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On the Relationship between Competition and Innovation in a Duopoly with a Single Innovator

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Abstract

This paper studies the incentive by a single firm in a differentiated goods duopoly to engage in cost-reducing innovations and how this incentive is affected by the level of competition in the product market. It is found that a firm's innovation effort has a U-shaped relationship with the level of competition. This result generally holds true for both the initially more efficient firm and the initially less efficient firm and in both the open loop model and the closed loop model. Consumers always benefit from innovations and fare the best when the initially more efficient firm is the innovator.

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1. Introduction

Ever since the pioneering works by Schumpeter (1942) and Arrow (1962), economists have been interested in the relationship between innovation and competition. The early literature debated on whether monopoly or competition is more conducive to innovation.[1] More recently, the focus of discussion shifted to oligopolistic market forms.[2] Along with this shift, the study of innovation has also moved from the examination of a single firm's incentive to innovate to one in which all firms engage in innovation activities. While this new focus is well justified, it is still an interesting issue to explore in the context of an oligopolistic market when a single firm innovates.[3]

The present paper studies innovation and its relationship to competition in a duopolistic market in which there is only one firm that engages in innovation. This seems to be an interesting and valuable topic to explore from both theoretical and empirical grounds. As mentioned above, the early literature focused mainly on the incentive of a single firm to innovate. For example, Arrow (1962) focused on the gain from innovation to a competitive firm that is the only one to undertake innovation, while Schumpeter (1942) spoke of the incentive to innovate by a monopolist. Empirically, it has been well documented that few firms engage in R&D activities. For example, Klette and Kortum (2004) report it as a stylized fact that the distribution of R&D intensity is highly skewed with a considerable fraction of firms reporting zero R&D.

We use the standard duopolistic model with differentiated goods and linear demand functions. In this model the product substitution parameter has frequently been used as a representation of the level of competition.[4] At one extreme, the firms' products are independent and the firms do not compete against each other. At the other extreme, their products are perfect substitutes. The two firms are initially asymmetric in that they have different levels of (constant) marginal costs. Our study proceeds by studying the incentive of each firm to innovate when it is the only firm undertaking innovation and then comparing the outcomes.[5] Innovation is in the form of marginal cost reduction. There is a fixed cost associated with innovation that is an increasing function of the level of innovation.

We examine innovation both in an open loop model and in a closed loop model. In the former, innovation choice is private information and unobservable by the competitor, while in the latter innovation choice becomes public information before output choices. The main result we find is that a firm's innovation effort has a U-shaped relationship with the level of competition.[6] This result generally holds true in both the open loop model and the closed loop model and for both the initially more efficient firm and the initially less efficient firm.

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[1] Demsetz (1969) and Gilbert and Newbery (1982) provide critiques and reviews of the early literature. See Chen and Schwartz (2013) for a recent research on the innovative incentive under monopoly and competition.
[2] Vives (2008) presents an extensive review of the literature.
[3] Among the sizable literature that studies innovation choices in an oligopolistic market, to our best knowledge, Yi (1999) is an exception that investigates innovation incentives by a single firm in a homogenous good Cournot oligopoly.
[4] Another often used parameter as a proxy for the level of competition in the oligopolistic market is the number of firms. For example, Yi (1999) uses the number of firms as measuring the competitiveness of the market. Vives (2008) considers both the substitution parameter and the number of firms as representing the level of competition.
[5] Klette and Kortum (2004) report it as a stylized fact that R&D intensity is independent of firm size. It is therefore justified to treat either the larger (i.e., the initially more efficient) firm or the smaller (initially less efficient) firm as the innovator.
[6] Tang (2006) documents empirically that innovation can be both increasing and decreasing in competition.

The U-shaped relationship between a firm's innovation and competition stipulates that the effect of increased competition on investment in innovation is strongest both when the level of competition is low and when it is high. This result is the consequence of two opposing forces. The effect on profit of a reduction in marginal cost is dependent on two components: the price/cost margin and the effect on output. An increase in competition has a negative effect on a firm's price/cost margin when the level of competition is weak; and this effect eventually turns positive as the level of competition becomes sufficiently strong. The effect of rising competition on the second component is always increasing in the level of competition. The interplay of these two forces leads to a U-shaped relationship between a firm's level of investment and the level of competition.

The paper that is closely related to our work is Sacco and Schmutzler (2011). Both papers study the incentive to innovate and the relationship of innovation to competition in a differentiated duopoly market with linear demand functions and constant marginal costs. The main difference is in focus. In Sacco and Schmutzler (2011), both firms engage in innovation at the same time, while the present paper examines innovation by a single firm. A U-shaped relationship between innovation and competition is also found by these authors. Hence, the present study complements the work of Sacco and Schmutzler (2011) by establishing the U-shaped relationship as a more general outcome and likely to arise in different environments.[7] Another study that is also closely related to our study is Yi (1999). Both papers share in common the focus on the incentive to innovate for a single firm in an oligopoly. While Yi (1999) examines the benefit from a small innovation and how this benefit is affected by the number of competitors in the product market, we deal with the choice of innovation and how the level of innovation is affected by the level of competition.

The remainder of the paper is organized as follows. The next section introduces the basic model setup and the solution for the no innovation situation. Section 3 covers innovation choices in the open loop model. Section 4 presents the closed loop model. Section 5 concludes the paper. Proofs of propositions and some derivations are presented in the appendix.

2. Model Setup

Consider a differentiated goods duopoly with the following inverse demand equations:

$$p_i = A - q_i - \gamma q_j \qquad i = 1,2, j = 1,2, i \neq j. \tag{1}$$

In (1), p_i and q_i denote, respectively, the price and quantity of firm i; A measures the maximum price consumers are willing to pay for either good; $\gamma \in [0,1]$ is the substitution parameter. The two goods are independent when $\gamma = 0$ and are perfect substitutes when $\gamma = 1$. As γ increases, each firm cuts more into the other firm's demand. Indeed, from (1), both firms' demand curves shift inward as γ increases. Hence, the market becomes more competitive as γ increases.

Initially, the firms' constant marginal costs of production are $c_1 = c_0$ and $c_2 = c_0 + \tau$, respectively, where $\tau > 0$ denotes the initial cost advantage of firm 1 over firm 2. For each firm i ($i = 1, 2$), when it engages in innovative activities in the amount of I_i, its marginal cost of production is reduced by I_i and it incurs a fixed investment cost of $(I_i)^2/2$. We assume in this paper that only one of the two firms is capable of innovation, namely the above described innovation technology is available to only one of the duopolists.

[7] Moreover, the present study explores both open and closed loop innovation models, whereas Sacco and Schmutzler (2011) work with a closed loop model only.

The two firms engage in Cournot quantity competition in the product market. Given arbitrary constant unit costs of production c_1 and c_2 for firm 1 and 2, respectively, it is straightforward to find that the firms' equilibrium quantities and profits are given by

$$q_1 = \frac{(2-\gamma)A-2c_1+\gamma c_2}{4-\gamma^2}, \quad q_2 = \frac{(2-\gamma)A-2c_2+\gamma c_1}{4-\gamma^2}, \tag{2}$$

$$\pi_1 = (q_1)^2, \quad \pi_2 = (q_2)^2. \tag{3}$$

It follows immediately that each firm's equilibrium output and profit rise as its own marginal cost decreases, pointing to an incentive to lower one's marginal cost of production.

For simplicity, we shall maintain the following assumption throughout the paper.

Assumption 1: $0 < \tau < (A - c_0)/2$.

Substituting $c_1 = c_0$ and $c_2 = c_0 + \tau$ into (2) yields that, without innovation by either firm, the two firms produce the following output levels (superscripts 0 denote equilibrium values when no firm innovates), respectively,

$$q_1^0 = \frac{(2-\gamma)(A-c_0)+\gamma\tau}{4-\gamma^2}, \quad q_2^0 = \frac{(2-\gamma)(A-c_0)-2\tau}{4-\gamma^2}. \tag{4}$$

Their respective profits are given by

$$\pi_1^0 = (q_1^0)^2, \quad \pi_2^0 = (q_2^0)^2. \tag{5}$$

Assumption 1 implies that $q_1^0 > q_2^0 > 0$ for all γ. Hence, the results above indicate that initially, given its cost advantage, the more efficient firm 1 produces a larger output and earns a greater profit than the less efficient firm 2.

From (4),

$$\frac{\partial q_1^0}{\partial \gamma} = \frac{-(4-4\gamma+\gamma^2)(A-c_0)+(4+\gamma^2)\tau}{(4-\gamma^2)^2}, \quad \frac{\partial q_2^0}{\partial \gamma} = \frac{-(4-4\gamma+\gamma^2)(A-c_0)+2\gamma^2\tau}{(4-\gamma^2)^2}.$$

Since $\frac{\partial q_1^0}{\partial \gamma}$ is negative for small values of γ and positive for large values of γ, q_1^0 is U-shaped in γ. On the other hand, $\frac{\partial q_2^0}{\partial \gamma}$ is negative for all values of γ, implying that q_2^0 is always decreasing in γ. It follows that firm 1's output (equal to its price/cost margin) decreases and then increases (i.e., U-shaped) as the level of competition increases while firm 2's output and price/cost margin always decrease in the level of competition. By (5), their profits as functions of γ follow the same respective patterns.

3. Open Loop Investment Choice

We start with an open loop model. In this approach, the innovating firm's choice of investment and both firms' choices of output are simultaneously made. The one-stage approach captures the information structure in which the innovating firm's choice on innovation is private information and is not observable by its competitor in the product market.

Since the two firms are initially asymmetric in their marginal costs of production, their innovation choices and the corresponding equilibrium may differ depending on which firm is the innovating firm. Our investigation in the following starts with the initially more efficient firm 1 as the innovator, then the initially less efficient firm 2 as the innovator, and finally a comparison of the corresponding outcomes.

3.1 The more efficient firm innovates

When the more efficient firm 1 is the only firm engaging in innovation activities, the marginal costs of the two firms are $c_1 = c_0 - I_1$ and $c_2 = c_0 + \tau$, respectively. Their profit functions are as follows:

$$\Pi_1 = [A - q_1 - \gamma q_2 - (c_0 - I_1)]q_1 - \frac{1}{2}(I_1)^2, \tag{6}$$

$$\Pi_2 = [A - q_2 - \gamma q_1 - (c_0 + \tau)]q_2. \tag{7}$$

Firm 1 maximizes (6) by choosing q_1 and I_1. Simultaneously, firm 2 chooses q_2 to maximize (7). The equilibrium is determined by the following first-order conditions:

$$\frac{\partial \Pi_1}{\partial q_1} = A - \gamma q_2 - (c_0 - I_1) - 2q_1 = 0,$$

$$\frac{\partial \Pi_1}{\partial I_1} = q_1 - I_1 = 0,$$

$$\frac{\partial \Pi_2}{\partial q_2} = A - \gamma q_1 - (c_0 + \tau) - 2q_2 = 0.$$

It is easy to verify that firm 1's profit function in (6) is strictly concave in (q_1, I_1) and firm 2's profit function in (7) is strictly concave in q_2. Hence, any solution to the above system of equations gives us the equilibrium. Straightforward algebra yields the following equilibrium values (superscripts 1 denote equilibrium values when firm 1 only innovates):

$$q_1^1 = I_1^1 = \begin{cases} \frac{(2-\gamma)(A-c_0)+\gamma\tau}{2-\gamma^2} \\ A - c_0 \end{cases} \text{for } \begin{cases} \gamma < \bar{\gamma} \\ \gamma \geq \bar{\gamma} \end{cases} \tag{8}$$

$$q_2^1 = \begin{cases} \frac{(1-\gamma)(A-c_0)-\tau}{2-\gamma^2} \\ 0 \end{cases} \text{for } \begin{cases} \gamma < \bar{\gamma} \\ \gamma \geq \bar{\gamma} \end{cases} \tag{9}$$

where $\bar{\gamma} = \frac{A-c_0-\tau}{A-c_0} \in (0,1)$. The firms' equilibrium profits are given by

$$\pi_1^1 = \frac{1}{2}(q_1^1)^2, \quad \pi_2^1 = (q_2^1)^2. \tag{10}$$

It follows from the above that if the market is sufficiently competitive (i.e., $\gamma \geq \bar{\gamma}$) then the more efficient firm 1, as the only innovating firm, will drive the less efficient firm 2 out of the market (although both firms are active if no one undertakes any innovation).

Note that Assumption 1 implies that $q_1^1 > q_2^1$ for all γ. Comparing the results in (8)-(10) with (4)-(5), we immediately reach the intuitive conclusion that, compared with no innovation by either firm, the innovating firm 1 produces more and gains from innovation while the non-innovating firm 2's output and profit are decreased.

Through a closer examination of firm 1's choice of innovation and output in equilibrium given in (8), we obtain the following conclusion about their behavior as the level of competition increases.

Proposition 1: As the level of competition increases, firm 1's innovation effort and output level first decrease and then increase. More specifically, there exists $\hat{\gamma}^1 \in (0, \bar{\gamma})$ such that (i) I_1^1 and q_1^1 decrease in γ for $\gamma < \hat{\gamma}^1$; and (ii) I_1^1 and q_1^1 increase in γ for $\gamma > \hat{\gamma}^1$.

This proposition implies that firm 1's innovation effort has a U-shaped relationship with the level of competition: It decreases with the level of competition when the level of competition is low, but the relationship is revered if the level of competition is high.

The U-shaped relationship is the result of two (mostly) opposing forces that are present when the level of competition rises. A firm's profit is the product of two terms: the price/cost margin and the output level. It follows that the effect on profit of a reduction in marginal cost is dependent on the product of two components: the price/cost margin and the effect on output. As noted in the preceding section, for firm 1, the price/cost margin is U-shaped in the level of competition. That is, an increase in γ has a negative effect on firm 1's price/cost margin when γ is small; and this effect eventually turns positive as γ becomes sufficiently large. The effect of an rising γ on the second component is always increasing in γ. The interplay of these two forces leads to a U-shaped relationship between firm 1's level of investment and the level of competition. When the competition is weak (small γ) the positive force is outweighed by the negative force, resulting in a decrease in innovation as γ increases. When the competition is strong (large γ) the opposite occurs, leading to more innovation as γ increases.

3.2 The less efficient firm innovates

When the less efficient firm 2 is the only firm engaging in innovation activities, the marginal costs of the firms are $c_1 = c_0$ and $c_2 = c_0 + \tau - I_2$, respectively. Their corresponding profit functions are as follows:

$$\Pi_1 = [A - q_1 - \gamma q_2 - c_0]q_1, \tag{11}$$

$$\Pi_2 = [A - q_2 - \gamma q_1 - (c_0 + \tau - I_2)]q_2 - \tfrac{1}{2}(I_2)^2. \tag{12}$$

Firm 1 maximizes (11) by choosing q_1. Simultaneously, firm 2 chooses q_2 and I_2 to maximize (12). The equilibrium is determined by the following first-order conditions:

$$\frac{\partial \Pi_1}{\partial q_1} = A - \gamma q_2 - c_0 - 2q_1 = 0,$$

$$\frac{\partial \Pi_2}{\partial q_2} = A - \gamma q_1 - (c_0 + \tau - I_2) - 2q_2 = 0,$$

$$\frac{\partial \Pi_2}{\partial I_2} = q_2 - I_2 = 0.$$

It is easy to verify that firm 1's profit function in (11) is strictly concave in q_1 and firm 2's profit function in (12) is strictly concave in (q_2, I_2). Hence, any solution to the above system of equations gives us the equilibrium. Straightforward algebra yields the following equilibrium values (superscripts 2 denote equilibrium values when firm 2 only innovates):

$$q_1^2 = \frac{(1-\gamma)(A-c_0)+\gamma\tau}{2-\gamma^2}, \quad q_2^2 = I_2^2 = \frac{(2-\gamma)(A-c_0)-2\tau}{2-\gamma^2}. \tag{13}$$

The firms' equilibrium profits are given by

$$\pi_1^2 = (q_1^2)^2, \quad \pi_2^2 = \tfrac{1}{2}(q_2^2)^2. \tag{14}$$

Applying Assumption 1, we have $q_1^2 > 0$ and $q_2^2 > 0$ for all γ. Hence, the initially less efficient firm 2 will never drive the initially more efficient firm 1 out of the market through innovation. Comparing the results in (13)-(14) with (4)-(5), we immediately reach the intuitive conclusion that, compared with no innovation by either firm, the innovating firm 2 produces more and gains from innovation while the non-innovating firm 1's output and profit are decreased.

Examining firm 2's equilibrium choice of innovation and output given in (13), we have the following conclusion about their behavior as the level of competition increases.

Proposition 2: (1) If the initial cost advantage of the more efficient firm 1 is sufficiently small ($\tau < (A - c_0)/4$), then firm 2's innovation effort and output have an U-shaped relation with the level of competition; that is, there exists $\hat{\gamma}^2 \in (0,1)$ such that (i) I_2^2 and q_2^2 decrease in γ for $\gamma < \hat{\gamma}^2$; and (ii) I_2^2 and q_2^2 increase in γ for $\gamma > \hat{\gamma}^2$.
(2) If the initial cost advantage of the more efficient firm 1 is large ($\tau > (A - c_0)/4$), then firm 2's innovation effort and output always decrease with the level of competition.

This proposition implies that, if firm 2 is not too disadvantaged in cost initially its innovation effort has also a U-shaped relation with the level of competition. The intuition for this result is similar to that discussed following Proposition 1. For firm 2, the two forces identified in the earlier discussion are always opposed. Their interplay leads to a U-shaped relationship. If firm 2 is highly disadvantaged in cost initially, the negative force overwhelms the positive one at any level of competition, leading to decreasing levels of innovation as competition intensifies.

3.3 Comparison

Comparing the level of innovation by firm 1 when it is the only firm innovating with that of firm 2 when it is the only firm innovating, we have the following result.

Proposition 3: The level of innovation by the more efficient firm 1 when it is the only firm innovating is always greater than that of the less efficient firm 2 when it alone innovates. That is, $I_1^1 > I_2^2$ for all possible values of γ.

The result that the more efficient firm invests more in innovation than the less efficient firm is well established in the literature when all firms engage in innovating activities. Proposition 3 shows that this result holds true in the comparison of different firms' innovating incentives when only one firm innovates. The main reason is that the more efficient firm benefits more from a reduction in marginal cost due to that it produces a greater output and enjoys a larger price/cost margin than the less efficient firm.

The next result compares the innovation patterns of the two firms.

Proposition 4: The more efficient firm 1 is more likely than the less efficient firm 2 to raise its innovation level as competition increases.

As shown by Propositions 1 and 2, both firms' innovation levels have a U-shaped relation with the level of competition, provided that firm 2 is not too disadvantaged in cost initially. Proposition 4 is the result of firm 1's innovation effort making a U-turn earlier than that of firm 2 (i.e., $\hat{\gamma}^1 < \hat{\gamma}^2$), as shown in the proof of Proposition 4.

We next examine how consumers fare under alternative innovation scenarios. It has been well established since the work of Dixit (1979) that the demand system (1) is derivable from a representative consumer model with a quasi-linear utility function that is quadratic in consumption quantities. Our measure of consumer surplus is based on the underlying utility function. The next proposition presents consumers' ranking of the different scenarios.

Proposition 5: Consumer surplus is the highest if the more efficient firm 1 is the innovator and the lowest if no firm innovates.

It is fairly intuitive that consumers benefit from innovations by firms. In the context of cost-reducing innovations, some of the cost reduction will be passed onto consumers in the form of lower prices. Lower prices in turn lead to higher consumptions, both factors contributing to increased consumer welfare. From the proof of Proposition 5, consumer surplus can be decomposed into a linear combination of output variance and average output squared. While either firm 1 innovating or firm 2 innovating leads to the same variance in output, the former gives rise to a higher average output. From this follows the result that consumers benefit more from innovation by the more efficient firm 1 than by the less efficient firm 2.

A cost-reducing innovation by one firm will hurt the other firm by making it relatively less competitive. Hence, when a single firm innovates, the innovating firm's profit will rise (otherwise it would choose zero innovation) but the non-innovating firm's profit will suffer. Total industry profits can be higher or lower than the situation with no firm innovating. Thus, total welfare (consumer surplus plus total profits) could conceivably be decreased by innovation.

4. Closed Loop Investment Choice

In this section we study innovation choices for individual firms and how they change with the level of competition by looking at a closed loop choice model. In this approach, the innovating firm's choice of investment in innovation is made before the firms make their simultaneous choices of output. This two-stage approach captures the information structure in which the innovating firm's choice on innovation is public information and observable by its competitor in the product market.

Our main conclusion in this section is that similar properties as in the open loop model hold under closed loop investment choices. Because of this, our presentation below is condensed as much as possible.

4.1 The more efficient firm innovates

When the more efficient firm 1 is the only firm engaging in innovation activities, the marginal costs of the two firms are $c_1 = c_0 - I_1$ and $c_2 = c_0 + \tau$, respectively. Substituting these values into (2) gives the firms' outputs in the second stage Cournot competition for any given I_1:

$$q_1 = \frac{(2-\gamma)(A-c_0)+2I_1+\gamma\tau}{4-\gamma^2}, \qquad q_2 = \frac{(2-\gamma)(A-c_0)-\gamma I_1-2\tau}{4-\gamma^2}. \tag{15}$$

Applying the values in (15) into firm 1's profit function $\Pi_1 = [A - q_1 - \gamma q_2 - (c_0 - I_1)]q_1 - \frac{1}{2}(I_1)^2$ and maximizing with respect to I_1 yields firm 1's optimal choice of investment in the first stage of the game, as given by

$$I_1^1 = \frac{4[(2-\gamma)(A-c_0)+\gamma\tau]}{8-8\gamma^2+\gamma^4}. \tag{16}$$

From (16), one can show that firm 1's innovation effort initially decreases and then increases in γ (derivation in the appendix). That is, firm 1's innovation effort is U-shaped as a function of the level of competition.

4.2 The less efficient firm innovates

When the initially less efficient firm 2 is the only firm engaging in innovation activities, the marginal costs of the two firms are $c_1 = c_0$ and $c_2 = c_0 + \tau - I_2$, respectively. Substituting these values into (2) gives the firms' output choices in the second stage Cournot competition for any given I_1, as given by

$$q_1 = \frac{(2-\gamma)(A-c_0)-\gamma I_2+\gamma\tau}{4-\gamma^2}, \quad q_2 = \frac{(2-\gamma)(A-c_0)+2I_2-2\tau}{4-\gamma^2}. \tag{17}$$

Substituting (17) into firm 2's profit function $\Pi_2 = [A - q_2 - \gamma q_1 - (c_0 + \tau - I_2)]q_2 - \frac{1}{2}(I_2)^2$ and maximizing with respect to I_2 yields firm 2's optimal choice of investment in the first stage of the game, as given by

$$I_2^2 = \frac{4[(2-\gamma)(A-c_0)-2\tau]}{8-8\gamma^2+\gamma^4}. \tag{18}$$

From (18), one can show that firm 2's innovation effort initially decreases and then increases in γ (derivation in the appendix). That is, firm 2's innovation effort is U-shaped as a function of the level of competition.

Direct comparison of (16) and (18) leads to a similar conclusion to that of Proposition 3; namely, the more efficient firm 1, when it is the only firm innovating, invests more than the less efficient firm 2, as the only innovating firm.

5. Concluding Remarks

We have studied in the context of a differentiated duopoly the relationship between innovation by a single firm and the level of market competition. Intensity of market competition is represented by the product substitution parameter. It is found to affect innovation through a U-shaped relationship. This relationship generally holds true for both the initially more efficient firm and less efficient firm as the sole innovator. It also holds both in an open loop model in which innovation is chosen simultaneously with outputs and in a closed loop model in which innovation is chosen prior to output choices. Innovation by either firm always raises consumer welfare. Moreover, innovation by the initially more efficient firm leads to more innovation and greater consumer welfare compared to innovation by the initially less efficient firm.

This study hopes to rekindle the debate on the relationship between innovation and competition by focusing on the innovation incentive for a single innovator as the original studies in this literature do. In this pursuit, much remains to be done in the oligopolistic context. In particular, one is naturally curious as to how robust the U-shaped relationship between innovation and competition is in a more general oligopoly setting and when other measures of competition are incorporated. Empirically, one interesting avenue to explore is the relationship between innovation and market competition at the firm level.

Appendix

Proof of Proposition 1:

Differentiating I_1^1 given in the first line of (8) with respect to γ yields

$$\frac{\partial I_1^1}{\partial \gamma} = \frac{1}{(2-\gamma^2)^2}[4\gamma(A - c_0) - (2 + \gamma^2)(A - c_0 - \tau)]. \tag{A1}$$

It follows that the sign of $\frac{\partial I_1^1}{\partial \gamma}$ is the same as the sign of the function $f(\gamma) = 4\gamma(A - c_0) - (2 + \gamma^2)(A - c_0 - \tau)$. Note that $f(\gamma)$ is an inverted-U shaped quadratic function in γ. It follows from the fact that $f(0) = -2(A - c_0 - \tau) < 0$ and $f(\bar{\gamma}) = (2 - \bar{\gamma}^2)(A - c_0 - \tau) > 0$ there exists a unique $\hat{\gamma}^1 \in (0, \bar{\gamma})$ such that $f(\hat{\gamma}^1) = 0$, $f(\gamma) < 0$ for all $\gamma < \hat{\gamma}^1$, and $f(\gamma) > 0$ for all $\gamma \in$

$(\hat{\gamma}^1, \bar{\gamma})$. Hence, $\frac{\partial I_1^1}{\partial \gamma} < 0$ for all $\gamma < \bar{\gamma}^1$, and $\frac{\partial I_1^1}{\partial \gamma} > 0$ for all $\gamma \in (\hat{\gamma}^1, \bar{\gamma})$.

The statements in Proposition 1 follow immediately from the above results and that $I_1^1 = q_1^1$.

Proof of Proposition 2:

Since q_2^2 has the same pattern as I_2^2, we will focus on I_2^2 in the following. Differentiating I_2^2 in (13) with respect to γ gives

$$\frac{\partial I_2^2}{\partial \gamma} = \frac{1}{(2-\gamma^2)^2}[4\gamma(A - c_0 - \tau) - (2 + \gamma^2)(A - c_0)]. \quad \text{(A2)}$$

It follows that the sign of $\frac{\partial I_2^2}{\partial \gamma}$ is the same as the sign of the function $g(\gamma) = 4\gamma(A - c_0 - \tau) - (2 + \gamma^2)(A - c_0)$. Note that $g(\gamma)$ is an inverted-U shaped quadratic function in γ with $g(0) = -2(A - c_0) < 0$. Consider first the case where $\tau < (A - c_0)/4$. In this case, $g(1) = A - c_0 - 4\tau > 0$. Hence, there exists a unique $\hat{\gamma}^2 \in (0,1)$ such that $g(\hat{\gamma}^2) = 0$, $g(\gamma) < 0$ for all $\gamma < \hat{\gamma}^2$, and $g(\gamma) > 0$ for all $\gamma > \hat{\gamma}^2$. It follows that $\frac{\partial I_2^2}{\partial \gamma} < 0$ for all $\gamma < \hat{\gamma}^2$, and $\frac{\partial I_2^2}{\partial \gamma} > 0$ for all $\gamma > \hat{\gamma}^2$. This proves part (1) of the proposition. Now consider the case where $\tau > (A - c_0)/4$. In this case, $g(1) = A - c_0 - 4\tau < 0$. Hence, it must be true that $g(\gamma) < 0$ for all $\gamma \in [0,1]$. Part (2) of the proposition follows immediately.

Proof of Proposition 3:

Applying the first line of (8) and (13), $I_1^1 - I_2^2 = \frac{(2-\gamma)(A-c_0)+\gamma\tau}{2-\gamma^2} - \frac{(2-\gamma)(A-c_0)-2\tau}{2-\gamma^2} = \frac{(2+\gamma)\tau}{2-\gamma^2} > 0$. Hence, $I_1^1 > I_2^2$ for all $\gamma \in [0, \bar{\gamma}]$. On the interval $[\bar{\gamma}, 1]$, $I_1^1 = A - c_0$ while I_2^2 is either always increasing or U-shaped. From (13), at $\gamma = \bar{\gamma}$, $I_2^2 = \frac{A-c_0-\tau}{2-\bar{\gamma}^2} < A - c_0$, and at $\gamma = 1$, $I_2^2 = A - c_0 - 2\tau < A - c_0$. It follows that $I_1^1 > I_2^2$ for all $\gamma \in [\bar{\gamma}, 1]$. The proposition follows by combining the above conclusions.

Proof of Proposition 4:

By Proposition 2, if $(\tau > (A - c_0)/4)$ then firm 2's innovation level is always decreasing in γ, implying immediately the statement in Proposition 4. We prove in the following that $\hat{\gamma}^1 < \hat{\gamma}^2$ assuming that $\tau < (A - c_0)/4$. From the proof of Proposition 1, $\hat{\gamma}^1$ is where the curve of $\frac{\partial I_1^1}{\partial \gamma}$ first intersects the horizontal axis (the γ axis) from below. From the proof of Proposition 2, $\hat{\gamma}^2$ is where the curve of $\frac{\partial I_2^2}{\partial \gamma}$ first intersects the γ axis from below. Applying (A1) and (A2),

$$\frac{\partial I_1^1}{\partial \gamma} - \frac{\partial I_2^2}{\partial \gamma} = \frac{1}{(2-\gamma^2)^2}[4\gamma\tau - (2 + \gamma^2)(-\tau)] = \frac{(2 + 4\gamma + \gamma^2)\tau}{(2-\gamma^2)^2} > 0$$

for all possible values of $\gamma \in [0, \bar{\gamma}]$. Hence, the curve of $\frac{\partial I_1^1}{\partial \gamma}$ lies everywhere above the curve of $\frac{\partial I_2^2}{\partial \gamma}$ for all $\gamma \in [0, \bar{\gamma}]$. It follows that we must have $\hat{\gamma}^1 < \hat{\gamma}^2$.

Since both I_1^1 and I_2^2 are U-shaped in γ, the fact that $\hat{\gamma}^1 < \hat{\gamma}^2$ implies that the curve of I_1^1 becomes rising at a lower value of γ than the curve of I_2^2. Hence, I_1^1 is increasing in γ for a larger range of γ than I_2^2 is, from which follows the statement in Proposition 4.

Proof of Proposition 5:
The utility function of the representing consumer corresponding to the demand system (1) is

$$U(q_1, q_2) = A(q_1 + q_2) - \frac{1}{2}[(q_1)^2 + (q_2)^2] - \gamma q_1 q_2.$$

Applying the demand equations in (1), consumer surplus becomes

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 = \frac{1}{2}[(q_1)^2 + (q_2)^2] + \gamma q_1 q_2. \tag{A3}$$

Let $\bar{q} = (q_1 + q_2)/2$ and $V = (q_1 - \bar{q})^2 + (q_2 - \bar{q})^2$ denote, respectively, the average output and output variance. Then we can rewrite the expression (A3) for CS as

$$CS = (1 + \gamma)(\bar{q})^2 + \frac{1-\gamma}{2} V. \tag{A4}$$

Applying (4), the average output and output variance when no firm innovates are given by

$$\bar{q}^0 = \frac{A - c_0 - \frac{\tau}{2}}{2+\gamma}, \quad V^0 = \frac{\tau^2}{2(2-\gamma)^2}. \tag{A5}$$

By (8) and (9), for $\gamma < \bar{\gamma}$, the average output and output variance when firm 1 innovates are given by

$$\bar{q}^1 = \frac{(\frac{3}{2} - \gamma)(A - c_0) - \frac{1-\gamma}{2}\tau}{2 - \gamma^2}, \quad V^1 = \frac{[(A - c_0) + (1+\gamma)\tau]^2}{2(2-\gamma^2)^2}. \tag{A6}$$

For $\gamma \geq \bar{\gamma}$, consumer surplus when firm 1 innovates is given by

$$CS^1 = \frac{(A - c_0)^2}{2}. \tag{A7}$$

Using (13), the average output and output variance when firm 2 innovates are given by

$$\bar{q}^2 = \frac{(\frac{3}{2} - \gamma)(A - c_0) - \frac{1-\gamma}{2}\tau}{2 - \gamma^2}, \quad V^2 = \frac{[(A - c_0) + (1+\gamma)\tau]^2}{2(2-\gamma^2)^2}. \tag{A8}$$

From (A5) and (A8), we can verify easily that $\bar{q}^2 > \bar{q}^0$ and $V^2 > V^0$. Hence, by (A4), we have $CS^2 > CS^0$. That is, consumer surplus when firm 2 innovates is higher than that under no firm innovating. Note that this is true for any value of γ. Comparing (A6) and (A8), we see that the variances are the same but the average output under firm 1 innovating is greater that under firm 2 innovating. Hence, for $\gamma < \bar{\gamma}$, consumer surplus when firm 1 innovates is higher than that under firm 2 innovating.

To complete the proof of the proposition, it remains to show that $CS^1 > CS^2$ for $\gamma \geq \bar{\gamma}$. In this case, CS^1 is given by (A7) and CS^2 is obtainable by substituting (A8) into (A4). Although we cannot prove the desired inequality algebraically, we have shown it by direct numerical calculations. More specifically, we use the fact that the ratio CS^2/CS^1 depends on only $\tau/(A - c_0)$ and γ and the fact that $\bar{\gamma} = \frac{A - c_0 - \tau}{A - c_0}$ depends only on $\tau/(A - c_0)$. Under Assumption 1, the feasible range for $\tau/(A - c_0)$ is $(0,1/2)$. Our calculations show that $\frac{CS^2}{CS^1} < 1$ for all $\frac{\tau}{A - c_0} \in (0,1/2)$ and all $\gamma \in [\bar{\gamma}, 1]$.

Derivation of statements in Section 4:
Proof of the statement that I_1^1 in (16) is U-shaped in γ: Rewriting I_1^1 in (16) gives

$$I_1^1 = \frac{4(2-\gamma)}{8-8\gamma^2+\gamma^4}(A-c_0) + \frac{4\gamma}{8-8\gamma^2+\gamma^4}\tau.$$

In this expression, the first term is U-shaped in γ while the second term is strictly increasing in γ. Moreover, the first term is dominant over the second term, resulting in a U-shaped function for the sum. More specifically, differentiating I_1^1 with respect to γ yields

$$\frac{\partial I_1^1}{\partial\gamma} = 4\frac{-(8-8\gamma^2+\gamma^4)(A-c_0-\tau)+4\gamma(4-\gamma^2)[(2-\gamma)(A-c_0)+\gamma\tau]}{(8-8\gamma^2+\gamma^4)^2}.$$

It is easily verified that this derivative is negative at $\gamma = 0$ or near 0, and positive at $\gamma = 1$ or near 1, confirming the above assertion.

Proof of the statement that I_2^2 in (18) is U-shaped in γ: Rewriting I_2^2 in (18) gives

$$I_2^2 = \frac{4(2-\gamma)}{8-8\gamma^2+\gamma^4}(A-c_0) - \frac{8}{8-8\gamma^2+\gamma^4}\tau.$$

In the above expression, the first term is U-shaped in γ while the second term is strictly decreasing in γ. It follows immediately that I_2^2 decreases in γ when γ is small. For large values of γ, the direction of movement of I_2^2 depends on the two opposing forces at work here. Specifically, differentiating I_2^2 with respect to γ yields

$$\frac{\partial I_2^2}{\partial\gamma} = 4\frac{-(8-8\gamma^2+\gamma^4)(A-c_0)+4\gamma(4-\gamma^2)[(2-\gamma)(A-c_0)-2\tau]}{(8-8\gamma^2+\gamma^4)^2}.$$

It is easily verified that this derivative is negative at $\gamma = 0$ and near 0. At $\gamma = 1$, $\frac{\partial I_2^2}{\partial\gamma} = 4[11(A-c_0) - 24\tau]$. Hence, $\frac{\partial I_2^2}{\partial\gamma}$ is positive at $\gamma = 1$ and near 1 if $\tau < 11(A-c_0)/24$. That is, I_2^2 is U-shaped in γ provided that $\tau < 11(A-c_0)/24$. For $\tau \in [11(A-c_0)/24, (A-c_0)/2)]$, $\frac{\partial I_2^2}{\partial\gamma}$ is negative for all γ, implying that I_2^2 is always decreasing in γ.

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