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A program for weakly decomposable inequality measures by population subgroups

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Abstract

This note explains the use of the program of the (α,β)-decomposition of the α-Gini measures available in free access on the LAMETA's website: http://www.lameta.univ-montp1.fr/online/gini.html. Inspired by the recent works published about the weak decomposition of inequality measures, this macro allows targeting accurately the groups mainly responsible for income disparities appraised within a distribution, whether subdivided into one or several partitions. This tool enables parameters of sensitivity reflecting the political decision-maker's preferences to be involved in the structure of the within- and between-group inequality components. The within-group component embodies a parameter of inequality aversion α whereas the between-group component includes both parameters α and β, such that β is the sensitivity towards between-group non-overlappings.

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1 Introduction

The literature outlines numerous methods of decomposition of income inequality measures. These methods aim at facilitating the understanding of the income disparities by focusing on one or several criteria of group partitions. The causes of the income inequalities are multiple. It is then essential to identify them. However to judge the unegalitarian nature of a population on the basis of partition criteria, exclusive and exhaustive groups must be carried out.

Various formulations were proposed during the last forty years. They are concerned with indicators of different families and are based on hypotheses. But all of them enable the within-group inequalities (*i.e.* between individuals who share a common criterion such as the gender, the age, or the educational level) and the between-group inequalities (*i.e.* between individuals who do not share this criterion like males and females) to be computed.

The inequality measures of the family of the generalized entropy as well as those of the family of the Gini index are known to be subgroups decomposable and appear among the most used. Both families of inequality measures do not meet the same decomposition requirements. Shorrocks' (1980) additive decomposition prevails for the generalized entropy measures. This method defines the within-group component as a weighted sum of the inequality observed within the same group whereas the between-group component corresponds to the discrepancies calculated between the arithmetic mean of incomes of each group. Such components cannot be defined from the structure of the Gini index, unless the distributions do no overlap (*see* Ebert, 1988). To mitigate this issue, Dagum (1997a) proposes an alternative formulation of the between-group component in order to decompose the Gini index. He suggests a between-group component based on pairwise comparisons of incomes.

At first sight, Dagum's subgroup decomposition was concerned only with the Gini index. However, Chameni (2006a, 2006b) demonstrates that some measures of the family of the generalized entropy such as the coefficient of variation squared can be decomposed according to this method. Ebert (2010) generalizes this technique of decomposition, the so-called *weak decomposition* and puts all the pair-based decomposable inequality measures within the same set. As a consequence the weak decomposition may be applied to any extensions of the Gini index as well as to the coefficient of variation squared, or even to the variance of logarithms.

In this note we focus on *regular*[1] inequality measures. The extensions of the variance of logarithms do not satisfy Pigou-Dalton's principle in the same way as the variance of logarithms. So only the extensions of the Gini index (absolute or relative), called the $\alpha-$Gini measures, are considered.

The purpose of this note is to propose guidance on the use of a program recently updated to facilitate the implementation of the weak decomposition of the $\alpha-$Gini measures. The program enables those measures to be decomposed by population subgroups by taking into account two parameters of sensitivity denoted by α and β. On one hand, the overall

[1] An inequality measure is regular when it satisfies Pigou-Dalton's principle of transfer, Dalton's principle of population, the axiom of symmetry and the axiom of normalization.

inequality aversion is captured by α and affects the main components of the decomposition (within- and between-group components). On the other hand, the sensitivity between-group towards non-overlappings is embodied in β and concerns only two specific between-group indicators, as this will be explained in details below.

The remainder of the paper is organized as follows. Section 2 aims at explaining the weak decomposition process resulting from several consecutive research works. In Section 3 the program of the (α,β)–decomposition of the $\alpha-$Gini measures is presented. An example is also proposed to guide the user in the interpretation of the empirical results. Finally some concluding remarks are made in Section 4.

2 The weak decomposition by subgroups

Before being formalized by Ebert (2010), the beginnings of the weak decomposition appeared in successive works: Dagum's (1997a, 1997b), Chameni's (2006b) and Mussard and Terraza's (2009).

Dagum's (1997a) works highlight a decomposition in three components for the Gini index. The total Gini index ($G(\mathbf{x}, n)$) appraised on an income distribution $\mathbf{x}$ related to a population $\mathscr{P}$ of size $n \equiv n(\mathbf{x})$ is given by the sum of the gross between-group component (G_{gb}) with the within-group component (G_w). Let K be the total number of groups in the whole population $\mathscr{P}$, $n(\mathbf{x}_k)$ the size of subgroup k, $\mu(\mathbf{x}_k)$ the mean income of distribution $\mathscr{P}_k$, $k \in \{1, \dots, K\}$. The gross between-group term depends on the between-group Gini indices that assess the income disparities between two subgroups k and h (*see* also Dagum, 1987):

$$G_{kh} := \frac{\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{r'=1}^{n(\mathbf{x}_h)} |x_{rk} - x_{r'h}|}{n(\mathbf{x}_k)n(\mathbf{x}_h)(\mu(\mathbf{x}_k) + \mu(\mathbf{x}_h))} \ ; \ \forall k \neq h \in \{1, \dots, K\} \ .$$

The within-group Gini index is computed on one group ($h = k$):

$$G_{kk} := \frac{\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{r'=1}^{n(\mathbf{x}_k)} |x_{rk} - x_{r'k}|}{2n^2(\mathbf{x}_k)\mu(\mathbf{x}_k)} \ .$$

The structure of the within- and between-group components are based on the calculation of binary income differences in absolute values. They may be rewritten in a more compact way:

$$G_w = \sum_{k=1}^{K} G_{kk} \ p_k \ s_k \quad \text{and} \quad G_{gb} = \sum_{k=2}^{K} \sum_{h=1}^{k-1} G_{kh} \ (p_k s_h + p_h s_k);$$

with, p_k [resp. p_h] and s_k [resp. s_h] the weighting functions such that $p_k := \frac{n(\mathbf{x}_k)}{n(\mathbf{x})}$ and $s_k := \frac{n(\mathbf{x}_k)\mu(\mathbf{x}_k)}{n(\mathbf{x})\mu(\mathbf{x})}$.

To define the third component, Dagum refers to Gini's (1916, 1921) seminal works about the phenomenon of *transvariation* often noticed between the income distributions. For instance, considering two subgroups k and h such that $\mu(\mathbf{x}_k) > \mu(\mathbf{x}_h)$. If some individuals' incomes of group h are higher than some individuals' incomes of group k (with

$\mu(\mathbf{x}_k) > \mu(\mathbf{x}_h)$) then a phenomenon of *transvariation* is observed between these groups. Such a phenomenon is also known as overlapping because of its graphical representation [*see* Figure 1 in Appendix as an example].

Dagum proposes to break down the gross between-group component (G_{gb}) in a net between-group component (G_{nb}) and a transvariation one (G_t), according to the following scheme of decomposition,

$$G(\mathbf{x}, n) = G_w + G_{nb} + G_t \quad \text{with} \quad G_{gb} = G_{nb} + G_t \ ;$$

such that:

$$G_{nb} := \sum_{k=2}^{K}\sum_{h=1}^{k-1} G_{kh} D_{kh}(1)(p_k s_h + p_h s_k) \quad \text{and} \quad G_t := \sum_{k=2}^{K}\sum_{h=1}^{k-1} G_{kh}[1 - D_{kh}(1)](p_k s_h + p_h s_k),$$

and,

$$D_{kh}(1) := \frac{\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{rk} > x_{r'h}} (x_{rk} - x_{r'h}) - \sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{r'h} \geqslant x_{rk}} (x_{r'h} - x_{rk})}{\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{r'=1}^{n(\mathbf{x}_h)} |x_{rk} - x_{r'h}|} \ , \ \mu(\mathbf{x}_k) > \mu(\mathbf{x}_h). \tag{1}$$

The net between-group component (G_{nb}) allows the discrepancies between $\mathscr{P}_k$ and $\mathscr{P}_h$ to be computed. It depends on the non-overlap area between those distributions. This component captures the disparities generated by the highest incomes of the groups whose the mean is higher than those of the other groups. The disparities of overlap between the subgroups (observed in the transvariation area) are embodied by the component of transvariation (G_t). The inequality components G_{nb} and G_t are complementary since they rely respectively on the economic directional distance ($D_{kh}(1)$) and the ratio of overlap ($1 - D_{kh}(1)$). The economic directional distance ($D_{kh}(1)$) is a key component of the decomposition of the Gini index. It provides an assessment of the affluence-gaps between individuals belonging to different groups. This indicator is included in $[0, 1]$ and is easy to interpret. When overlappings between distributions are perfect (*i.e.* both distributions are superimposed) the economic directional distance is nil ($D_{kh}(1) = 0$). On the contrary the economic directional distance is equal to 1 when the distributions do not overlap ($D_{kh}(1) = 1$).[2]

Afterward Chameni (2006a, 2006b) demonstrates that the coefficient of variation squared (CV^2) may be also decomposed according to Dagum's process, until now reserved for the Gini index. He adapts the structure of each component by using the following relation:

$$\frac{1}{n^2(\mathbf{x})} \sum_{r=1}^{n(\mathbf{x})} \sum_{r'=1}^{n(\mathbf{x})} |x_r - x'_r|^2 = \frac{1}{n(\mathbf{x})} \sum_{r=1}^{n(\mathbf{x})} x_r^2 - \mu^2(\mathbf{x}).$$

Consequently the decomposition of the coefficient of variation squared according to Dagum's

[2] *Cf.* Dagum (1987) for more details.

method is given by:

$$\begin{aligned} CV^2(\mathbf{x},n) &= \sum_{k=1}^{K} \frac{\sum_{r=1}^{n(\mathbf{x}_k)}\sum_{r'=1}^{n(\mathbf{x}_k)} |x_{rk}-x_{r'k}|^2}{2n^2(\mathbf{x}_k)\mu^2(\mathbf{x}_k)} \cdot \frac{n(\mathbf{x}_k)}{n(\mathbf{x})} \cdot \frac{n(\mathbf{x}_k)\mu^2(\mathbf{x}_k)}{n(\mathbf{x})\mu^2(\mathbf{x})} \\ &+ \sum_{k=2}^{K}\sum_{h=1}^{k-1} \frac{\sum_{r=1}^{n(\mathbf{x}_k)}\sum_{r'=1}^{n(\mathbf{x}_h)} |x_{rk}-x_{r'h}|^2}{n(\mathbf{x}_k)n(\mathbf{x}_h)(\mu^2(\mathbf{x}_k)+\mu^2(\mathbf{x}_h))} \left(\frac{n(\mathbf{x}_k)}{n(\mathbf{x})} \cdot \frac{n(\mathbf{x}_h)\mu^2(\mathbf{x}_h)}{n(\mathbf{x})\mu^2(\mathbf{x})} + \frac{n(\mathbf{x}_h)}{n(\mathbf{x})} \cdot \frac{n(\mathbf{x}_k)\mu^2(\mathbf{x}_k)}{n(\mathbf{x})\mu^2(\mathbf{x})} \right) \\ &= \sum_{k=1}^{K} CV_{kk}^2 \; p_k \; s_k^2 + \sum_{k=2}^{K}\sum_{h=1}^{k-1} CV_{kh}^2 \; (p_k s_h^2 + p_h s_k^2) \;, \end{aligned}$$

where, $s_k^2 := \frac{n(\mathbf{x}_k)\mu^2(\mathbf{x}_k)}{n(\mathbf{x})\mu^2(\mathbf{x})}$ and $s_h^2 := \frac{n(\mathbf{x}_h)\mu^2(\mathbf{x}_h)}{n(\mathbf{x})\mu^2(\mathbf{x})}$.

This new expression allows putting forward a common basic structure between the standard Gini index and the coefficient of variation squared, denoted $G^2(\mathbf{x},n)$ from now onwards. This coefficient may also be decomposed in three components according to Dagum's economic directional distance.

A few years later, Mussard and Terraza (2009) propose a concept of pair-based inequality measures, that is, a *decomposition by subgroups based on pairwise comparisons*, aiming at introducing the Gini mean ratio. The pair-based decomposable inequality measures comprise the Gini index and the coefficient of variation squared. Formally, the pairwise comparisons replace the arithmetic means used by Shorrocks (1980) in the definition of the between-group component.

Finally, in 2010 Ebert associates the within- and between-group components with a parameter of sensitivity that Chameni (2011) defines as a parameter of inequality aversion. This parameter affects only the binary income differences in absolute values as well as the mean incomes, under the shape of an exponent α:

$$G^\alpha(\mathbf{x},n) = G_w^\alpha + G_{gb}^\alpha \;;\; \forall \alpha \in [1,\infty[\;.$$

That is equivalent to (*see* Chameni, 2011):

$$G^\alpha(\mathbf{x},n) = \sum_{k=1}^{K} G_{kk}^\alpha \, p_k \, s_k^\alpha + \sum_{k=2}^{K}\sum_{h=1}^{k-1} G_{kh}^\alpha D_{kh}(1)(p_k s_h^\alpha + p_h s_k^\alpha) + \sum_{k=2}^{K}\sum_{h=1}^{k-1} G_{kh}^\alpha [1-D_{kh}(1)](p_k s_h^\alpha + p_h s_k^\alpha) \;; \tag{2}$$

where, for all $k \neq h \in \{1;\ldots;K\}$,

$$G_{kh}^\alpha := \frac{\sum_{r=1}^{n(\mathbf{x}_k)}\sum_{r'=1}^{n(\mathbf{x}_h)} |x_{rk}-x_{r'h}|^\alpha}{n(\mathbf{x}_k)n(\mathbf{x}_h)(\mu^\alpha(\mathbf{x}_k)+\mu^\alpha(\mathbf{x}_h))}, \; s_k^\alpha := \frac{n(\mathbf{x}_k)\mu^\alpha(\mathbf{x}_k)}{n(\mathbf{x})\mu^\alpha(\mathbf{x})}, \; s_h^\alpha := \frac{n(\mathbf{x}_h)\mu^\alpha(\mathbf{x}_h)}{n(\mathbf{x})\mu^\alpha(\mathbf{x})}.$$

The evaluation of the income comparisons between the individuals depends on the degree of inequality aversion α felt by a decision-maker, α being included in the interval $[1,\infty[$. It enables a large number of inequality indexes to be decomposed according to the same method of decomposition, the so-called *weak decomposition*, providing the Gini index and the coefficient of variation squared (*see* Ebert, 2010).

Chameni's technique, relying on Dagum's economic directional distance ($D_{kh}(1)$), may be generalized. One can think, for instance, that it is possible to include a parameter of sensitivity towards between-group non-overlappings β in the economic directional distance such that:

$$D_{kh}(\beta) := \frac{\left(\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{rk} > x_{r'h}} (x_{rk} - x_{r'h})\right)^{\beta} - \left(\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{r'h} \geqslant x_{rk}} (x_{r'h} - x_{rk})\right)^{\beta}}{\left(\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{rk} > x_{r'h}} (x_{rk} - x_{r'h})\right)^{\beta} + \left(\sum_{r=1}^{n(\mathbf{x}_k)} \sum_{x_{r'h} \geqslant x_{rk}} (x_{r'h} - x_{rk})\right)^{\beta}}\ , \quad (3)$$

$\forall \beta \geqslant 1$ and $\mu(\mathbf{x}_k) > \mu(\mathbf{x}_h)$.

Then, the inequality aversion α may be different from the sensitivity towards between-group non-overlappings β. These parameters represent the decision-maker's preferences with regard to the nature of the inequality. They play a large part in the implementation of redistributive actions and permit to target the most unegalitarian groups. The higher the value of α or β is, the more important the redistributive actions will be.[3] When such a parametrization is used to break down a weakly decomposable inequality measure, the method is called the (α, β)–decomposition by population subgroups. In the next Section, we present our program and show how it integrates the interplay between α and β.

3 The program and the results

Following Dagum's works (1997a, 1997b) several softwares were conceived to facilitate the application of the Gini index decomposition by subgroups. An Excel's macro was programmed in 2001 by Dagum, Mussard, Seyte and Terraza in collaboration with the SOCREES.[4] Recently, in association with this group of researchers, we decide to enlarge the configuration of this program to all the weakly decomposable measures compatible with Equations (2) and (3). The Excel workbook containing the macro of the (α, β)–decomposition is available in free access on the LAMETA's website at the following address: `http://www.lameta.univ-montp1.fr/online/gini.html`.

3.1 Preliminaries

Three Excel spreadsheets appear automatically while opening the workbook.

• *Sheet1*: it is a descriptive sheet that explains briefly how to load the data and to execute the macro.

• *Formulae*: this sheet recalls the formulation of some components of the (α, β)–decomposition.

• *Sheet2*: is reserved to the sample of data from which the disparities are estimated. It is imperative that the sampled individuals are beforehand numbered and classified according to the various criteria considered for the analysis of the inequalities. The macro command obeys a particular procedure of computation. A digital code has to be associated with the individuals in column A. The group to which individuals belong must be indicated in column B (always in digital format). The amount of the variable subject to the analysis of the within and between-group disparities must appear in column C. The title of each column must be the most synthetic possible, (*e.g.* in A1: Individuals' code;

[3]The reader is referred to Mornet, Zoli *et al.* (2013) for more details on the properties of this generalized distance and the notions of inequality aversion or sensitivity towards between-group non-overlappings.

[4]Société de réalisation d'études économiques et statistiques

in B1: Gender; in C1: Wages, *see* Figure 2 in Appendix).

• *Run the macro*: The user has to indicate the number of subgroups. Then, he has to enter a value included between 1 and ∞ for the parameter α, the inequality aversion degree. Finally, the user has to indicate the value of β, the sensitivity towards between-group non-overlappings. The Excel workbook displays automatically the results on a new sheet entitled *Characteristics* which layout is similar to the one depicted on the sheet *Formulae.*

3.2 An example

Let us imagine that the decision-maker is interested in the income disparities observed between males and females in France in 2005. All the elementary statistics calculated on the sample of data are provided by the program in Table 1. Note that the first illustration corresponds to the (1,1)–decomposition which is actually Dagum's Gini index decomposition by subgroups. The main components of the decomposition are represented in Tables 5 and 7. When the value of α the inequality aversion varies from the value 1 to 2, *ceteris paribus*, the components become those defined by Chameni (2006b) within the scope of the decomposition of the coefficient of variation squared. The values of the various coefficients summarized in Table 10 must be interpreted cautiously. Since $\alpha > 1$, the $\alpha-$Gini is included in $[0, \infty[$.

The main components cannot be directly compared one with another. Comparisons can only be made between the $\beta-$directional distances for various values of β since these indicators remain included in $[0, 1]$, for all $\beta \geqslant 1$. To judge the more or less unegalitarian nature of income distributions when the decision-maker's preferences vary, ratio can be calculated for all $\alpha \geqslant 1$ as suggested in Tables 10 and 11.

For instance, when the decision-maker's preferences are such that $\alpha = 2$, the impact of the contribution of the income disparities between males and females seems to be more important with regard to lower preferences for the redistribution *i.e.*, $\alpha = 1$ ($G^1_{gb}/G^1 = 51.63\% < G^2_{gb}/G^2 = 52.28\%$). The opposite result is noticed when the contribution of the within-group inequalities is considered ($G^1_w/G^1 = 48.37\% > G^2_w/G^2 = 47.72\%$). Similar conclusions may be drawn when the contributions of the Gini index components are compared with the $3-$Gini ones. Since the $3-$Gini tends to be tail-sensitive[5] these conclusions do not hold when the contributions of the $2-$Gini – that is the coefficient of variation squared – are compared with the $3-$Gini ones. Besides the value of β impacts directly the net between-group term and *de facto* the β-directional distance, as shown in Table 11. It appears that the less sensitive to transvariation the decision-maker is, the more important the net between-group contribution to the overall inequality is.

Indeed the net between-group inequality component focuses on discrepancies between males' and females' highest incomes belonging to the most affluent groups on average. By definition these income-gaps are not appraised in transvariation area. So, the higher the value of β is, the more sensitive to these income-gaps the decision-maker is. This is the reason why the net between-group contribution increases twofold when β raises from 1 to 3. The effects of the sensitivity parameter in terms of transvariation are deducted

[5] see Mornet, Zoli *et al.* 2013.

from the value of the $\beta-$directional distance. The closer to 0 the distance is, the more the distributions overlap. On the contrary the closer to 1 the distance is, the less the distributions overlap.

4 Conclusion

This note presents the program of the (α, β)–decomposition of the $\alpha-$Gini measures. With free access and easy to use such a program provides the opportunity of decomposing any regular inequality measure belonging to the family of the $\alpha-$Gini measures. The Gini index (when $\alpha = 1$ and $\beta \geqslant 1$) and the coefficient of variation squared (for $\alpha = 2$ and $\beta \geqslant 1$) are embedded in this family of weakly decomposable inequality measures.

As illustrated, the value of the components is henceforth a function of the values of the parameters α and β, fixed at the beginning of analysis. While α represents the decision-maker's degree of inequality aversion, β embodies his sensitivity towards non-overlappings. The β parameter is integrated into the structure of the well-known economic directional distance of Dagum (1980) allowing its normative dimension to be spread. A wider set of preferences regarding redistribution than the one initially defined (for $\beta = 1$) is taken into consideration thanks to this new parametrization.

However, it is not the only possible parametrization. Considering the fact that the β parameter impacts directly the between-group components without affecting the within-group one, this opens up the way to other parametrizations of the various components in order to distinguish the degree of within-group inequality aversion from the degree of between-group inequality aversion.

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Appendix

Illustration of a phenomenon of transvariation:

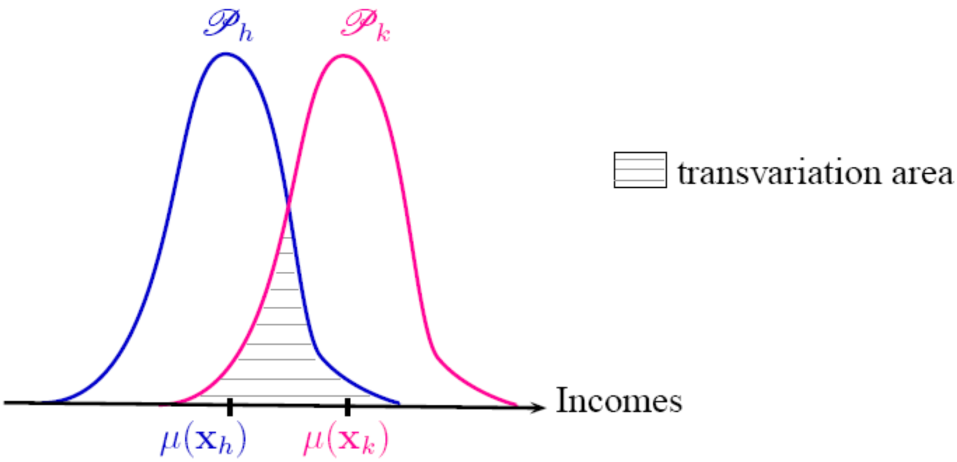
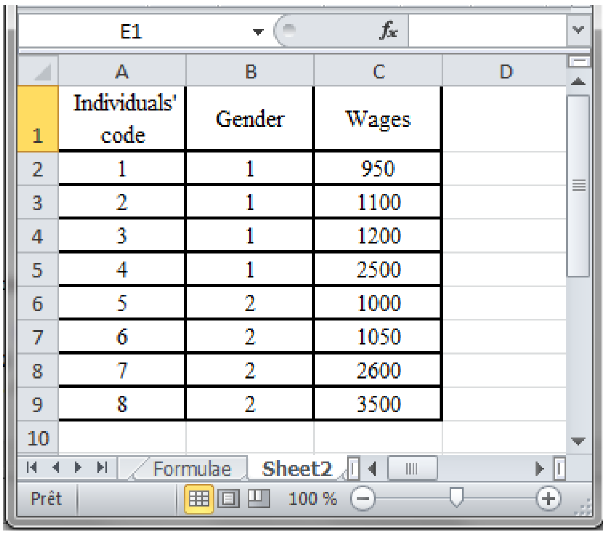


Figure 1: **Transvariation between income distributions**

Example of data entry on Sheet2:



Individuals' code	Gender	Wages
1	1	950
2	1	1100
3	1	1200
4	1	2500
5	2	1000
6	2	1050
7	2	2600
8	2	3500

Figure 2: **Example of data entry**

Output available on the sheet Characteristics provided by the program: [6]

Segmentation variable	Gender			
Name of the analysis	(alpha,beta)-Decomposition			
Number of groups	2			
Value of alpha	1	Value of beta	1	
Name of the group		GTT	G2	G1
Description of the group		Total	Modality 2	Modality 1
Modality code			2	1
Size of the group	n_k	14 281	7 122	7 159
Total income of the group	R_k	256 284 219	102 179 336	154 104 883
Mean income of the group	M_k	17 945.82	14 347.00	21 526.03
$\frac{\text{Share of the group}}{\text{Total}}$	$P_k = n_k/n$	1	0.4987	0.5013
$\frac{\text{Income of the group}}{\text{Total income}}$	$S_k = R_k/R$	1	0.3987	0.6013
Variance		274 751 318.3	122 298 480	400 675 349.4
Filter		Gender	Gender	Gender
			2	1

Table 1: **Elementary statistics relating to the sample of data**

L5:Beta-directional economic distance D		G2	G1
	G2	0.000000	
	G1	0.495567	0.000000

Table 2: **Dagum's economic directional distance $D_{kh}(1)$**

Delta Matrix		G2	G1
	G2	11171.53418	
	G1	14486.49414	15947.29883

Delta Vector (DELTA kk)	11171.53418	15947.29883

Table 3: **Sum of males' and females' affluence-gaps**

[6] The males' and females' incomes are due to a french survey intitled "Budget des Familles - 2005-2006"

L6:Within-group GINI ratio (G_{kk} vector)	0.390875	0.389333	0.370419

Table 4: **Global Gini coefficients assessed in each group**

L7:Weighted Within-group GINI ratio (PSGkk vector)	0.189067	0.077412	0.111656

Table 5: **Within-group Gini component $G_w^1(\mathrm{x}, n)$**

L8:Between-group GINI ratio (Gkh Matrix)		G2	G1
	G2	0.389333	
	G1	0.403827	0.370419

Table 6: **Between-group Gini ratios $G_{kh}^1(\mathrm{x}, n)$**

L8a:Gross between-group GINI ratio contribution (Gbbkh Matrix)	0.201807	G2	G1
	G2	0.077412	
	G1	0.201807	0.111656

Table 7: **Gross between-group Gini component $G_{gb}^1(\mathrm{x}, n)$**

L9:Net between-group GINI ratio contribution (Gbkh Matrix)	0.100009	G2	G1
	G2	0.000000	
	G1	0.100009	0.000000

Table 8: **Net between-group Gini component $G_{nb}^1(\mathrm{x}, n)$**

L10:Between-group transvariation (Gtkh Matrix)	0.101798	G2	G1
	G2	0.154823	
	G1	0.101798	0.223312

Table 9: **Transvariation Gini component $G_t^1(\mathrm{x}, n)$**

	$\alpha = 1, \beta = 1$	$\alpha = 2, \beta = 1$	$\alpha = 2, \beta = 2$
$G^\alpha(\mathbf{x}, n)$	0.3909	0.8531	0.8531
$G^\alpha_w(\mathbf{x}, n)$	0.1891	0.4071	0.4071
G^α_w/G^α	**48.37%**	**47.72%**	47.72%
$G^\alpha_{gb}(\mathbf{x}, n)$	0.2018	0.4460	0.4460
G^α_{gb}/G^α	**51.63%**	**52.28%**	52.28%
$G^\alpha_{nb}(\mathbf{x}, n)$	0.1000	0.2210	0.3549
G^α_{nb}/G^α	25.59%	25.91%	41.60%
$G^\alpha_t(\mathbf{x}, n)$	0.1018	0.2250	0.0911
G^α_t/G^α	26.04%	26.37%	10.68%
$D_{kh}(\beta)$	0.4956	**0.4956**	**0.7957**

Table 10: **Decomposition of the Gini index and the coefficient of variation squared when $\boldsymbol{\beta} \in \{1, 2\}$**

	$\alpha = 3, \beta = 1$	$\alpha = 3, \beta = 2$	$\alpha = 3, \beta = 3$
$G^\alpha(\mathbf{x}, n)$	7.0995	7.0995	7.0995
$G^\alpha_w(\mathbf{x}, n)$	3.4285	3.4285	3.4285
G^α_w/G^α	48.29%	48.29%	48.29%
$G^\alpha_{gb}(\mathbf{x}, n)$	3.6710	3.6710	3.6710
G^α_{gb}/G^α	51.71%	51.71%	51.71%
$G^\alpha_{nb}(\mathbf{x}, n)$	1.8192	2.9211	3.3997
G^α_{nb}/G^α	**25.62%**	**41.14%**	**47.89%**
$G^\alpha_t(\mathbf{x}, n)$	1.8518	0.7499	0.2713
G^α_t/G^α	**26.08%**	**10.56%**	**3.82%**
$D_{kh}(\beta)$	**0.4956**	**0.7957**	**0.9261**

Table 11: **Decomposition of the 3–Gini index when $\boldsymbol{\beta} \in \{1, 2, 3\}$**