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Tullock contests under committee administration

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Abstract

Much of the Tullock contest (Tullock 1980) literature analyzes rent-seeking efforts under the assumption of a single prize administrator. Here, I allow a committee to decide the winner of the rent-seeking contest according to simple majority voting rule, and study the impact of committee size on rent-seeking expenditures. I find that increasing the size of the committee produces an ambiguous impact on total rent-seeking efforts, with the heterogeneity of the contestants acting as the factor determining a resultant increase or decrease in efforts.

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1. Introduction

A contest is a game in which players expend non-recoverable effort to win a given prize. The Tullock contest (Tullock 1980) and its variants have been well-studied. Much of Tullock contest literature focuses on factors affecting rent-seeking expenditures, such as the number of contestants, the degree of contestants' heterogeneity level, or the sensitivity level of the rent-giver. However, most of these models are studied under the assumption that the prize is awarded by a single rent-giver. In this paper, I allow a committee to decide the winner of the rent-seeking contest according to simple majority voting rule, and study the impact of committee size on rent-seeking expenditures.

Models most closely related to the one in this paper are Amegashie (2002) and Klumpp & Polborn (2006), who both also assume multiple prize administrators; however, they do not explicitly analyze variations in committee size.[1] Amegashie (2002) does compare rent-seeking expenditures under a 3-member committee versus under a single administrator, and finds rent-seeking expenditures to be higher under 3-member committees than under a single rent-giver.[2] I consider the more general case by introducing heterogeneity between contestants and allowing variation in committee size. Allowing different levels of heterogeneity between contestants leads to different results when committee size varies. Klumpp & Polborn (2006) study campaign effort expended in different districts. Similar to my model, the campaign winner is decided by simple majority rule, and each district can be interpreted as a committee member. As in Amegashie, their main focus is not on variation of committee size. Further, both papers assume homogeneous contestants, whereas I allow for heterogeneity.

Below I analyze the impact on aggregate rent-seeking expenditure from varying committee size and the heterogeneity of the contestants. I find that increasing the committee size produces an ambiguous impact on total rent-seeking efforts. This is because, *a priori*, there are two counter-acting forces acting upon rent-seeking efforts when the size of committee increases. First, given there are more committee members to direct efforts towards, we expect an increase in aggregate rent-seeking efforts. Second, however, an increase in committee size means that a contestant must secure the support

[1] Somewhat related is Clark & Konrad (2007), who present a game with a single prize administrator but where two identical contestants compete in multiple tasks, where the increase in tasks can be interpreted analogously as increasing the number of prize administrators.

[2] While Amegashie (2002) employs a modified majority voting rule, under only two contestants, it simplifies to the simple majority voting rule, creating a comparable setting with my model.

of a greater number of them in order to win the prize, decreasing the chance that the contestant's efforts towards a given committee member will produce a pivotal outcome. These counter-acting forces are such that the latter will dominate the former as committee size increases. The committee size at which this dominance happens is falling in the level of heterogeneity between contestants.

2. Rent-Seeking Model with Heterogeneous Contestants

There are two risk-neutral contestants competing to win a prize, which each contestant values individually. Contestant 1 has a valuation of the prize $V_1 > 0$, and Contestant 2 has a valuation of the prize $V_2 > 0$. Without loss of generality, we assume $V_1 \geq V_2$. The degree of heterogeneity between the contestants is represented by $t \equiv V_2/V_1 \leq 1$.

The prize is awarded to one of the two contestants by a committee of $2m+1$ members, where m is any non-negative integer, and m=0 represents a single prize administrator. In order to win the prize, a contestant must win the support of a majority of the committee.

Contestants compete by expending non-recoverable and non-negative effort directed toward specific committee members. Contestants 1 and 2 simultaneously choose their efforts, x_i and y_i, respectively, where i represents a given committee member toward whom effort is directed. Therefore each contestant's strategy is a vector, represented by $\mathrm{X} = (x_1, x_2 \cdots x_{2m+1})$ and $\mathrm{Y} = (y_1, y_2 \cdots y_{2m+1})$, respectively. The cost function is assumed to be linear.

Each committee member makes his or her decision to support a contestant according to the Tullock success function, where r is sensitivity to rent-seeking effort. We assume $0 < r \leq 1$ so that the second-order condition is globally fulfilled. Let q_i denote the probability of committee member i vote for contestant 1. Therefore, q_i can be expressed as: $q_i(x_i, y_i) = \frac{{x_i}^r}{{x_i}^r + {y_i}^r}$ if $x_i + y_i \neq 0$; $q_i = \frac{1}{2}$ otherwise.

Let P be the probability that Contestant 1 wins the contest, with (1 – P) the probability that Contestant 2 wins the contest. Each contestant seeks to maximize his or her expected utility, subject to his or her respective effort expended. Hence Contestant 1 solves $\max_{x_i \geq 0} \mathrm{E}(\mathrm{U}_1) = PV_1 - \sum_{i=1}^{2m+1} x_i$, and Contestant 2 solves $\max_{\mathrm{y_i} \geq 0} \mathrm{E}(\mathrm{U}_2) = (1 - \mathrm{P})\mathrm{V}_2 - \sum_{\mathrm{i}=1}^{2\mathrm{m}+1} \mathrm{y_i}$.

Contestant 1's bidding choice x_i to committee member *i* indicates that the marginal benefit of effort x_i is the product of the probability for committee member *i* to be the pivotal prize administrator, times the marginal effect on probability to win committee member *i*:

$$\frac{\partial P}{\partial x_i} = Q_i \cdot \frac{\partial q_i(x_i, y_i)}{\partial x_i} ,$$

where Q_i represents the probability that committee member *i* becomes the pivotal decision maker. This happens when exactly m of the committee members (out of the remaining 2m committee members besides *i*), vote for Contestant 1.

The following discussion considers pure-strategy equilibrium condition such that m satisfies $m <= \overline{m}(r, t)$, which we shall define below. As such, given any pure strategy profile (X,Y), the probability for the committee member *i* to be the pivotal prize administrator must be same between the two contestants. We obtain F.O.C. for Contestants 1 and 2, respectively:

$$V_1 \cdot Q_i \cdot \frac{\partial q_i(x_i, y_i)}{\partial x_i} - 1 = 0 \qquad (1)$$

$$V_2 \cdot Q_i \cdot \left(-\frac{\partial q_i(x_i, y_i)}{\partial y_i}\right) - 1 = 0 \qquad (2)$$

Equation (1) divided by equation (2) is $-\frac{V_1}{V_2} \cdot \frac{q_x(x,y)}{q_y(x,y)} = 1$. Since $q_x(x,y)/q_y(x,y)$ is equivalent to $-y/x$, $\frac{y_i}{x_i} = \frac{V_2}{V_1} \equiv t \leq 1$; and thus $q_i = \frac{1}{1+t^r} \geq \frac{1}{2}$, regardless of the bidding profile directed towards other committee members. Since this reasoning can be applied to all committee members, each is won with the same probability, $q = 1/(1+t^r)$. Note that since $\frac{y_i}{x_i} = t$ and $q = 1/(1+t^r)$ are properties of pure strategy equilibrium, Q_i can be represented as $C_{2m}^{m} q^m (1-q)^m$. When we substitute these results back into our F.O.C.s, a unique solution of equations (1) and (2) can be obtained. This leads to Proposition 1.

<u>Proposition 1</u>: If the committee size is relatively small ($m \leq \overline{m}(r,t)$), there exists a unique pure strategy equilibrium, and in this pure strategy equilibrium, both contestants use uniform bidding strategies which satisfies $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$.

The unique solution of equation (1) and (2) can be obtained:

$$\mathrm{x}(m) = \ V_1 \cdot \ C_{2m}^{m} \frac{r(t^r)^{m+1}}{(1+t^r)^{2m+2}} \qquad (3),$$

and $$\mathrm{y}(m) = \ V_2 \cdot \ C_{2m}^{m} \frac{r(t^r)^{m+1}}{(1+t^r)^{2m+2}} \qquad (4),$$

with total rent-seeking effort T defined as: $T(m) = (2m+1)(1+t)\,\mathrm{x}(m)$.

Note that the above analysis is based on contestants playing the pure strategy equilibrium, the existence of which is dependent on contestants receiving non-negative payoffs under the pure strategy equilibrium (else they could abstain playing and get zero payoffs). Since Contestant 1 always wins with probability more than one-half, we only need to guarantee the participation of the weak contestant. The participation constraint can be written as $\mathrm{r} \leq \frac{1+\alpha}{(m+1)} \cdot [\sum_{i=0}^{m} \frac{C_{2m+1}^{m+1+i} \cdot \alpha^i}{C_{2m+1}^{m+1}}]$.[3] The smaller is t or the higher is r, the more difficult it is for the participation constraint to hold; likewise, for a set of given values of r and t, as m grows, the participation constraint becomes more difficult to hold. Hence $\overline{m}(r,t)$ is defined as the largest value of m that satisfies this inequality. Then the participation constraint always hold for $m \leq \overline{\mathrm{m}}$, and does not hold for $m > \overline{\mathrm{m}}$.

3. Rent-Seeking Effort Provision: Varying Committee Size

Equations (3) and (4) are functions of m, enabling us to consider the impact on rent-seeking expenditure as committee size varies. We compare the ratio of x(m+1)/x(m) in the following corollary.

Corollary 1: For a set of given values of t and r, the effort level supplied by each contestant to each individual committee member decreases in m.

As m increases, the marginal benefit of additional effort directed to a committee member is decreasing, since the probability for that committee member to be pivotal is decreasing; thus contestants would have the incentive to reduce their investment. Proof: Define $\mathrm{f}(\alpha) = \frac{\alpha}{(1+\alpha)^2}$, $\alpha \equiv t^r$. Then the ratio of the effort supplied by

[3] Note that if $\alpha \to 1$, the RHS converges to $\frac{2^{2m+1}}{(m+1) \cdot C_{2m+1}^{m+1}}$, and if $\alpha \to 0$, the RHS converges to $\frac{1}{(m+1)}$.

Contestant 1 when m increases by one is $\frac{x(m+1)}{x(m)} = \frac{C_{2m+2}^{m+1}}{C_{2m}^{m}} \cdot f(\alpha)$, which can be transformed into $\frac{x(m+1)}{x(m)} = f(\alpha) \cdot (4 - \frac{2}{m+1})$,[4] which is increasing in m. Hence, $2f(\alpha) \leq \frac{x(m+1)}{x(m)} < 4f(\alpha)$. Since f(α) is increasing in α, $f(\alpha) \leq 1/4$. Therefore, under pure strategy equilibrium, $\frac{x(m+1)}{x(m)} < 1$, which implies $\frac{\partial x(m)}{\partial \mathrm{m}} < 0$.

Note however that the aggregate rent-seeking expenditure can increase since the committee size is bigger. Which effect is stronger depends on the level of heterogeneity as defined by *t*. Specifically, $\frac{T(m+1)}{T(m)} = \frac{2m+3}{2m+1} \cdot \frac{\mathrm{x}(m+1)}{\mathrm{x}(m)} = (4 + \frac{2}{m+1}) \cdot \mathrm{f}(\alpha)$, which is decreasing in *m*, so $T(m+1)/T(m)$ is bounded by $(4\mathrm{f}(\alpha), 6\mathrm{f}(\alpha)]$. It is clear then that the actual degree to which total rent-seeking efforts change depends on the value of t^r.

This finding deserves discussion as it adds to the understanding within the literature. Amegashie (2002), for instance, proves that aggregate rent-seeking efforts are higher under 3-member committee than under single administrator; while this result is correct, it is only correct in so far as it is restricted to the case of homogeneous contestants. When we allow for the more general case of heterogeneous contestants, we find that aggregate rent-seeking efforts are not necessarily increasing when we increase *m*, instead depending on the value of t^r. The specific nature of the relationship depends on the interaction between the two effects, which is discussed in the following Corollary.

<u>Corollary 2</u>: Given the existence of pure strategy equilibrium ($m \leq \bar{m}(r,t)$), we have the following properties:

(a) *When contestants are identical (when t=1, or $\alpha \equiv t^r = 1$), the aggregate rent-seeking effort is increasing in committee size m, at a decreasing rate.*

(b) *Under conditions of sufficiently high heterogeneity between contestants (i.e., when $0 < \alpha < 2 - \sqrt{3}$), aggregate rent-seeking effort is decreasing in committee size m, at an increasing rate.*

(c) *Under conditions of weak or moderate heterogeneity between contestants (i.e., when $2 - \sqrt{3} \leq \alpha < 1$), aggregate rent-seeking effort is initially increasing (or*

[4] Note since $C_{2m+2}^{m+1} = C_{2m+1}^{m} + C_{2m+1}^{m+1}$ and $C_{2m+1}^{m} = C_{2m+1}^{m+1}$, $C_{2m+2}^{m+1} = 2C_{2m+1}^{m}$; $C_{2m+1}^{m} = C_{2m}^{m-1} + C_{2m}^{m}$, and therefore $C_{2m+2}^{m+1} = 2C_{2m+1}^{m} = 2(C_{2m}^{m-1} + C_{2m}^{m})$.

non-decreasing) in committee size m, until some critical value $\tilde{m}(r,t)$*, at which it reaches its maximum; thereafter, it is decreasing in committee size m, provided* $\tilde{m}(r,t) < \bar{m}(r,t)$*, else aggregate rent-seeking efforts are increasing in m.*

Proof:

(a) when t=1, $f(\alpha) = \frac{1}{4}$, hence $\frac{T(m+1)}{T(m)} = 1 + \frac{1}{2m+2} > 1$;

(b) when $0 < \alpha < 2 - \sqrt{3}$, $0 < f(\alpha) < \frac{1}{6}$, therefore $\frac{T(m+1)}{T(m)} \leq 6f(\alpha) < 1$;

(c) when $2 - \sqrt{3} \leq \alpha < 1$, $\frac{1}{6} \leq f(\alpha) < \frac{1}{4}$, we have $\frac{T(1)}{T(0)} = 6f(\alpha) \geq 1$, since $\frac{T(m+1)}{T(m)}$ is decreasing in m, there exists a critical value $\tilde{m}(r,t)$ such that $\frac{T(m+1)}{T(m)} \geq 1$ for all $m \leq \tilde{m}(r,t)$, and $\frac{T(m+1)}{T(m)} < 1$ for all $m > \tilde{m}(r,t)$. Suppose $\tilde{m}$ is the greatest value that satisfies the following inequality $\frac{T(m+1)}{T(m)} = (4 + \frac{2}{m+1}) \cdot f(\alpha) \geq 1$. Since $\frac{T(m+1)}{T(m)}$ is increasing in α, when we increase the value of α (which is equivalent either to an increase in *t*, or a decrease in *r*), the left hand side of the inequality becomes easier to hold; therefore, $\tilde{m}$ is non-decreasing in *t*, and non-increasing in *r*. The greater the heterogeneity between the two contestants is (the smaller *t* is, and hence the smaller is $\tilde{m}$), the faster the dominance will happen.

4. Conclusion

Prizes are often awarded by committees rather than a single individual. As demonstrated in this paper, anticipating whether aggregate rent-seeking efforts will be larger or smaller as the size of the prize-awarding committee grows depends upon both the level of rent-seeking sensitivity of the committee members, and the degree of heterogeneity between the contestants, with heterogeneity exacerbating the degree to which aggregate rent-seeking effort is attenuated by increasing committee size.

Note that the analysis presented in this paper represents an examination of the pure strategy equilibrium. While the existence of such is assured when the committee size is relatively small, when the committee size m is greater than the critical value $\bar{m}(r,t)$, a pure strategy equilibrium does not exist. Klumpp & Polborn (2006) prove that when $m > \bar{m}(r,1)$, there exists a symmetric, uniform mixed strategy equilibrium when both contestants have the same valuations; subsequently, Alcade & Dahm (2010) prove the existence of mixed-strategy equilibrium when $r \geq 2$ in Tullock-contests under a single administrator. Following their analysis, it is possible for us to show the existence of a mixed uniform bidding strategy[5], similar to the equilibrium result found in the all-pay auction framework. This has serious implications regarding the nature of the interaction between committee size and the provision of aggregate rent-seeking efforts. Such complexities are left to consideration in future research.

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[5] Under such mixed-strategy equilibrium(s), the expected payoff of Contestant 1 can be shown to be $V_1 - V_2$, with expected rent-seeking efforts $V_2/2$, with Contestant 2 receiving expected payoff of 0 and expected rent-seeking efforts $(V_2)^2/(2V_1)$.