour cost share.

The associated optimal pricing rule is then $p_{E,j}(\varphi) = (1+\theta) W_j^{\beta_E} R_j^{1-\beta_E} \varphi^{-1}$. Substituting this into (2) results in

$$P_{E,j} = (1+\theta) W_j^{\beta_E} R_j^{1-\beta_E} \left(\tilde{\varphi}_j N_{E,j}^{\theta}\right)^{-1}, \tag{3}$$

where $\tilde{\varphi}_j$ is the average productivity in Region j defined by

$$\tilde{\varphi}_j = \left[\int_0^\infty \varphi^{\frac{1}{\theta}} g_j^*(\varphi) d\varphi\right]^{\theta}. \tag{4}$$

In the following, I assume that the entrepreneurial productivity φ follows a Pareto distribution with coefficient δ and a lower bound $\underline{\varphi}$. Under the assumption that $\delta > 1/\theta$, the individual variable profit $\pi_j^V(\varphi)$ and output $q_j(\varphi)$ are expressed as functions of the productivity ratio $\varphi/\tilde{\varphi}_j$ and the average variables as in Melitz (2003):

$$\pi_j^V(\varphi) = \left(\frac{\varphi}{\tilde{\varphi}_j}\right)^{\frac{1}{\theta}} \pi_j^V(\tilde{\varphi}_j), \qquad \pi_j^V(\tilde{\varphi}_j) = \theta W_j^{\beta_E} R_j^{1-\beta_E} \tilde{\varphi}_j^{-1} N_{E,j}^{-(1+\theta)} \int_{\mathbb{S}_j} M_{E,j}(s)ds, \quad (5)$$
$$q_j(\varphi) = \left(\frac{\varphi}{\tilde{\varphi}_j}\right)^{\sigma} q_j(\tilde{\varphi}_j), \qquad q_j(\tilde{\varphi}_j) = N_{E,j}^{-(1+\theta)} \int_{\mathbb{S}_j} M_{E,j}(s)ds.$$

2.4 Individuals

Individuals are ex-ante heterogeneous in their entrepreneurial productivity φ. Depending on this productivity, each individual chooses her occupation and location freely in order to maximize her utility. Let $U_j(\varphi)$ and $e_j(\varphi)$ denote the utility and income of an individual having the productivity of φ and residing in Region j.

2.4.1 Occupational Choice

Suppose that an individual chooses to reside in Region j. Then she chooses the occupation which maximizes her income. Thus, her income $e_j(\varphi)$ is given by $e_j(\varphi) = \max\{\pi_j(\varphi), W_j\}$. This suggests that there exists a cut-off level φ_j^* such that

$$W_j = \sigma^{-1} \tilde{A}_j {\varphi_j^*}^{\frac{1}{\theta}} - R_j f, \quad (6)$$

where $\tilde{A}_j$ denotes the per-capita market size of the skill-intensive intermediate goods sector in Region j normalized by the regional average productivity, i.e., $\alpha\Gamma_j|\mathbb{S}_j|E/N_{E,j}$ divided by $\tilde{\varphi}^{\frac{1}{\theta}}$, which is derived in Appendix A.2.

For the given income $e_j(\varphi)$ as well as the given location j, each individual then consumes final goods and housing services:

$$\begin{aligned} U_j(\varphi) \quad = \quad & \max_{\{c_j(s,\varphi)\}_{s\in[0,1]}, h_j(\varphi)} \exp\left[\alpha \int_0^1 \ln(c_j(s,\varphi))ds\right] h_j(\varphi)^{1-\alpha}, \quad \alpha \in (0,1), \\ & s.t. \\ & \int_0^1 P(s)c_j(s,\varphi)ds + R_j h_j(\varphi) = e_j(\varphi), \end{aligned}$$

where α is the expenditure share of the consumption goods. $c_j(s,\varphi)$ and $h_j(\varphi)$ denote the quantities of goods and housing services, respectively, consumed by an individual with φ residing in Region j.

2.4.2 Residential Choice

Finally, each individual chooses her location in order to maximize her utility.

The following result then states that if both regions host a positive measure of production activities, or, stated more weakly, if there exists a threshold $\bar{\varphi}$ such that entrepreneurs with productivity of $\bar{\varphi}$ are indifferent between the two location choices, the sorting of entrepreneurs is always associated:[2]

Proposition 1. *Suppose that there exists $\bar{\varphi}$ such that $\bar{\varphi} > \max_j\{\varphi_j^*\}$ and*

$$u(\bar{\varphi}) = \frac{\pi_2(\bar{\varphi})/(P^\alpha R_2^{1-\alpha})}{\pi_1(\bar{\varphi})/(P^\alpha R_1^{1-\alpha})} = \frac{\pi_2(\bar{\varphi})/\pi_1(\bar{\varphi})}{(R_2/R_1)^{1-\alpha}} = 1.$$

Then, if $R_1 < R_2$, it holds that

$$\left(\frac{R_2}{R_1}\right)^{1-\alpha} < \frac{\tilde{A}_2}{\tilde{A}_1} < \frac{R_2}{R_1}, \ and$$

$\pi_2(\varphi)/\pi_1(\varphi)$ is monotonically increasing in a well-defined region. Or, if $R_1 = R_2$, it must hold that $\tilde{A}_1 = \tilde{A}_2$ and thus $u(\varphi) = 1$ for all $\varphi \geq \underline{\varphi}$.

3 Equilibrium Analysis

3.1 Basic Mechanism

In the following, I focus on the case where regions are ex-post heterogeneous. Specifically, without loss of generality, I focus on equilibria in which the land rent in Region 2 is greater than that in Region 1, i.e., $R_1 < R_2$. Therefore, given Proposition 1, an interior equilibrium is associated with a unique threshold $\bar{\varphi}$ such that $\bar{\varphi} > \max_j\{\varphi_j^*\}$ and there is a spatial sorting of entrepreneurs. In this case, individuals with φ higher than or equal to $\bar{\varphi}$ reside in Region 2 and work as entrepreneurs. Those with φ less than $\bar{\varphi}$ but higher than or equal to φ_1^* reside in Region 1 and also work as entrepreneurs. Workers consist of individuals with φ less than φ_1^*. Since workers' income is independent of φ, the following free-migration condition or compensated differential for workers must be satisfied:

$$\frac{W_1}{P^\alpha R_1^{1-\alpha}} = \frac{W_2}{P^\alpha R_2^{1-\alpha}}, \quad \text{or} \quad \frac{W_2}{W_1} = \left(\frac{R_2}{R_1}\right)^{1-\alpha}, \tag{7}$$

which states that utility levels are equalized across regions.

In order to compute an equilibrium, I use the next result which is obtained immediately:

Proposition 2. *Suppose an asymmetric interior equilibrium exists. Then, the spatial distribution of final goods sectors is summarized by a threshold $S_1 \in (0, 1)$ such that $\mathbb{S}_1 = [0, S_1)$ and $\mathbb{S}_2 =$* $[S_1, 1]$.

The implication of this result for the computation of an equilibrium is that the system of an equilibrium can be now interpreted as a fixed-point problem of S_1. The discussion, which is described in Appendix A, proceeds in two steps. (i) Given the spatial distribution S_1, the system of an equilibrium is consolidated into two simultaneous equations with two unknowns: the ratio of $\bar{\varphi}$ to φ_1^* and the ratio of φ_1^* to $\underline{\varphi}$. All other variables except S_1 are given as functions of these two and

[2] The proof is straightforward and is thus omitted.

S_1. (ii) The rest of the computation is to search for an S_1 that is consistent with the comparative advantage of regions. Stated differently, S_1 must be a solution to the nonlinear equation

$$\frac{\chi_2(S_1)}{\chi_1(S_1)} = \left(\frac{P_{L,2}}{P_{L,1}}\right)^{1-\gamma(S_1)} \left(\frac{P_{E,2}}{P_{E,1}}\right)^{\gamma(S_1)} = 1 \tag{8}$$

and the Region 2-1 ratio $\chi_2(s)/\chi_1(s)$ of offer prices must be decreasing in s. If the latter condition does not hold, all ss greater than or equal to S_1 reside in Region 1, not Region 2, clearly contradicting the assumption.

The intuitive mechanism which can work in the model is summarized as follows: Suppose that some shock hits the economy consisting of two ex-ante identical regions in a way that expenditures concentrate on one of the regions (here Region 2), i.e., $|\mathbb{S}_1| < |\mathbb{S}_2|$. Since both regions have the same amount of land, it then holds that the land rent in Region 2 becomes higher than in Region 1, i.e., $R_1 < R_2$.[3] Because of the free migration of workers or the compensating differential, i.e., (7), the wage rate in Region 2 also becomes higher than that in Region 1, i.e., $W_1 < W_2$. Thus, unit costs or prices of non-differentiated goods are higher in Region 2 than in Region 1, i.e., $P_{L,1} < P_{L,2}$.[4] Instead, because of the sorting of entrepreneurs, i.e., (3), Region 2 has a comparative advantage in producing skill-intensive intermediate goods, i.e., $P_{E,1} > P_{E,2}$. Reflecting these regional advantages, the spatial distribution of final goods sectors settles down in such a way that the reallocation of expenditures caused by the initial shock is actually preserved as an equilibrium outcome.

3.2 Numerical Exercise

In order to verify the mechanism in the previous subsection, I resort to a numerical exercise. The result shows that an equilibrium with such a mechanism actually exists. It is also verified that the equilibrium is unique in the sense that there is only one interior sorting equilibrium with the assumed regional rankings of variables.

In this exercise, parameters are set as follows: the elasticity of substitution σ between skill-intensive intermediate goods is set to 3. The expenditure share α of final goods is set to 0.7. The lower bound $\underline{\varphi}$ of the Pareto distribution of entrepreneurial productivities is set to 1. The coefficient δ of the Pareto distribution is set to 4.2. The labour share parameter of the labour-intensive intermediate goods sector is set to 0.6. The same number is used for the labour share in variable costs of the skill-intensive intermediate goods sector. The fixed requirement f of land is set to 1. This value is chosen in a way that the demand of entrepreneurs for land does not substantially affect land prices, and these prices are mainly determined by housing expenditures and housing demands associated with variable inputs. As for the specification of $\gamma(s)$, I simply assume that $\gamma(s) = s$ for $s \in [0, 1]$.

The equilibrium is summarized by Figures 1 and 2. The lower panel of Figure 1 depicts the relationship between the entrepreneurial productivity φ and the wage and entrepreneurs' profit schedules for each region. As already mentioned, the wage schedule is flat since workers' income

[3]Strictly speaking, this relationship between the two rankings holds only if expenditures are the most important determinant of land rents, as suggested by the land market clearing condition (18) derived in Appendix A.

[4]Note that the price of homogeneous labour-intensive intermediate goods is a weighted geometric mean of the wage rate and land rent.

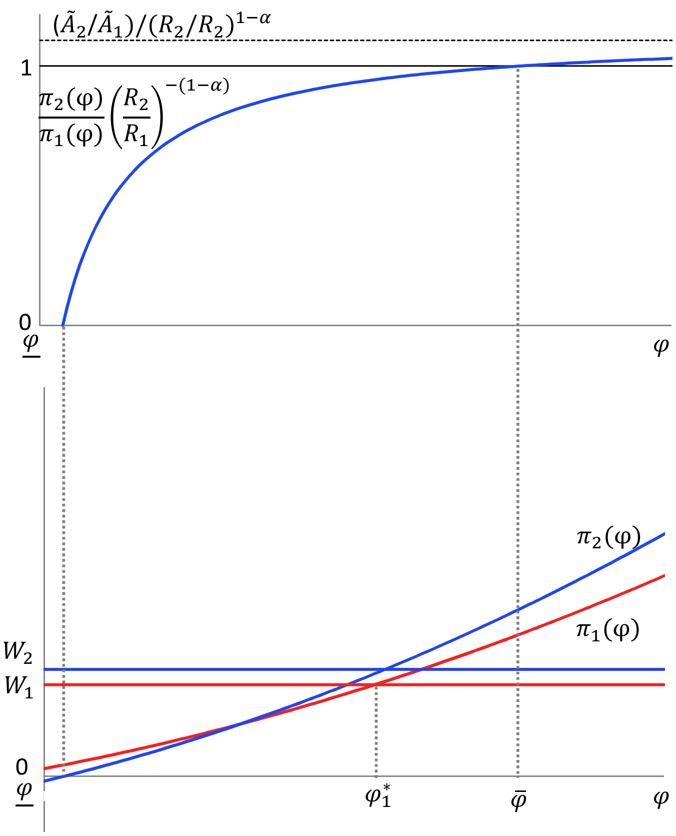

Figure 1: Region 2-1 Ratio Utility Conditional on Choosing to Become an Entrepreneur (The Upper Panel) and Entrepreneurs' and Workers' Incomes (The Lower Panel)

is independent of their entrepreneurial productivity φ. Meanwhile, entrepreneurs' income $\pi_j(\varphi)$ is monotonically increasing in φ. That $W_1 < W_2$ is implied by the free-migration condition for workers together with the ranking $R_1 < R_2$. As for $\pi_j(\varphi)$, it is not always the case that $\pi_1(\varphi) < \pi_2(\varphi)$ for all φ. What is important here is that $u(\varphi)$ is monotonically increasing in φ (Proposition 1) and that $u(\varphi) = 1$ at $\bar{\varphi}$, which are shown in the upper panel of Figure 1.

Figure 2 shows that the Region 2-1 ratio $\chi_2(s)/\chi_1(s)$ of offer prices is monotonically decreasing in s, and there actually exists a threshold S_1 which summarizes the spatial distribution of final goods sectors. Since $|\mathbb{S}_j|$ is proportional to the regional GDP, the result that $|\mathbb{S}_1| < |\mathbb{S}_2|$ implies that the size of Region 2 is greater than that of Region 1 in terms of income. Importantly, the monotonicity of $\chi_2(s)/\chi_1(s)$ is the consequence of two results: $P_{L,2}/P_{L,1} > 1$ and $P_{E,2}/P_{E,1} < 1$. The former result is simply due to the fact that $R_1 < R_2$ and $W_1 < W_2$ as discussed before. The latter result suggests that there actually exists a case where the cost-reducing effect of product differentiation and sorting on the aggregate price level dominates the cost push because of higher land rents and thus higher wage rates.[5]

[5]It should be noted that the numerical exercise also shows that in equilibrium, Region 1 has a greater number of entrepreneurs than Region 2, i.e., $N_{E,1} > N_{E,2}$. This suggests that for the theory considered in this paper, what is important for Region 2 to have a comparative advantage in producing skill-intensive goods is not the number of entrepreneurs, but the average productivity.

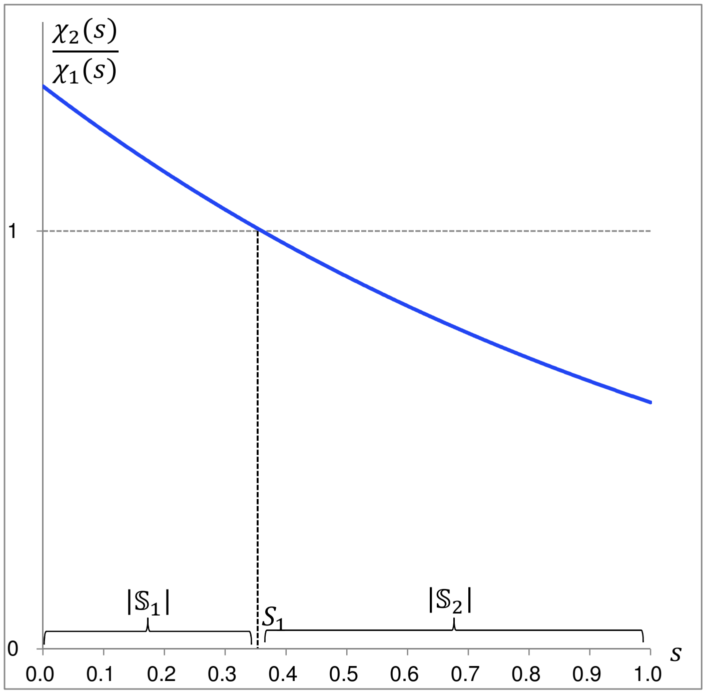

Figure 2: Region 2-1 Ratio of Offer Prices of Final Goods

4 Conclusion

A positive correlation is observed between skill premium and the size of regions, which are measured by the income ratio of high-skilled and low-skilled workers and regional income, respectively. The paper theoretically investigates one possible explanation for this fact by providing a model with heterogeneous individuals and final and intermediate goods sectors, in which ex-ante identical regions specialize in different sectors, and interactions between individuals' location-occupation choices and regional comparative advantage result in the positive correlation between the skill premium and income of regions. The theory can also accommodate the interpretation that the regional difference in skill premium is caused by specialization in task trade, not industries. Although perfect sorting featuring the equilibrium itself is not a crucial element of the theory, filling the gap between the model and reality could be an important direction for future research.

References

Davis, D. R. and Dingel, J. I. (2012) A spatial knowledge economy, NBER Working Papers 18188, National Bureau of Economic Research, Inc.

Dixit, A. K. and Stiglitz, J. E. (1977) Monopolistic competition and optimum product diversity, *American Economic Review*, **67**, 297–308.

Dornbusch, R., Fischer, S. and Samuelson, P. A. (1977) Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods, *American Economic Review*, **67**, 823–839.

Forslid, R. and Ottaviano, G. I. (2003) An analytically solvable core-periphery model, *Journal of Economic Geography*, **3**, 229–240.

Matsuyama, K. (2013) Endogenous ranking and equilibrium lorenz curve across (ex ante) identical countries, *Econometrica*, forthcoming.

Melitz, M. J. (2003) The impact of trade on intra-industry reallocations and aggregate industry productivity, *Econometrica*, **71**, 1695–1725.

A Equilibrium System as a Fixed-Point Problem of S_1

In this section, I show that the equilibrium system of an equilibrium of interest is summarized as a fixed-point problem of S_1.

A.1 Number of Entrepreneurs, Conditional Densities, and Average Productivities

First of all, given the Pareto distribution $g(\varphi)$ of entrepreneurial productivity φ and the ranking of thresholds, i.e., $\varphi_1^* < \bar{\varphi}$, the number $\{N_{E,j}\}_{j=1}^2$ of entrepreneurs in each region, densities $\{g_j^*\}$ of productivity conditional on sorting, and average productivities $\{\varphi_j^*\}_{j=1}^2$ of regions are given as functions of thresholds $(\varphi_1^*, \bar{\varphi})$:

$$N_{E,1} = G(\bar{\varphi}) - G(\varphi_1^*) = \left(\varphi_1^{*-\delta} - \bar{\varphi}^{-\delta}\right)\underline{\varphi}^{\delta}, \tag{9}$$

$$N_{E,2} = 1 - G(\bar{\varphi}) = \left(\frac{\underline{\varphi}}{\bar{\varphi}}\right)^{\delta}, \tag{10}$$

$$g_1^*(\varphi) = \frac{1}{N_{E,1}}\mathbf{1}\{\varphi_1^* \leq \varphi < \bar{\varphi}\}g(\varphi) = \left(\varphi_1^{*-\delta} - \bar{\varphi}^{-\delta}\right)^{-1}\mathbf{1}\{\varphi_1^* \leq \varphi < \bar{\varphi}\}\delta\varphi^{-(\delta+1)}, \tag{11}$$

$$g_2^*(\varphi) = \frac{1}{N_{E,2}}\mathbf{1}\{\bar{\varphi} \leq \varphi\}g(\varphi) = \bar{\varphi}^{\delta}\mathbf{1}\{\bar{\varphi} \leq \varphi\}\delta\varphi^{-(\delta+1)}, \tag{12}$$

$$\tilde{\varphi}_1 = \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta}\left[\frac{(\varphi_1^*/\bar{\varphi})^{\frac{1}{\theta}-\delta} - 1}{(\varphi_1^*/\bar{\varphi})^{-\delta} - 1}\right]^{\theta}\bar{\varphi}, \text{ or } \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta}\left[\frac{1 - (\bar{\varphi}/\varphi_1^*)^{\frac{1}{\theta}-\delta}}{1 - (\bar{\varphi}/\varphi_1^*)^{-\delta}}\right]^{\theta}\varphi_1^*, \tag{13}$$

$$\tilde{\varphi}_2 = \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta}\bar{\varphi}, \tag{14}$$

where $\mathbf{1}\{\cdot\}$ is the indicator function which is equal to one if the statement in the braces is true and zero otherwise.

A.2 Factor Prices as Functions of Three Thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$

Next, $\tilde{A}_j$ is computed as a function of thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$ with the help of market clearing conditions: the Cobb-Douglas preference suggests that the economy-wide expenditure for final goods is given by αE, where E denotes the economy-wide income excluding land rents. If the $\mathbb{S}_j$ set of industries locates in Region j, equal weights of industries in preference and the production technology of the final goods sectors then imply that two market clearing conditions, one for the final goods and the other for the skill-intensive intermediate goods, are consolidated into

$$\int_{\mathbb{S}_j} P_{E,j} M_{E,j}(s)ds = \alpha\Gamma_j|\mathbb{S}_j|E, \quad \text{or} \quad \int_{\mathbb{S}_j} M_{E,j}(s)ds = P_{E,j}^{-1}\alpha\Gamma_j|\mathbb{S}_j|E,$$

where $\Gamma_j \equiv |\mathbb{S}_j|^{-1}\int_{\mathbb{S}_j}\gamma(s)ds$, implying that $\alpha\Gamma_j|\mathbb{S}_j|E = \int_{\mathbb{S}_j}\alpha E\gamma(s)ds$, the sum of expenditures for the skill-intensive intermediate goods in Region j, which in turn is related to consumers' demand. Substituting (3) into this equation, I get

$$\int_{\mathbb{S}_j} M_{E,j}(s)ds = (1+\theta)^{-1}\left(W_j^{\beta_H}R_j^{1-\beta_H}\right)^{-1}\tilde{\varphi}_j N_{E,j}^{\theta}\alpha\Gamma_j|\mathbb{S}_j|E.$$

Finally, substituting this equation into (5) results in

$$\pi_j^V(\tilde{\varphi}) = \sigma^{-1}\frac{\alpha\Gamma_j|\mathbb{S}_j|E}{N_{E,j}},$$

which gives

$$\tilde{A}_j = \frac{\alpha\Gamma_j|\mathbb{S}_j|E}{\tilde{\varphi}_j^{\frac{1}{\theta}}N_{E,j}}. \tag{15}$$

That is, $\tilde{A}_j$ is the normalized average market size of skill-intensive intermediate goods in Region j. Note that given (9)-(14) and Proposition 2, $\tilde{A}_j$ is a function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$.

This derivation of $\tilde{A}_j$ is useful for the computation of factor prices $\{(W_j, R_j)\}_{j=1}^2$ in relating labour and land market clearing conditions with thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$, to which I turn next.

Since the sales of an entrepreneur with productivity φ are $\tilde{A}_j\varphi^{\frac{1}{\theta}}$ and since the variable profit $\pi_j^V(\varphi) = \sigma^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$, the variable cost is equal to $(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$. Thus the Cobb-Douglas technology implies that the associated variable labour and land costs are given by $\beta_h(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$ and $(1-\beta_H)(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$, respectively. Factor market clearing conditions, which aggregate these firm-level costs, then pin down factor prices and the spatial distribution of workers.

The labour market clearing condition for each region is given as follows:

$$N_{E,j}\int_{\underline{\varphi}}^{\infty}\beta_H(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}g_j^*(\varphi)d\varphi + \beta_L\alpha(1-\Gamma_j)|\mathbb{S}_j|E = W_j\lambda_j N_W,$$

where $\lambda_j \in (0,1)$ denotes the share of Region j in workers, and N_W the total number of workers, i.e., $N_W = G(\varphi_1^*) = 1 - (\underline{\varphi}/\varphi_1^*)^{\delta}$. Together with (4) and (15), the first term becomes $\beta_H(1+\theta)^{-1}\alpha\Gamma_j|\mathbb{S}_j|E$, i.e., the clearing condition simplifies to

$$\beta_H(1+\theta)^{-1}\alpha\Gamma_j|\mathbb{S}_j|E + \beta_L\alpha(1-\Gamma_j)|\mathbb{S}_j|E = W_j\lambda_j N_W \quad \text{for all } j = 1, 2. \tag{16}$$

The second term on the left-hand side is the demand from the labour-intensive sector, where the total sales $\alpha(1-\Gamma_j)|\mathbb{S}_j|E$ are derived in a similar way as in the case of the skill-intensive intermediate goods sector, and the Cobb-Douglas technology then implies that a β_L fraction of these must be distributed to workers.

Thus, noting that N_W is a function of φ_1^* and that both Γ_j and $|\mathbb{S}_j|$ are functions of S_1, the labour market clearing condition together with the free-migration condition for workers, i.e., (7), gives the wage rate and the spatial distribution of workers as functions of two thresholds (φ_1^*, S_1) and the land rents ratio R_2/R_1:

$$W_1 = \frac{\tilde{\beta}_1 A_1}{(1-\lambda_2)N_W}, \quad W_2 = W_1 \left(\frac{R_2}{R_1}\right)^{1-\alpha}, \quad \lambda_2 = \frac{\frac{\tilde{\beta}_2 A_2}{\tilde{\beta}_1 A_1}}{\frac{\tilde{\beta}_2 A_2}{\tilde{\beta}_1 A_1} + \left(\frac{R_2}{R_1}\right)^{1-\alpha}}, \quad \lambda_1 = 1 - \lambda_2,$$

where

$$A_j \equiv \alpha|\mathbb{S}_j|E, \quad \tilde{\beta}_j \equiv \Gamma_j \frac{\beta_H}{1+\theta} + (1-\Gamma_j)\beta_L.$$

As for the land market clearing condition, an argument similar to that in the case of the labour market gives land prices as a function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$: the demands for land consists of not only those from firms in both skill-intensive and labour-intensive sectors but also those from individuals, i.e., $(1-\alpha)E_j$, where E_j is Region-j income excluding land rents given by

$$E_j = N_{E,j} \int_{\underline{\varphi}}^{\infty} \pi_j(\varphi) g_j^*(\varphi) d\varphi + W_j \lambda_j N_W = \frac{\theta}{1+\theta} \alpha \Gamma_j |\mathbb{S}_j| E - R_j f N_{E,j} + W_j \lambda_j N_W. \quad (17)$$

Noting that the demands from the skill-intensive sector are further divided into those related to variables costs and those related to fixed costs, the market clearing condition is specified by

$$\begin{aligned} R_j &= N_{E,j} \int_{\underline{\varphi}}^{\infty} (1-\beta_H)(1+\theta)^{-1} \tilde{A}_j \varphi^{\frac{1}{\theta}} g_j^*(\varphi) d\varphi + R_j f N_{E,j} + (1-\beta_L)(1-\Gamma_j)\alpha|\mathbb{S}_j|E + (1-\alpha)E_j, \\ &= (1-\beta_H)(1+\theta)^{-1} \alpha \Gamma_j |\mathbb{S}_j| E + R_j f N_{E,j} + (1-\beta_L)(1-\Gamma_j)\alpha|\mathbb{S}_j|E + (1-\alpha)E_j, \end{aligned}$$

where the second equation follows from the definitions of $\tilde{\varphi}_j$ and $\tilde{A}_j$, i.e., (4) and (15). Together with the labour market clearing condition (16) and equation (17) for the local income E_j, this equation is solved for R_j in order to interpret R_j as a function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$:

$$R_j = \frac{1}{1-\alpha f N_{E,j}} \eta_j A_j, \quad \text{where} \quad \eta_j \equiv \Gamma_j \frac{1-\alpha\beta_H + (1-\alpha)\theta}{1+\theta} + (1-\Gamma_j)(1-\alpha\beta_L). \quad (18)$$

Given this result, wage rates $\{W_j\}_{j=1}^2$ and the spatial distribution $\{\lambda_j\}_{j=1}^2$ of workers are now functions of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$.

A.3 Productivity Thresholds $(\varphi_1^*, \bar{\varphi})$ as Functions of S_1

I now show that two productivity thresholds $(\varphi_1^*, \bar{\varphi})$ are functions of S_1. For this purpose, two conditions are used: one is for φ_1^* and the other for $\bar{\varphi}$. The first condition states that an individual with productivity φ_1^* is indifferent between becoming a worker and working as an entrepreneur in Region 1, i.e., (6) with $j = 1$. Together with (13) and (15), this reduces to

$$\frac{W_1 + R_1 f}{\sigma^{-1}\alpha\Gamma_1|\mathbb{S}_1|E/N_{E,1}} \frac{\delta}{\delta - 1/\theta} \frac{1-(\bar{\varphi}/\varphi_1^*)^{-(\delta-\frac{1}{\theta})}}{1-(\bar{\varphi}/\varphi_1^*)^{-\delta}} \varphi_1^* = 1.$$

Further, substituting the labour and land market clearing conditions, (16) and (18), into this equation results in

$$\frac{\delta}{\delta - 1/\theta}\frac{\sigma}{\Gamma_1}\frac{1-(\bar{\varphi}/\varphi_1^*)^{-\left(\delta-\frac{1}{\theta}\right)}}{1-(\bar{\varphi}/\varphi_1^*)^{-\delta}}\left[\frac{\tilde{\beta}_1 N_{E,1}}{(1-\lambda_2)N_W}+\eta_1\frac{fN_{E,1}}{1-\alpha f N_{E,1}}\right]=1.$$

Finally, using (9), the first condition is written as follows

$$\frac{\delta}{\delta - 1/\theta}\frac{\sigma}{\Gamma_1}\frac{1-(\varphi_1^*/\bar{\varphi})^{\delta-\frac{1}{\theta}}}{1-(\varphi_1^*/\bar{\varphi})^{\delta}}\left\{\frac{\tilde{\beta}_1}{1-\lambda_2}\frac{1}{1-\left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta}}+\eta_1\frac{f}{1-\alpha f\left[1-\left(\frac{\varphi_1^*}{\bar{\varphi}}\right)^{\delta}\right]\left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta}}\right\}$$
$$\times\left[1-\left(\frac{\varphi_1^*}{\bar{\varphi}}\right)^{\delta}\right]\left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta}=1. \tag{19}$$

If I define x and y by $x \equiv \varphi_1^*/\bar{\varphi} \in (0,1)$ and $y \equiv \underline{\varphi}/\varphi_1^* \in (0,1)$, respectively, this equation adds a restriction to the relationship between x and y for a given S_1. Note that λ_2 is a function of $(\varphi_1^*, \bar{\varphi}, S_1)$ and that $(\varphi_1^*, \bar{\varphi})$ corresponds to (x, y) equivalently for any given lower bound $\underline{\varphi}$ of productivity.

The second condition is $u(\bar{\varphi}) = 1$, where $\bar{\varphi}$ is assumed to be greater than $\max_j\{\varphi_j^*\}$, or

$$\frac{\sigma^{-1}\tilde{A}_2\bar{\varphi}^{\frac{1}{\theta}}-R_2 f}{\sigma^{-1}\tilde{A}_1\bar{\varphi}^{\frac{1}{\theta}}-R_1 f}=\left(\frac{R_2}{R_1}\right)^{1-\alpha}.$$

After some calculations which use (9), (10), (15), and (18), this equation is restated as follows:

$$\frac{\frac{\delta-1/\theta}{\delta}\frac{\Gamma_2}{\eta_2\sigma}\frac{(xy)^{-\delta}-\alpha f}{f}-1}{\frac{\delta-1/\theta}{\delta}\frac{1}{x^{1/\theta}-x^{\delta}}\frac{\Gamma_1}{\eta_1\sigma}\frac{1-\alpha f(1-x^{\delta})y^{\delta}}{fy^{\delta}}-1}=\left[\frac{\frac{1-\alpha f(xy)^{\delta}}{1-\alpha f(1-x^{\delta})y^{\delta}}}{\frac{\eta_2}{\eta_1}\frac{1-S_1}{S_1}}\right]^{\alpha}, \tag{20}$$

which adds another restriction to the relationship between x and y for a given S_1.

Therefore, for a given S_1, there are two unknowns, x and y, and two equations, (19) and (20). This system of equations, if solved, implies that x and y are obtained as functions of S_1. Of course, there might exist multiple solutions for the system, and thus it is more appropriate to state that the system gives x and y as correspondence of S_1. However, in the numerical computation considered in the paper, the system actually gives a unique solution.

A.4 Determination of S_1 through Comparative Advantage

In the above discussion, S_1 is fixed at some point. Stated differently, I considered an interior sorting equilibrium where the spatial distribution of final goods industries is fixed in a particular manner. Thus, finally, I discuss how to pin down the value of S_1.

The condition which determines the value of S_1 is the comparative advantage condition, i.e., (8), which states that prices of final goods sector S_1, if posted by two regions, are equalized.

Focusing on the case considered in the numerical analysis, i.e., $\beta_H = \beta_L$, this condition is written as follows:

$$\frac{\chi_2(S_1)}{\chi_1(S_1)} = \left(\frac{R_2}{R_1}\right)^{1-\alpha\beta} \left[\frac{N_{E,2}^{\theta}\tilde{\varphi}_2}{N_{E,1}^{\theta}\tilde{\varphi}_1}\right]^{-\gamma(S_1)} = 1.$$

Since all the ratios in parentheses and brackets are functions of S_1, as discussed above, this is a single equation determining the value of S_1. Thus, the computation of an equilibrium can be interpreted as a fixed-point problem with respect to S_1, which nests a system of nonlinear equations for (x, y). Once the value of S_1 which satisfies the above equation is found, the values of the other variables are computed. Without loss of generality, the economy-wide income E excluding land rents is normalized to unity.