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Equivalence in the internal and external public debt burden

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Abstract

Whether public debt is internal or external, the burden is equivalent in the OLG model. This equivalence holds, regardless of whether the definition of burden reflects Modigliani's view or Lerner's perspective. It results from the assumption of perfect substitutability between public debt and productive capital.

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1. Introduction

According to Modigliani (1961), the debt burden is a reduction in the aggregate stock of private capital, which will lead to a reduction in the flow of goods and services for future generations. Debt is a burden, because it crowds out capital. But does this burden differ, depending on whether the debt is "internal" (held by domestic agents) or "external" (held by foreign agents)? Conflicting arguments persist in relation to the burden of debt, according to whether it is internal or external.

Lerner (1948) argues that external public debt is a burden, whereas internal public debt is not. The sale of bonds abroad generates a charge for the nation, whose servicing represents a perpetual trade surplus that future generations must bear. According to Lerner, an "external debt burden" is the trade surplus needed to finance the debt service. Mutoh (1985) contests this argument, using the principle of Ricardian equivalence. In this view, the debt allows a reduction in present taxes, which leads the representative agent to save (and invest) the equivalent of the tax reduction, from the perspective of future tax increases. There is no crowding out of capital, and neither does the agent change its chronic consumption such that debt is neutral, and there is no burden. Mutoh affirms these relations, whether the debt is held nationally or by foreigners. If a country's public debt is held by foreigners, the national savings will support further investments in foreign securities (foreigners conversely hold more debt and fewer securities). That is, if foreigners buy national debt, residents buy foreign securities. The capital stock of the two countries thus remains unchanged, and there is no burden, contrary to the Lerner's assertion.

However, this proof is obvious only in the context of Ricardian equivalence, according to which debt is neutral, and there is no burden, either internal or external. In this context, the internal and external public debt burden are equivalent: it does not exist. But what about in the Overlapping Generations (OLG) model where there is no Ricardian equivalence and public debt implies a genuine burden?

The reference for the OLG model is Diamond (1965), who shows that internal debt crowds out capital, but this effect does not hold for debt held by foreigners. Contrary to Lerner's assertions, internal detention imposes burden, while external debt does not. The purpose of Diamond's famous article is to show that, surprisingly, internal holding of public debt is worse than external holding. As Diamond points out in his introduction, *"External debt reduces the utility of an individual ... surprisingly, internal debt is seen to cause an even larger decline in this utility level"* (Diamond 1965, p. 1126). His famous equilibrium equation for capital market $S_t = K_{t+1} + B_{t+1}$, also leads him to comment: *"internal debt has a further effect in that it substitutes pieces of paper for physical capital in the portfolios of wealth owners, thus reducing output."* (Diamond 1965, p. 1141). For proof, Diamond uses the difference between the two equilibrium conditions for capital market in case of internal detention $S_t = K_{t+1} + B_{t+1}$ versus external detention $S_t = K_{t+1}$. The first equation involves crowding out of capital, which is not present in second equation. In Diamond's view, all national savings is available for the national productive capital when foreigners who hold the debt. Yet Diamond's discussion of the superiority of external debt relies on a partial equilibrium framework. As Thompson[1] (1967) and Carlberg (1985) recognize, Diamond does not work in a general equilibrium framework in open economy.

[1]"Diamond's analysis is inconsistent with the existence of an equibrating international market for real capital" (Thompson 1976, p. 922).

With Diamond's OLG model, we show that in an open economy, in a general equilibrium framework, internal and external public debt burden are equivalent. This equivalence holds as long as we retain the definition of burden in the sense of Modigliani or Lerner. Such equivalence arises because the assumption of perfect substitutability between public debt and productive capital.

2. The model in close economy

We assume two overlapping generations economies. At each date t, in each country, a generation of size N_t emerges and works during the first period of life, then consumes savings in the second period. Population growth is realized in each country at the rate n.

In each country, the production function is Cobb-Douglas, with $k_t = K_t/N_t$ revealing the capital per worker, such that:

$$f(k_t) = q_t = A{k_t}^{\alpha} \tag{1}$$

Also in each country, the government budget identity is $T_t + B_{t+1} = G_t + (1 + r_t)B_t$, where T is tax, G refers to public spending, and B is public debt. Dividing by N_t, we obtain the constraint by worker $\tau_t + (1 + n)b_{t+1} = g_t + (1 + r_t)b_t$. The government of each country sets the tax rate to maintain a constant long-term debt ratio:

$$\frac{b_t}{q_t} = \beta \quad \forall t \tag{2}$$

For simplicity and because we are interested only in the consequences of debt, we assume that $g_t = 0$, so the value of the tax that balanced the budget in the steady state is $\tau = \beta(r - n)q$. We assume that the tax is on capital income which represents a twofold assumption. The theoretical goal is the ability to highlight only the crowding-out effect of debt by eliminating its effect on the wage and savings. The practical objective is to allow for a solution to the recurrence equation, to determine the steady-state capital stock.

For each country, we assume that an agent's utility function is a logarithmic function of consumption in both periods of life and that each agent born in t maximizes his utility under the budget constraints that affect the young and old age:

$$Max \;\; V_t = \ln c_t^y + \frac{\ln c_{t+1}^o}{1+\rho} \quad \text{s.t.} \quad c_t^y + s_t = w_t \quad \text{and} \quad c_{t+1}^o = (1 + r_{t+1})\, s_t - \tau_t \tag{3}$$

Maximizing the producer's profit under the production constraint, equation (1), yields the following factor prices determination:

$$w_t = (1 - \alpha)A{k_t}^{\alpha}, \quad r_t = \alpha A{k_t}^{\alpha-1} - \delta \tag{4}$$

By solving the consumer's problem in equation (3), the savings of the young is:

$$s_t = \frac{w_t}{2+\rho} \tag{5}$$

In an autarky, in each country, the private wealth of agents A_t consists of private domestic assets K_t and national government bonds B_t:

$$A_t = K_t + B_t \tag{6}$$

The increase in wealth thus is equal to the savings of young $N_t s_t$, minus the spending by the old A_t, who consume all their wealth (they are egoistic). Therefore, $A_{t+1} - A_t = N_t s_t - A_t$ or $A_{t+1} = N_t s_t$; dividing by N_t, we obtain $(1+n)\, a_{t+1} = s_t$ such that for each country:

$$a_{t+1} = \frac{s_t}{(1+n)} \tag{7}$$

Equilibrium conditions in the capital market of each closed economy equalize supply in equation (7) to demand in equation (6). Written as a variable per worker $a_{t+1} = k_{t+1} + b_{t+1}$, and:

$$\frac{s_t}{1+n} = k_{t+1} + b_{t+1} \tag{8}$$

If we replace saving from equation (5) and wage from equation (4), we obtain:

$$k_{t+1} = \frac{(1-\alpha)\, A k_t^{\alpha}}{(1+n)\,(2+\rho)} - b_{t+1} \tag{9}$$

Thus, we obtain the classical result shown by Diamond (1965), namely, crowding out of capital by public debt.

In this model with a single good, we consider two countries exchanging capital as representative of the world economy. That is, the world is composed of two countries, ”Tilde” and ”Hat”, such that each variable and parameter used a tilde or a hat to specify the country to which it refers. We assume that the two economies differ on two points.

First, to exchange capital, the two countries must differ in their accumulation behavior. As in Buiter (1981), we could anticipate a difference in rate of time preference $\tilde{\rho} \neq \hat{\rho}$, but it more interesting for our purposes[2] to assume that the two economies differ only in their public debt ratio $\tilde{\beta} \neq \hat{\beta}$. We assume that Hat Country has no public debt. Only Tilde Country has incurred public debt, so we analyze the debt burden for it.

$$\tilde{\beta} > 0, \quad \hat{\beta} = 0 \tag{10}$$

Second, the size of these economies differs, $\tilde{N} \neq \hat{N}$. This assumption enables us to model a case in which the Tilde economy is a small open economy, and the two economies constitute the world economy (case of general equilibrium framework ignored by Diamond).

Applying equations (10),(2) and (1), for each country we can apply equation (9):

$$\hat{k}_{t+1} = \frac{(1-\alpha) A \hat{k}_t^{\alpha}}{(1+n)\,(2+\rho)} \quad \text{and} \quad \tilde{k}_{t+1} = \frac{(1-\alpha)\, A \tilde{k}_t^{\alpha}}{(1+n)\,(2+\rho)} - \tilde{\beta} A \tilde{k}_t^{\alpha} \tag{11}$$

Solving at a steady state:

$$\hat{k}^* = \left(\frac{(1-\alpha)\, A}{(1+n)\,(2+\rho)}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \tilde{k}^* = \left(\frac{(1-\alpha)\, A}{(1+n)\,(2+\rho)} - \tilde{\beta} A\right)^{\frac{1}{1-\alpha}} \tag{12}$$

In autarky, the Tilde Country has crowded out its capital through debt. The decrease in capital stock when $\tilde{\beta} > 0$ is the burden of public debt according Modigliani, for Tilde Country. We now consider this burden in an open economy.

[2]Differences in time preference do not change our results, but complicates the interpretation.

3. Accounting framework of an open economy

In an open economy, agents are able to hold domestic and foreign securities, as well as domestic and foreign government bonds. By hypothesis, the domestic and foreign securities, and domestic and foreign government bonds are perfectly substitutable assets. This assumption is traditional in an OLG model, in which savings transform into capital and bonds. The net wealth of agents consists of the amounts held, national and foreign securities, and domestic and foreign government bonds. The private wealth of agents in Tilde Country and in Hat Country is:

$$\tilde{A}_t = \tilde{\gamma}\tilde{K}_t + \tilde{\varepsilon}\tilde{B}_t + (1-\hat{\gamma})\hat{K}_t \quad \text{and} \quad \hat{A}_t = \hat{\gamma}\hat{K}_t + (1-\tilde{\varepsilon})\tilde{B}_t + (1-\tilde{\gamma})\tilde{K}_t \tag{13}$$

where :

$\tilde{\gamma}\tilde{K}$	:	capital of Tilde Country held by the tildians,
$\tilde{\varepsilon}\tilde{B}_t$	:	public debt of Tilde Country held by the tildians,
$(1-\hat{\gamma})\hat{K}$	:	capital of Hat Country held by tildians,
$\hat{\gamma}\hat{K}$	:	capital of Hat Country held by hatese,
$(1-\tilde{\varepsilon})\tilde{B}_t$	:	public debt of Tilde Country held by hatese, and
$(1-\tilde{\gamma})\tilde{K}$	:	capital of Tilde Country held by the hatese.

The exogenous parameter $\tilde{\varepsilon}$ specifies the identity of the holders of Tilde Country's public debt, such that it can be wholly owned by national agents $\tilde{\varepsilon} = 1$ or by foreigners $\tilde{\varepsilon} = 0$ or by both $0 < \tilde{\varepsilon} < 1$.

Definition 1 *$\tilde{\varepsilon}\tilde{B}_t$ is internal public debt, and $(1-\tilde{\varepsilon})\tilde{B}_t$ is external public debt.*

Tilde Country lends $(1-\hat{\gamma})\hat{K}$ and borrows $(1-\tilde{\varepsilon})\tilde{B}_t + (1-\tilde{\gamma})\tilde{K}$. Its net loan thus is:

$$\tilde{E}_t = (1-\hat{\gamma})\hat{K}_t - (1-\tilde{\varepsilon})\tilde{B}_t - (1-\tilde{\gamma})\tilde{K}_t \tag{14}$$

Hat Country lends $(1-\tilde{\varepsilon})\tilde{B}_t + (1-\tilde{\gamma})\tilde{K}$ and borrows $(1-\hat{\gamma})\hat{K}$. Its net loan thus is:

$$\hat{E}_t = (1-\tilde{\varepsilon})\tilde{B}_t + (1-\tilde{\gamma})\tilde{K}_t - (1-\hat{\gamma})\hat{K}_t \tag{15}$$

Definition 2 *E_t is the net international investment position (NIIP). The variation of E is equal to the current account F, or trade balance $(X - M)$ plus net international investment income:*

$$E_{t+1} - E_t = F_t = (X_t - M_t) + r_t E_t \tag{16}$$

Because there are only two countries, net lending by one is net borrowing by the other, so by construction:

$$\tilde{E}_t = -\hat{E}_t \quad \forall t \tag{17}$$

Proposition 1 *Agents' wealth is independent of $\tilde{\varepsilon}$, that is, of the identity on the debt holders.*

$$\tilde{A}_t = \tilde{\gamma}\tilde{K}_t + \tilde{\varepsilon}\tilde{B} + (1-\hat{\gamma})\hat{K} = \tilde{\gamma}\tilde{K}_t + \tilde{\varepsilon}\tilde{B} + \tilde{E}_t + (1-\tilde{\varepsilon})\tilde{B}_t + (1-\tilde{\gamma})\tilde{K}_t = \tilde{K}_t + \tilde{B} + \tilde{E}_t$$

$$\hat{A}_t = \hat{\gamma}\hat{K}_t + (1-\tilde{\varepsilon})\tilde{B} + (1-\tilde{\gamma})\tilde{K} = \hat{\gamma}\hat{K}_t + (1-\tilde{\varepsilon})\tilde{B} + \hat{E}_t - (1-\tilde{\varepsilon})\tilde{B}_t + (1-\hat{\gamma})\hat{K}_t = \hat{K}_t + \hat{E}_t$$

In turn, agents' wealth can be written independently of $\tilde{\varepsilon}$ by identity:

$$\tilde{A}_t \equiv \tilde{K}_t + \tilde{B}_t + \tilde{E}_t \quad \text{and} \quad \hat{A}_t \equiv \hat{K}_t + \hat{E}_t \tag{18}$$

The private agents' wealth does not depend on porfolio composition, as expressed in equation (13), because by assumption, the domestic and foreign assets and domestic and foreign bonds are perfect substitutes assets. Therefore, only NIIP is important for assessing private agents' wealth.

4. Modigliani burden of internal and external debt

The behaviors of agents in each country are described by equations (4) and (5). In each country, the wealth increase is equal to the savings of the young minus the dissaving, or spending of the old, who consume all their wealth (are egoistic). For Tilde Country, $\tilde{A}_{t+1} - \tilde{A}_t = \tilde{N}_t\tilde{s}_t - \tilde{A}_t$ or $\tilde{A}_{t+1} = \tilde{N}_t\tilde{s}_t$, and for the two countries:

$$\tilde{N}_t\tilde{s}_t = \tilde{A}_{t+1} \quad \text{and} \quad \hat{N}_t\hat{s}_t = \hat{A}_{t+1} \tag{19}$$

The equilibrium condition for the capital market in an open economy is based on equations (19) and (13):

$$\tilde{N}_t\tilde{s}_t + \hat{N}_t\hat{s}_t = \tilde{\gamma}\tilde{K}_{t+1} + \tilde{\varepsilon}\tilde{B}_{t+1} + (1-\hat{\gamma})\hat{K}_{t+1} + \hat{\gamma}\hat{K}_{t+1} + (1-\tilde{\varepsilon})\tilde{B}_{t+1} + (1-\tilde{\gamma})\tilde{K}_{t+1} = \tilde{K}_{t+1} + \tilde{B}_{t+1} + \hat{K}_{t+1}$$

Or written in variables per worker:

$$\tilde{N}_t\tilde{s}_t + \hat{N}_t\hat{s}_t = \tilde{N}_{t+1}\left(\tilde{k}_{t+1} + \tilde{b}_{t+1}\right) + \hat{N}_{t+1}\left(\hat{k}_{t+1}\right) \tag{20}$$

Because the population grows in each country at the same rate n,

$$\tilde{N}\tilde{s}_t + \hat{N}\hat{s}_t = (1+n)\left(\tilde{N}\left(\tilde{k}_{t+1} + \tilde{b}_{t+1}\right) + \hat{N}\left(\hat{k}_{t+1}\right)\right)$$

Dividing by $\tilde{N}$, we can replace saving by (5) and wage by equation (4) to determine

$$\frac{(1-\alpha)A\tilde{k}_t^{\alpha}}{(1+n)(2+\rho)} + \frac{\hat{N}}{\tilde{N}}\frac{(1-\alpha)A\hat{k}_t^{\alpha}}{(1+n)(2+\rho)} = \tilde{k}_{t+1} + \tilde{b}_{t+1} + \frac{\hat{N}}{\tilde{N}}\hat{k}_{t+1}$$

In an open economy, whatever t is $\hat{k}_t = \tilde{k}_t = k_t$, therefore,

$$\left(1 + \frac{\hat{N}}{\tilde{N}}\right)\frac{(1-\alpha)Ak_t^{\alpha}}{(1+n)(2+\rho)} = \left(1 + \frac{\hat{N}}{\tilde{N}}\right)k_{t+1} + \tilde{b}_{t+1}$$

By applying equations (1) and (2) we obtain

$$\left(1 + \frac{\hat{N}}{\tilde{N}}\right)\frac{(1-\alpha)Ak_t^{\alpha}}{(1+n)(2+\rho)} = \left(1 + \frac{\hat{N}}{\tilde{N}}\right)k_{t+1} + \tilde{\beta}Ak_{t+1}^{\alpha}$$

At a steady state $k_{t+1} = k_t = k^*$, we find the steady state capital value for all countries, that is,

$$k^* = \left(\frac{(1-\alpha)A}{(1+n)(2+\rho)} - \tilde{\eta}\tilde{\beta}A\right)^{\frac{1}{1-\alpha}} \tag{21}$$

where $\tilde{\eta} = \frac{\tilde{N}}{\tilde{N}+\hat{N}}$ is the relative weight of Tilde Country in the world. Comparing equations (12) and (21) shows that $\tilde{k}^* < k^*$ for $0 < \tilde{\eta} < 1$ and for $0 < \tilde{\beta}$. Crowding out is lower in the open economy. It even declines to nothing, as in Diamond's case, if $\tilde{\eta} = 0$, that is in the case of a small open economy. From equation (21), we can derive the following proposals:

Proposition 2 *Public debt crowds out capital, not only Tilde Country's capital, but also the capital of all countries. This crowding-out effect increases with the size of Tilde Country. When Tilde country is smaller, the crowding-out effect is diluted.*

Proposition 3 *Steady-state capital value and crowding-out effect do not depend on $\tilde{\varepsilon}$,that is, on whether public debt is internal or external. The Modigliani burden is equivalent whether public debt is internal or external.*

In an open economy, the crowding-out effect is independent of the internal or external holding of public debt. The market equilibrium condition of capital in equation(20) is independent of $\tilde{\varepsilon}$, (i.e., identity of debt holders), so the value of the steady-state capital is independent of the identity of the holders of debt as well.

5. Lerner burden of internal and external debt

Following Lerner, now define the external debt burden by the steady-state trade surplus needed to finance public debt service. Denote $\tilde{Z}_t = \tilde{X}_t - \tilde{M}_t$ as the trade balance of Tilde Country. Expressing equation (16) in variables per worker $(1+n)\tilde{e}_{t+1} - \tilde{e}_t = \tilde{f}_t = \tilde{z}_t + r_t\tilde{e}_t$, and at a steady state, we obtain

$$n\tilde{e} = \tilde{f} = \tilde{z} + r\tilde{e} \tag{22}$$

from which we can extract the burden value of debt, according to Lerner's meaning:

$$\tilde{z} = (n - r)\,\tilde{e} \tag{23}$$

Definition 3 *There is a burden, in Lerner's definition, if $\tilde{z}$ is positive.*

The burden exists, according to Lerner's definition, if the country supports a trade surplus at steady state. In dynamic efficiency $(r > n)$, there is a burden if $\tilde{e} < 0$, such that the country is a net borrower. This steady-state debt logically is "paid for" by a perpetual trade surplus. To calculate the sign of $\tilde{e}$ at steady state, we can write equation (18) in the steady state:

$$\tilde{e} = \tilde{a} - \tilde{b} - k^* \tag{24}$$

By replacing wealth by saving value and debt by capital value, we recognize:

$$\tilde{e} = \frac{(1-\alpha)\,Ak^{*\alpha}}{(1+n)\,(2+\rho)} - \tilde{\beta}Ak^{*\alpha} - k^*, \text{and,}$$

$$\tilde{e} < 0 \quad \Leftrightarrow \quad \left(\frac{(1-\alpha)\,A}{(1+n)\,(2+\rho)} - \tilde{\beta}A\right) < k^{*1-\alpha}$$

The left-hand side is $\tilde{k}^{*1-\alpha}$ (see Equation 12). We have shown already that if $0 < \tilde{\beta}$, then $\tilde{k}^* < k^*$ for $0 < \tilde{\eta} < 1$, so $\tilde{e} < 0$. We thus conclude that if $r > n$, $\tilde{z}$ is positive. (Note that if $\tilde{\beta} = 0$, then $\tilde{k}^* = k^*$ and $\tilde{e} = 0$ because $\tilde{\rho} = \hat{\rho}$). We accordingly make the following proposals:

Proposition 4 *In dynamic efficiency, there is a Lerner burden of public debt.*

We also acknowledge that dynamic efficiency is a condition for the existence of a Modigliani burden as well.

Proposition 5 *Both $\tilde{e}$ and $\tilde{z}$ are independent of $\tilde{\varepsilon}$, that is, whether, the debt is internal or external. The Lerner burden is equivalent, whether public debt is internal or external.*

The burden of debt as a trade surplus, is independent of its internal or external holding. As established by Proposition 1 and equation (18), the wealth of agents is independent of $\tilde{\varepsilon}$. If foreigners hold more government debt, they hold less capital, and only the net international investment position (NIIP) is important for assessing the Lerner burden. As a result, the trade surplus is independent of $\tilde{\varepsilon}$. This independence results from the assumption of perfect substitutability between securities and bonds, domestic and foreign.

6. Conclusion

In an OLG model in an open economy, public debt generates a burden, according both a Modigliani meaning and a Lerner meaning. Public debt crowds out capital (which causes international debt) and requires the export of goods to pay for the debt service. But these burdens are independent of the internal or external holding of public debt. This equivalence arises because in an open economy, holding domestic and foreign assets and bonds is equivalent, under the assumption of perfect substitutability retained in the OLG model.

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