

Volume 33, Issue 4**Increases in skewness and insurance**

Thomas Eichner
University of Hagen

Abstract

The present paper analyzes how the welfare state, i.e., social insurance that works through redistributive taxation, should respond to increases in the skewness of the risk distribution. Income risks can be hedged either by individual self-insurance or by social insurance. It is shown that skewness-affine agents reduce both self-insurance and social insurance in response to an increase in income skewness. Thus countries with a more right-skewed income distribution have less redistribution.

1. Introduction

Insurance losses and insurance claims exhibit skewed and heavy tailed distributions (Vernic 2006, Furman 2008). The World Economic Forum (2012) reports that some types of risks have significantly increased over the last years, e.g. the risks related to macroeconomic imbalances, risks from rising greenhouse gas emissions or risks from severe income disparities. Not only the risks have increased but also the asymmetry of their distribution has changed. E.g. the impacts of climate change are not uniform. Tol et al. (2004)'s empirical estimates show that climate change impacts more severely the poorer people making the income risk distribution more skewed. Furthermore, Gordon and Dew-Becker (2007) point to the increased income skewness in the United States over the last 30 years.

In the present paper we analyze the impact of increased skewness - as measure of the asymmetry of the risk distribution - on individual self-insurance activities and on the demand for social insurance. For that purpose we introduce in the social insurance model of Sinn (1995, 1996) and Eichner and Wagener (2004) the skewness of the risk distribution. In that framework agents face life-time risks concerning their human capital, health status or occupational careers. Life-time risks can be hedged either by self-insurance efforts, e.g. by investments in human capital, or by social insurance. In the context of life-time risks private insurance markets are imperfect and do not appropriately deal with these specific risks. For that reason our focus lies on social insurance which is considered here as an all-inclusive insurance. Social insurance is designed as redistributive taxation and reduces ex ante income risk volatility.¹ The income volatility translates ex post into income inequality.

Decisions on self-insurance and social insurance take place under uncertainty and agents are assumed to have three-moment preferences. Since the problem at hand satisfies Chiu (2010)'s skewness comparability condition, which requires that all lotteries which an agent can choose differ by at most the first three moments of their distributions, three-moment preferences and expected utility approach are consistent.² Our analysis reveals that skewness affinity recently introduced by Eichner and Wagener (2011) characterizes the comparative static effects of increases in the skewness on self-insurance and on optimal social insurance. An agent is called skewness affine if and only if her marginal willingness to accept a risk increases when the distribution of the risk becomes more skewed to the right. Skewness-affine agents reduce self-insurance when the skewness of the income distribution is increased. Since lower self-insurance raises the variance of pre-tax income and increases the marginal willingness to accept risks, it renders social insurance at the margin less attractive. As a consequence skewness-affine agents desire to cut back social insurance.

¹While nothing is known about the impact of skewness in this context, the role of social insurance to cope with income risks is well understood in the public economics literature (see Eaton and Rosen 1980, Varian 1980 or Rochet 1991).

²Under this compatibility an increase in skewness is equivalent to an increase in downside risk (Menezes et al. 1980).

2. The model

The present paper builds on a model of self-insurance and social insurance à la Sinn (1995, 1996). To keep the exposition brief, we refer the reader to these papers for a thorough discussion of the original assumptions.

Let m and n be the exogenously given maximum values of market and non-market life-time incomes. The market income decreases by a random loss θ which has continuous support on the interval $[0, m]$. Agents can mitigate the random loss by undertaking costly self-insurance efforts e . The loss-reducing effect of e is captured by the function $\lambda : \mathbb{R}_+ \rightarrow (0, 1]$ which satisfies $\lambda(0) = 1$, $\lambda'(e) < 0$, and $\lambda''(e) \geq 0$. Then the post-tax income is given by

$$Y = n + m - \lambda(e)\theta - e - T + p, \quad (1)$$

where T and p denote taxes and transfers, respectively. The market income is linearly taxed at rate $\tau \in [0, 1)$. The agent's tax liability amounts to

$$T = \tau [m - \lambda(e)\theta]. \quad (2)$$

In (2) we assume that self-insurance efforts are unobservable by the government and hence are not deductible from the tax bill.³ The government redistributes tax revenues back to the tax payers. Following Eichner and Wagener (2004) we assume that redistribution causes costs, e.g. administrative costs, shadow costs of public funds or governmental waste of resources. Presupposed the law of large numbers applies to θ the fiscal budget is balanced if

$$p = (1 - c)\mathbf{E}_\theta T = (1 - c)\tau [m - \lambda(e)\mu_\theta], \quad (3)$$

where $c\mathbf{E}_\theta T$, $c \in (0, 1]$, reflects the deadweight costs of taxation, and where $\mu_\theta \equiv \mathbf{E}_\theta \theta > 0$ is the expected or average loss of market income. In (1)-(3) the tax rate τ and the transfer p are the components of the social insurance contract.

Next, we phrase the model in terms of the first three moments. For notational convenience we introduce the pre-tax standard deviation

$$\sigma_G := \lambda(e)\sigma_\theta, \quad (4)$$

where σ_θ is the standard deviation of the random loss, and the self-insurance function⁴

$$\bar{\mu}(\sigma_G) := \mathbf{E}_\theta [n + m - \lambda(e)\theta - e] = n + m - k\sigma_G - \lambda^{-1}\left(\frac{\sigma_G}{\sigma_\theta}\right), \quad k := \frac{\mu_\theta}{\sigma_\theta}. \quad (5)$$

Using (4) and the definition of k in (3) the fiscal budget constraint can be written as

$$p = (1 - c)\tau(m - k\sigma_G). \quad (6)$$

³Sinn (1995) assumes that some self-insurance efforts may be deductible. However, introducing a deductible rate as in Sinn (1995, equation (3)) would make the notation more clumsy, the analysis more complex, but leaves our results unaltered.

⁴ λ^{-1} is the inverse function of λ .

Calculating the moments of Y and taking advantage of the definitions of the pre-tax standard deviation and self-insurance function in (4) and (5), respectively, we obtain

$$\mu_Y = \bar{\mu}(\sigma_G) - \tau(m - k\sigma_G) + p, \quad (7)$$

$$\sigma_Y = (1 - \tau)\sigma_G, \quad (8)$$

$$s_Y = -s_\theta. \quad (9)$$

In (7) and (8), μ_Y and σ_Y are the mean and standard deviation of post-tax income. The standard deviation σ_Y displays the post-tax income inequality in society. $s_Y := \int \frac{(y - \mu_Y)^3}{\sigma_Y^3} dF(y)$ is the third standardized central moment of the distribution $F(y)$ of the random variable Y and measures the skewness of Y .⁵ For right-skewed income distributions which are empirically relevant it holds $s_Y > 0$.

Finally, the model is closed by introducing the agent's preferences over post-tax incomes which are represented by a three-parameter utility function $U(\mu_Y, \sigma_Y, s_Y)$. Throughout the paper we assume that U has the following properties⁶

$$U_\mu(\mu_Y, \sigma_Y, s_Y) > 0 \quad \text{for all } (\mu_Y, \sigma_Y, s_Y), \quad (10a)$$

$$U_\sigma(\mu_Y, \sigma_Y, s_Y) < 0 \quad \text{for all } (\mu_Y, \sigma_Y, s_Y), \quad (10b)$$

$$\text{for all } s_Y : \quad U(\mu_Y, \sigma_Y, s_Y) \text{ is strictly quasi-concave in } (\mu_Y, \sigma_Y). \quad (10c)$$

These properties are standard for mean-standard deviation preferences; here we transfer them to a three-parameter framework. Condition (10a) is a non-satiation property and condition (10b) reflects risk aversion. Together, (10a) and (10b) imply that (μ_Y, σ_Y) -indifference curves are upward-sloped. Denote by

$$\alpha(\mu_Y, \sigma_Y, s_Y) := -\frac{U_\sigma(\mu_Y, \sigma_Y, s_Y)}{U_\mu(\mu_Y, \sigma_Y, s_Y)} > 0 \quad (11)$$

for all (μ_Y, σ_Y, s_Y) , the marginal rate of substitution between σ_Y and μ_Y (i.e., the slope of an indifference curve of U in (μ_Y, σ_Y) -space). α is the mean-standard deviation analogue of the Arrow-Pratt measure of absolute risk aversion (Lajeri and Nielsen 2000). In what follows, we need the convexity of the indifference curves in the (μ_Y, σ_Y) -space, formally,

$$\left. \frac{d\alpha(\mu_Y, \sigma_Y, s_Y)}{d\sigma_Y} \right|_U = \alpha_\sigma(\mu_Y, \sigma_Y, s_Y) + \alpha(\mu_Y, \sigma_Y, s_Y) \cdot \alpha_\mu(\mu_Y, \sigma_Y, s_Y) > 0 \quad (12)$$

for all (μ_Y, σ_Y, s_Y) . Quasi-concavity (10c) of the three-moment utility function U ensures that (12) is indeed satisfied. In addition, the sign of the derivative α_s plays a decisive role in the subsequent analysis. We call agents whose preferences display

$$\alpha_s(\mu_Y, \sigma_Y, s_Y) < 0 \quad (13)$$

for all (μ_Y, σ_Y, s_Y) *skewness affine*. Skewness affinity has been introduced by Eichner and Wagener (2011) and captures the idea that an agent's marginal willingness to accept a risk increases when the distribution of the risk becomes more skewed to the right.

⁵Analogous s_θ is the third standardized central moment associated to the random variable θ .

⁶Subscripts to the functions U and α denote partial derivatives.

3. Self-insurance, social insurance and increases in skewness

In this section we turn to the agent's decisions and investigate the comparative static effects of increases in the skewness. Throughout the paper we assume that agents "see-through" the fiscal budget constraint, i.e. they take into account (6) in their optimization problems. The individual choice of self-insurance efforts e is rephrased in terms of the pre-tax standard deviation σ_G . Inserting (6) in (7) the agent's optimization problem is given by

$$\begin{aligned} \max_{\sigma_G} U(\mu_Y, \sigma_Y, s_Y) \quad \text{s.t.} \quad & \mu_Y = \bar{\mu}(\sigma_G) - c\tau(m - k\sigma_G), \\ & \sigma_Y = (1 - \tau)\sigma_G, \\ & s_Y = -s_\theta. \end{aligned} \quad (14)$$

The restrictions in (14) determine the set of feasible allocations. The geometrical locus associated to these feasible allocations is the redistribution curve. Solving (14) yields the first-order condition for an optimal level σ_G^* of pre-tax income risk⁷

$$\Omega := \bar{\mu}'(\sigma_G^*) + c\tau k - (1 - \tau)\alpha(\mu_Y, \sigma_Y, s_Y) = 0. \quad (15)$$

According to (15) the agent chooses the pre-tax standard deviation such that the marginal benefits⁸ of reducing the post-tax income volatility ($(1 - \tau)\alpha > 0$) are equal to the marginal costs of reducing expected post-tax income ($\bar{\mu}'(\sigma_G^*) + c\tau k > 0$). If we divide (15) by $(1 - \tau)$, then it graphically requires the slope α of the indifference curve being equal to the slope of the redistribution curve.

Next, we analyze parametric changes in the skewness of the income risk distribution s_Y . Implicit differentiation of (15), μ_Y , σ_Y from (14), and then using the envelope theorem we get

$$\frac{\partial \sigma_G^*}{\partial s_Y} = -\frac{\Omega_{s_Y}}{\Omega_{\sigma_G}} = \frac{(1 - \tau)\alpha_s}{\bar{\mu}''(\sigma_G^*) - (1 - \tau)^2(\alpha_\sigma + \alpha \cdot \alpha_\mu)}, \quad (16)$$

$$\frac{\partial \mu_Y}{\partial s_Y} = [\bar{\mu}'(\sigma_G^*) + c\tau k] \frac{\partial \sigma_G^*}{\partial s_Y}, \quad (17)$$

$$\frac{\partial \sigma_Y}{\partial s_Y} = (1 - \tau) \frac{\partial \sigma_G^*}{\partial s_Y}. \quad (18)$$

Since $\bar{\mu}(\sigma_G)$ is concave, which follows from the convexity of $\lambda(e)$, and $\alpha_\sigma + \alpha \cdot \alpha_\mu > 0$ (see (12)), we infer from (16)

$$\text{sign} \frac{\partial \sigma_G^*}{\partial s_Y} = -\text{sign} \alpha_s. \quad (19)$$

(17)-(19) give rise to

⁷We omit the arguments of the functions U and α when there is no risk of confusion.

⁸Marginal benefits and costs are expressed in units of average income.

Proposition 1. *An increase in the skewness of the income risk s_Y leads to an increase in the variance of pre-tax incomes, to lower self-insurance efforts,⁹ to an increase in the variance of post-tax incomes and to an increase in expected income if and only if preferences are skewness affine.*

The interpretation of Proposition 1 is straightforward. Skewness-affine agents' willingness to take a risk enhances when the distribution of the income risk becomes more skewed to the right. As response to increases in the skewness, skewness-affine agents reduce self-insurance efforts (higher risk-taking), which in turn raises both pre-tax and post-tax inequality and mean income.

Finally, we turn to the optimal social insurance and its dependence on skewness. The optimal tax rate follows from maximizing the indirect utility function, formally¹⁰

$$\max_{\tau} U^* = U(\bar{\mu}(\sigma_G^*) - c\tau(m - k\sigma_G^*), (1 - \tau)\sigma_G^*, s_Y). \tag{20}$$

Restricting our attention to an interior solution and taking into account (15) the first-order condition is given by

$$\frac{\partial U^*}{\partial \tau} = U_{\mu} \cdot [\sigma_G^* \alpha - c(m - k\sigma_G^*)] = 0. \tag{21}$$

Equation (21) determines the optimal τ^* which is chosen such that marginal benefits and marginal costs of taxation are balanced.¹¹ Marginal benefits consist in reducing the variance of post-tax incomes ($\sigma_G^* \alpha > 0$). Marginal costs ($c(m - k\sigma_G^*) > 0$) are incurred because an increase in τ reduces expected post-tax income. Closer inspection shows that the deadweight costs cause the loss of expected income. Further making use of (15) the first-order condition (21) can be rearranged to

$$\Upsilon := -cm(1 - \tau) + ck\sigma_G^* + \sigma_G^* \bar{\mu}'(\sigma_G^*) = 0. \tag{22}$$

Υ can be interpreted as marginal *net* benefits of taxation. Implicit differentiation of (22) gives

$$\frac{\partial \tau^*}{\partial s_Y} = -\frac{\Upsilon_{s_Y}}{\Upsilon_{\tau}} = -\frac{\Upsilon_{\sigma_G} \cdot \frac{\partial \sigma_G^*}{\partial s_Y} \Big|_{\tau=\tau^*}}{cm + \Upsilon_{\sigma_G} \cdot \frac{\partial \sigma_G^*}{\partial \tau} \Big|_{\tau=\tau^*}}, \tag{23}$$

where

$$\Upsilon_{\sigma_G} = ck + \bar{\mu}'(\sigma_G^*) + \sigma_G^* \bar{\mu}''(\sigma_G^*), \tag{24}$$

$$\frac{\partial \sigma_G^*}{\partial \tau} = -\frac{\Omega_{\tau}}{\Omega_{\sigma_G}} = -\frac{ck + \alpha + (1 - \tau)[c(m - k\sigma_G^*)\alpha_{\mu} + \sigma_G^* \alpha_{\sigma}]}{\bar{\mu}''(\sigma_G^*) - (1 - \tau)^2(\alpha_{\sigma} + \alpha \cdot \alpha_{\mu})}. \tag{25}$$

⁹ $\frac{\partial e}{\partial s_Y} < 0$ follows from differentiation of (4) which yields $\frac{\partial e}{\partial s_Y} = \frac{1}{\lambda'(e)\sigma_{\theta}} \cdot \frac{\partial \sigma_G^*}{\partial s_Y}$, and making use of $\lambda'(e) < 0$.

¹⁰ In (20) the star indicates that agents have optimally chosen the pre-tax standard deviation.

¹¹ Marginal benefits and costs are measured in average income.

Evaluating (25) at $\tau = \tau^*$ and thus taking advantage of (21) yields

$$\left. \frac{\partial \sigma_G^*}{\partial \tau} \right|_{\tau=\tau^*} = -\frac{ck + \alpha + (1 - \tau^*)\sigma_G^* (\alpha \cdot \alpha_\mu + \alpha_\sigma)}{\bar{\mu}''(\sigma_G^*) - (1 - \tau^*)^2(\alpha_\sigma + \alpha \cdot \alpha_\mu)} > 0. \quad (26)$$

Using the information of (26) in (23) the second-order condition $\Upsilon_\tau < 0$ can only be satisfied if $\Upsilon_{\sigma_G} < 0$. Hence we infer from (23)

$$\frac{\partial \tau^*}{\partial s_Y} = -\text{sign} \frac{\partial \sigma_G^*}{\partial s_Y} = \text{sign} \alpha_s. \quad (27)$$

Suppose the second-order condition is satisfied ($\Upsilon_\tau < 0$), then (27) gives rise to

Proposition 2. *An increase in the skewness of the income risk s_Y reduces the optimal social insurance if and only if preferences are skewness affine.*

The rationale behind Proposition 2 is as follows. Recall that increases in the skewness induce skewness-affine agents to lower self-insurance. This raises the volatility of pre-tax incomes ($\partial \sigma_G^* / \partial s_Y > 0$) according to Proposition 1. Increases in σ_G affect both marginal benefits and marginal costs of taxation. Its impact on marginal net benefits is specified in (24). A higher variance of pre-tax income (lower e) reduces the tax base from (2) and hence lowers the deadweight costs of taxation. This effect is formally captured by $ck > 0$ in (24) and reduces the marginal costs of taxation. Concerning the marginal benefits there emerge two effects: First, increases in σ_G enhance post-tax income risk and hence strengthen the risk-reducing effect of taxation. This effect is formally reflected by $\bar{\mu}'(\sigma_G^*) > 0$ in (24) and enlarges marginal benefits of taxation. Second, increases in σ_G reduce α - the marginal rate of substitution between risk and mean income.¹² The marginal rate of substitution between mean and standard deviation measures the compensation that an agent requires to accept a higher risk. Then a lower α means that for given compensation payment agents are willing to take higher risks. This effect is formally displayed by $\sigma_G^* \bar{\mu}''(\sigma_G^*) < 0$ in (24) and reduces the marginal benefits of taxation. To summarize the *change* of marginal *net* benefits comprises two positive effects and one negative effect when σ_G ceteris paribus is increased. However, it turns out that the negative effect overcompensates the positive effects and hence marginal net benefits become negative ($\Upsilon < 0$ due to $\Upsilon_{\sigma_G} < 0$). Since the second-order condition is assumed to be satisfied and marginal net benefits are decreasing in the tax rate ($\Upsilon_\tau < 0$), a lower tax rate becomes desirable (in order to arrive at the "new" optimal tax rate which in turn is characterized by $\Upsilon = 0$). Roughly speaking, increases in the skewness reduce self-insurance, raise the variance of pre-tax income, increase the marginal willingness to accept risks and hence render social insurance at the margin less attractive. To put it differently, self-insurance and social insurance turn out to be complements and social insurance should be cut back when the society encounters an increase in income skewness.

¹²Recall that the marginal benefits of taxation are given by $\sigma_G^* \alpha$ where $\alpha = \frac{\bar{\mu}'(\sigma_G^*) + \tau k}{(1 - \tau)}$ according to (15). The concavity of the function $\bar{\mu}(\sigma_G)$ implies that α decreases when σ_G ceteris paribus increases.

4. Extensions

The results of the previous section can be extended to higher moments and higher-moment preferences. For that purpose define

$$m_Y(n) := \int \frac{(y - \mu_Y)^n}{\sigma_Y^n} dF(y) \quad n \geq 4 \quad (28)$$

as the n^{th} (standardized) moment of the random variable Y .¹³ In addition, suppose that the agent's preferences are represented by the n -moment utility function

$$U(\mu_Y, \sigma_Y, s_Y, m_Y(4), \dots, m_Y(n)). \quad (29)$$

Analogous to the notion of skewness affinity an agent is called n^{th} moment affine if and only if

$$\alpha_{m_Y(n)}(\mu_Y, \sigma_Y, s_Y, m_Y(4), \dots, m_Y(n)) < 0. \quad (30)$$

Since in the optimization problems (14) and (20) all n^{th} moments of Y are unaffected by the decision variables σ_G and τ , respectively, we are in the position to transfer Proposition 1 and 2 to the n^{th} moment and to n -moment preferences.

Corollary 1. *An increase in the n^{th} moment ($n \geq 4$) of the income risk $m_Y(n)$*

(a) leads to an increase in the variance of pre-tax incomes, to lower self-insurance efforts, to an increase in the variance of post-tax incomes and to an increase in expected income if and only if preferences are n^{th} moment affine;

(b) reduces the optimal social insurance if and only if preferences are n^{th} moment affine.

Finally, we clarify the relationship between expected utility and moment preferences, and increases in moments and increases in n^{th} degree risk in more detail. According to Ekern (1980) the risk \tilde{Y} is an increase in n^{th} degree risk over Y if \tilde{Y} dominates Y via n^{th} -order stochastic dominance and all $n - 1$ moments of the distributions of Y and \tilde{Y} coincide. An increase in third-degree risk is tantamount to Menezes et al. (1980)'s increase in downside risk. Under the compatibility of three-moment preferences and expected utility approach an increase in skewness is equivalent to an increase in third-degree risk, and skewness affinity translates into $-yu^4(y)/u^3(y) \leq 3$, where $-yu^4(y)/u^3(y)$ is the third degree of relative risk aversion - and u is the von Neumann-Morgenstern (vNM) utility function (see Eichner and Wagener 2011). These results can be generalized to n -moment affinity and n^{th} degree of relative risk aversion as follows¹⁴

$$\alpha_{m_Y(n)}(\mu_Y, \sigma_Y, s_Y, m_Y(4), \dots, m_Y(n)) < 0 \quad \iff \quad -y \frac{u^{n+1}(y)}{u^n(y)} \leq n \quad \forall y. \quad (31)$$

In view of (31) and Corollary 1 we get

¹³ $m_Y(4)$ is the kurtosis.

¹⁴The proof of (31) follows from applying Proposition 2 in Eichner and Wagener (2011) to higher moments and increases in higher degree risks.

Corollary 2. Suppose that n -moment preferences and expected utility approach are compatible. Then an increase in n^{th} degree risk ($n \geq 4$)

(a) leads to lower self-insurance efforts if and only if the vNM utility function satisfies $-yu^{n+1}(y)/u^n(y) \leq n$;

(b) reduces the optimal social insurance if and only if the vNM utility function satisfies $-yu^{n+1}(y)/u^n(y) \leq n$.

References

- Chiu, W.H. (2010) "Skewness preference, risk taking and expected utility maximization" *The Geneva Risk and Insurance Review* **38**, 108-129.
- Eaton J. and H.S. Rosen (1980) "Optimal redistributive taxation and uncertainty" *Quarterly Journal of Economics* **95**, 357-364.
- Eichner, T. and A. Wagener (2011) "Increases in skewness and three-moment preferences" *Mathematical Social Sciences* **61**, 109-113.
- Eichner, T. and A. Wagener (2004) "The welfare state in a changing environment" *International Tax and Public Finance* **61**, 109-113.
- Ekern, S. (1980) "Increasing N^{th} degree risk" *Economics Letters* **6**, 329-333.
- Furman, E. (2008) "On a multivariate gamma distribution" *Statistics and Probability Letters* **78**, 2533-2360.
- Gordon, R.J. and I. Dew-Becker (2007) "Selected issues in the rise of income inequality" *Brooking Papers on Economic Activity* **38**, 169-192.
- Lajeri, F. and L.T. Nielsen (2000) "Parametric characterizations of risk aversion and prudence" *Economic Theory* **15**, 469-576.
- Lane, M.N. (2000) "Pricing risk transfer transactions" *ASTIN Bulletin* **30**, 259-293.
- Menezes, C., Geiss, C. and J. Tressler (1980) "Increasing downside risk" *American Economic Review* **70**, 921-932.
- Meyer, J. (1987) "Two-moment decision models and expected utility maximization" *American Economic Review* **77**, 421-430.
- Rochet, J.C. (1991) "Incentives, redistribution and social insurance" *Geneva Papers on Risk and Insurance Theory* **16**, 143-165.
- Sinn, H.-W. (1995) "A theory of the welfare state" *Scandinavian Journal of Economics* **97**, 495-526.
- Sinn, H.-W. (1996) "Social insurance, incentives and risk taking" *International Tax and Public Finance* **3**, 259-280.
- Tol, R.S.J., Downing T.E., Kuik O.J. and J.B. Smith (2004) "Distributional aspects

of climate change impacts" *Global Environmental Change* **14**, 259-272.

Varian, M. (1980) "Redistributive taxation as social insurance" *Journal of Public Economics* **14**, 49-68.

Vernic, R. (2006) "Multivariate skew-normal distributions with applications to insurance" *Insurance: Economics and Mathematics* **38**, 413-426.

World Economic Forum (2012) *Global risks 2012* (seventh Edition): Geneva, Switzerland.