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Uzawa´s theorem (Uzawa (1961)) is extended to allow for adjustment costs in the process of capital accumulation. A new steady-state growth theorem with adjustment costs establishes that capital-augmenting technical change may arise in steady state. This is in sharp contrast to Uzawa´s original finding. In a growing economy this possibility arises since diminishing returns in the production of capital cause a gap between the growth of gross capital investments and the growth of capital. In steady state, capital-augmenting technical change has the role to fill this gap. The discussion of the new theorem characterizes the conditions under which a steady-state path with capital-augmenting technical change exists.

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1 Introduction

Uzawa's steady-state growth theorem (Uzawa (1961)) establishes one of the most important structural properties of neoclassical growth models: technical change must be labor-augmenting along a steady state starting in finite time.[1] The theorem exploits two fundamental components of neoclassical growth theory, namely, the aggregate production function with constant returns to scale in capital and labor and the role of capital accumulation.

The present paper studies a variant of Uzawa's theorem where adjustment costs interfere with the process of capital accumulation. In a new steady-state growth theorem with adjustment costs I establish that capital-augmenting technical change occurs in the steady-state of a non-stationary economy.[2] Here, adjustment costs drive a wedge between the evolution of gross capital investments and the evolution of capital. As a consequence, capital does not inherit the trend growth of total output but grows at a slower rate. The task of steady-state capital-augmenting technical change is to bridge this gap.

The applicability of this finding is shown to be subject to a few qualifications. First, it requires that the functional relationship between current gross capital investment and the amount of future capital generated by it gives rise to diminishing returns. Second, the economy must be growing in steady state. Third, the definition of the steady state requires output, consumption, gross capital investment, capital, currently installed capital, and the labor endowment to grow at a constant rate whereas the growth rate of adjustment costs may vary.

This paper is organized as follows. Section 2 presents the neoclassical economy with adjustment costs. Section 3 establishes the new steady-state growth theorem

[1]Arguably, the renewed interest in this theorem is due to the elegant and intuitive proof invented by Schlicht (2006). The variant of Uzawa's steady-state growth theorem alluded to in the title of the present paper refers to the setup studied by Schlicht. His proof was successively adopted by Jones and Scrimgeour (2008) and Acemoglu (2009). The proof of Theorem 1 below builds on and extends Schlicht's proof strategy. In the same vein, the contribution of Irmen (2013) is related to Schlicht's analysis. See, e. g., Russel (2004) for an alternative proof strategy based on advective equations.

[2]See, e. g., Klump, McAdam, and Willman (2007) for a recent empirical study that reports capital-augmenting technical for the US economy between 1953 to 1998.

with adjustment costs (Theorem 1). Section 4 discusses qualifications to and extensions of the new theorem. Section 5 concludes. All proofs are relegated to the Appendix.

2 The Model

Consider a closed economy in continuous time, i. e., $t \in (-\infty, +\infty)$. There is a single good produced according to the production function

$$Y(t) = \tilde{F}\left[K(t), L(t), \mathbf{A}_F(t)\right], \tag{2.1}$$

where $\tilde{F} : \mathbb{R}_+^2 \times \mathfrak{A}_F \rightarrow \mathbb{R}_+$, $Y(t)$ is total output, $K(t) > 0$ is the capital stock, $L(t) > 0$ is the labor endowment, and $\mathbf{A}_F(t) \in \mathfrak{A}_F$ represents the components of technological knowledge available at t. Here, $\mathfrak{A}_F$ is an arbitrary set. Assume that $\tilde{F}$ is increasing in $K(t)$ and $L(t)$ and exhibits constant returns to scale in these arguments.

Total output may either be consumed or invested in the capital stock. Hence, with $C(t)$ denoting consumption and $I(t)$ gross capital investment the resource constraint of the economy is

$$Y(t) = C(t) + I(t). \tag{2.2}$$

When capital accumulates without adjustment costs, then each unit of gross capital investment generates one additional unit of future capital. Adjustment costs drive a wedge between these variables. To model this, let $\Phi\left(I(t)\right)$ denote the amount of future capital generated by a gross capital investment of $I(t)$ units of period t output where[3]

$$\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \quad \text{and} \quad \Phi(0) = 0. \tag{2.3}$$

Henceforth, I shall refer to $\Phi\left(I(t)\right)$ as the "installation function of capital". The presence of adjustment costs is captured by the assumption that for all $I(t) > 0$

$$0 < \Phi\left(I(t)\right) < I(t). \tag{2.4}$$

[3]With only minor modifications Theorem 1 carries over to a setup where adjustment costs hinge on the investment-capital ratio, $I(t)/K(t)$, rather than on gross investment.

Hence, some units of current output are used up in the process of capital installation. I refer to these units as adjustment costs, i. e.,

$$AC\left(I(t)\right) \equiv I(t) - \Phi\left(I(t)\right). \tag{2.5}$$

Here, $AC : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is strictly increasing and strictly convex for all $I(t) > 0$. This requires

$$1 > \frac{d\Phi\left(I(t)\right)}{dI(t)} > 0 > \frac{d^2\Phi\left(I(t)\right)}{dI(t)^2}, \tag{2.6}$$

i. e., the marginal effect of increasing $I(t)$ must not be too pronounced and the installation of capital is subject to diminishing returns. The evolution of the capital stock is then given by

$$\dot{K}(t) = \Phi\left(I(t)\right) - \delta K(t), \quad \delta \in \mathbb{R}_+. \tag{2.7}$$

where δ is the instantaneous depreciation rate of capital.

Finally, the labor endowment evolves exponentially with a time-invariant instantaneous growth rate that may be positive, zero, or negative, i. e.,

$$L(t) = L(0)e^{g_L t}, \quad L(0) > 0, \quad g_L \in \mathbb{R}. \tag{2.8}$$

In what follows, I will denote by $g_x(t) \in \mathbb{R}$ the instantaneous growth rate of any variable $x(t)$ at t.

3 Steady-State Growth with Adjustment Costs

Definition 1 *A steady state for the economy of Section 2 is a path along which the growth rates of $Y(t)$, $C(t)$, $I(t)$, $K(t)$, $\Phi\left(I(t)\right)$, and $L(t)$ are constant for all $t \geq \tau \geq 0$.*

The following theorem uses Definition 1 and extends Uzawa's result to the neoclassical economy with adjustment costs of the previous section.

Theorem 1 *Consider the economy described by equations (2.1), (2.2), (2.7), and (2.8). Suppose there exists $\tau < \infty$ such that the economy is in steady state with $Y(t) > C(t) > 0$ for all $t \geq \tau$. Then, the following holds:*

I. $g_Y = g_C = g_I$.

II. *Either*

$$g_K = g_\Phi = \phi g_Y \neq 0,$$

in conjunction with

$$\Phi\left(I(t)\right) = c \cdot I(t)^{\phi}, \quad c > 0, \quad 0 < \phi < 1,$$

or

$$g_K = g_\Phi = g_Y = 0.$$

III. *For any* $t \geq \tau$*, total output has a representation as*

$$Y(t) = F\left[B(t)K(t), A(t)L(t)\right],$$

where $B(t) = e^{g_Y(1-\phi)(t-\tau)} \in \mathbb{R}_{++}$ *and* $A(t) = e^{(g_Y - g_L)(t-\tau)} \in \mathbb{R}_{++}$.

Capital per worker grows at rate $\phi g_Y - g_L$*. Output, consumption, and investment per worker grow at rate* $g = g_Y - g_L$.

The main message of Theorem 1 is that capital-augmenting technical change may arise in a steady state of a neoclassical economy with adjustment costs. The explanation relies on the three parts of the theorem.

Part I shows that output, consumption, and gross investment must grow at the same rate. This follows directly from the resource constraint (2.2) and the requirement that all three variables must be strictly positive.

The findings of Part II are derived from the capital accumulation equation (2.7) where adjustment costs play a crucial role. For an economy with a growing or a shrinking capital stock it is established that the installation function of capital must be a power function. This is the only functional relationship under which both $\Phi\left(I(t)\right)$ and $I(t)$ can grow at constant rates (different from zero) while being strictly positive. Positive but diminishing returns require $\phi \in (0, 1)$. The key implication of diminishing returns for the growth process is that a doubling of gross capital investments increases the capital stock by only $\phi \times 100$ percent. Hence, the capital stock grows slower than $Y(t)$, $C(t)$, and $I(t)$, and $g_K = g_\Phi = \phi g_Y \neq 0$. For an economy with $g_K = g_\Phi = g_Y = 0$, there is no net capital accumulation and the level of adjustment cost incurred in each period remains constant. As a consequence, diminishing returns in the production of capital have no bite for the steady state and $g_Y = g_C = g_I = g_K = g_\Phi = 0$.[4]

Part III reveals that capital augmenting technical change at rate $g_Y(1-\phi)$ occurs in steady state. With capital growing at rate ϕg_Y this is what is needed to have both arguments of the installation function, $F[B(t)K(t), A(t)L(t)]$, grow at the same rate. In other words, capital-augmenting technical change is called for to bridge the gap that arises since diminishing returns in the installation function of capital cause gross capital investments and capital to grow at different rates. To see this in an intuitive way consider the production function (2.1). Dividing both sides by $Y(t)$ delivers for all $t \geq \tau$

$$1 = \tilde{F}\left[\frac{K(t)}{Y(t)}, \frac{L(t)}{Y(t)}, \mathbf{A}_F(\tau)\right].$$

In steady state, the first two arguments must remain time-invariant. The capital-output ratio declines over time since $g_K = \phi g_Y < g_Y$. Capital-augmenting technical change at rate $g_Y(1-\phi)$ exactly offsets this tendency. The second argument may grow, shrink, or remain constant depending on whether $g_L \gtreqless g_Y$. Here, labor-augmenting technical change at rate $g_Y - g_L$ is the balancing force.

The final statement of Part III refers to the growth rates of per-worker variables. Since capital grows (and shrinks) slower than the remaining economic aggregates, the growth rate of capital per worker differs from the growth rate of labor-augmenting technical change. Due to constant returns to scale of $F[\cdot,\cdot]$, the latter is also the growth rate of output, consumption, and investment per worker.

There are two scenarios in which capital-augmenting technical change does not arise. Theorem 1 suggests the first scenario of a stationary economy with $g_Y = g_I = g_K = 0$. Here, the capital-output ratio remains constant so that there is no gap to be filled by capital-augmenting technical change. The second scenario has $\phi = 1$ and $0 < c < 1$, i. e., the installation function of capital is linear.[5] Then,

[4]In fact, in a stationary environment the installation function of capital does not have to be a power function but may take on any functional form consistent with the properties (2.3), (2.4), and (2.6).

[5]This scenario is not explicitly included in Theorem 1 since it violates (2.6). This modification affects Part II of Theorem 1 as explained in the main text. Part I remains unchanged whereas Part III applies for $\phi = 1$.

capital accumulation of (2.7) requires in steady state that $(g_K + \delta) K(t) = c \cdot I(t)$. Hence, $g_K = g_\Phi = g_I = g_Y$ where the last equality follows from Part I. Intuitively, the gap between gross capital investments and capital disappears if the installation function of capital exhibits constant instead of diminishing returns. As a consequence, there is no role for capital-augmenting technical change to play in steady state.

Overall, these two scenarios lead to the conclusion that adjustment costs per se are not sufficient for steady-state capital-augmenting technical change. For the latter to occur two interdependent conditions must be satisfied. First, there must be diminishing returns in the installation function of capital. Second, the economy must not be stationary. In fact, the second condition implies that the first one has bite.

Finally, observe that the second scenario above brings us closest to Uzawa's original theorem. In fact, with $\phi = 1$ and $c = 1$ adjustment costs vanish and the economy under scrutiny here coincides with the one for which Schlicht (2006) proves his variant of Uzawa's theorem.

4 Discussion

Theorem 1 presupposes the existence of a steady state. The following proposition reveals that existence requires either a stationary or a strictly growing economy.

Proposition 1 *Consider the economy of Theorem 1. A steady-state exists only if $g_Y \geq 0$.*

To grasp the intuition behind Proposition 1 suppose a steady state exists and $g_Y < 0$. Then, Part II of Theorem 1 applies, and, in view of (2.4), a steady state has to satisfy $I(t) > c \cdot I(t)^\phi$ for all $t \geq \tau$. However, as $I(t)$ declines at rate $g_I = g_Y < 0$ this inequality is violated in finite time, and, afterwards, adjustment costs will become negative. Obviously, this problem cannot arise in a stationary or a strictly growing economy.

Theorem 1 applies to a steady state as defined in Definition 1. This definition does not require adjustment costs to grow at a constant rate. But how do adjustment costs evolve in steady state? From (2.5) one readily verifies that the growth rate of adjustment costs may be expressed as

$$g_{AC}(t) = g_I(t)\varepsilon_{AC}(t), \tag{4.1}$$

where

$$\varepsilon_{AC}(t) \equiv \frac{d \ln AC\,(I(t))}{d \ln I(t)} = \frac{(1 - d\Phi\,(I(t))\,/dI(t))\,I(t)}{I(t) - \Phi\,(I(t))} > 1 \tag{4.2}$$

is the elasticity of adjustment costs with respect to gross capital investments. From (2.3) and (2.6) this elasticity is strictly greater than unity for all $I(t) \in (0, \infty)$. The evolution of $g_{AC}(t)$ may the be characterized as follows.

Proposition 2 *Consider Theorem 1 with $g_Y \geq 0$. Then, for all $t \geq \tau$ the following holds.*

1. *If $g_Y > 0$ then $\dot{g}_{AC}(t) < 0$ and $\lim_{t\to\infty} g_{AC}(t) = g_Y$.*
2. *If $g_Y = 0$ then $g_{AC}(t) = 0$.*

The explanation is straightforward. If $g_Y > 0$, then, $g_I > 0$, and due to diminishing returns in the installation function of capital, adjustment costs grow faster than the economy. This follows from Part I of Theorem 1 and equations (4.1) and (4.2). However, as $I(t)$ increases, $\varepsilon_{AC}(t)$ declines and approaches unity as $\lim_{t\to\infty} I(t) = \infty$. This reflects the tendency that the effect of a growing $I(t)$ on $AC\,(I(t))$ that operates through the installation function of capital peters out as $I(t)$ gets very large. If $g_Y = 0$ then the economy is stationary. Adjustment costs are also constant over time which is immediate from (4.1) with $g_I = 0$.

Proposition 1 and 2 suggest a link between the role of adjustment costs for the presence of steady-state capital-augmenting technical change and the definition of a steady state. To develop this link, let me replace Definition 1 by[6]

[6] Arguably, Definition 2 is more consistent with the classical definition of a steady state requiring that in steady state all relevant variables of the model grow at constant rates (see, e. g., Hahn and Matthews (1964), p. 781). However, whether Definition 1 or Definition 2 is more appropriate in the present context is most likely a matter of taste.

Definition 2 *A steady state for the economy of Section 2 is a path along which* $Y(t)$, $C(t)$, $K(t)$, $\Phi(I(t))$, $I(t)$, $L(t)$, *and also* $AC(I(t))$ *grow at a constant rate for all* $t \geq \tau \geq 0$.

Proposition 3 *Under Definition 2 any steady state satisfying Theorem 1 has* $g_Y = 0$ *and* $A(t) = e^{-g_L(t-\tau)}$. *There is no capital-augmenting technical change.*

The argument that proves Proposition 3 is simple. By Proposition 1 a steady state exists only for a strictly growing or a stationary economy. By Proposition 2, if $g_Y > 0$ then $g_{AC}(t)$ is constant only in the limit $t \to \infty$. Hence, a steady state satisfying Definition 2 cannot start in finite time. This possibility arises only if $g_Y = g_{AC} = 0$. Then, in accordance with Theorem 1, all variables are stationary. In particular, the capital-output ratio is constant so that there is no role for capital-augmenting technical change. Labor-augmenting technical change balances any growth of the labor endowment to keep $L(t)/Y(t)$ constant. Hence, the steady-state of Proposition 3 is consistent with the predictions of Uzawa's original theorem.

5 Conclusion

This paper shows that capital-augmenting technical change may arise in the steady state of a neoclassical economy with adjustment costs. This is the case if adjustment costs cause a gap between the evolution of gross capital investments and of capital so that the former grows strictly faster than the latter. It is then the role of steady-state capital-augmenting technical change to bridge this gap. This intuition is captured in a new steady-state growth theorem with adjustment costs.

The discussion of the theorem develops three necessary conditions under which adjustment costs imply steady-state capital-augmenting technical change. First, the installation function of capital must exhibit diminishing returns. Second, the growth rate of the economy must be strictly positive. Third, the definition of the steady state must allow for adjustment cost to grow at a time-varying rate.

6 Appendix: Proofs

6.1 Proof of Theorem 1

Observe that $Y(t) > C(t) > 0$ implies $I(t) > 0$. Moreover, without loss of generality, let $\tau = 0$.

Part I Given time-invariant growth rates I have $I(t) = I(0)e^{g_I t}$, $Y(t) = Y(0)e^{g_Y t}$, and $C(t) = C(0)e^{g_C t}$. Hence, the resource constraint (2.2) delivers for all $t \geq 0$

$$I(0)e^{g_I t} = Y(0)e^{g_Y t} - C(0)e^{g_C t}.$$

Dividing both sides by $e^{g_I t}$ gives

$$I(0) = Y(0)e^{(g_Y - g_I)t} - C(0)e^{(g_C - g_I)t}.$$

Differentiation with respect to t delivers

$$0 = (g_Y - g_I)\, Y(0)e^{(g_Y - g_I)t} - (g_C - g_I)\, C(0)e^{(g_C - g_I)t}.$$

The latter can hold for all t if any of the following conditions are satisfied; a) $g_Y = g_I = g_C$, b) $g_Y = g_C$ and $Y(0) = C(0)$, c) $g_Y = g_I$ and $C(0) = 0$, and d) $g_C = g_I$ and $Y(0) = 0$. Alternatives b) - d) contradict $Y(0) > C(0) > 0$. Hence, $g_Y = g_C = g_I$ must apply as claimed.

Part II Capital accumulation of (2.7) can be written as

$$(g_K + \delta_K)\, K(t) = \Phi\left(I(t)\right).$$

Hence, in steady state $g_K = g_\Phi$. Taking time derivatives delivers

$$(g_K + \delta_K)\, \dot{K}(t) = \frac{d\Phi\left(I(t)\right)}{dI(t)} \dot{I}(t).$$

Dividing the latter by the former gives

$$g_K = \frac{d\Phi\left(I(t)\right)}{dI(t)} \frac{I(t)}{\Phi\left(I(t)\right)} g_I.$$

This can only be satisfied for all t if either $g_K = g_I = 0$, or $g_K \neq 0$, $g_I \neq 0$, and

$$\Phi\left(I(t)\right) = c \cdot I(t)^{\phi},$$

where $c > 0$ is a constant of integration and $\phi = g_K/g_I$. To satisfy (2.6) it must be that $0 < \phi < 1$. Part I delivers $g_I = g_Y$. Hence, I also have $g_K = \phi g_Y$ as claimed.

Part III For any $t \geq 0$, output at time 0 may be written as

$$e^{-g_Y t} \cdot Y(t) = \tilde{F}\left[e^{-g_K t} \cdot K(t), e^{-g_L t} \cdot L(t), \mathbf{A}_F(0)\right].$$

Multiplying both sides by $e^{g_Y t}$ and using constant returns of $\tilde{F}$ gives

$$Y(t) = \tilde{F}\left[e^{(g_Y - g_K)t} \cdot K(t), e^{(g_Y - g_L)t} \cdot L(t), \mathbf{A}_F(0)\right]. \quad (6.1)$$

Then, in light of Part II one either has $g_K = g_Y = 0$ or $g_K = \phi g_Y \neq 0$.

- If $g_K = g_Y = 0$ then (6.1) delivers

$$Y(t) = \tilde{F}\left[K(t), e^{-g_L t} \cdot L(t), \mathbf{A}_F(0)\right].$$

  Since the latter equation is true for all $t \geq 0$ and $\tilde{F}$ is linear homogenous in the first two arguments, there exists a linear homogeneous function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ such that

$$Y(t) = F\left[K(t), e^{-g_L t} \cdot L(t)\right] = F\left[K(t), A(t)L(t)\right]$$

  with $A(t) = e^{-g_L t} \in \mathbb{R}_{++}$.

- If $g_K = \phi g_Y \neq 0$ then

$$Y(t) = \tilde{F}\left[e^{g_Y(1-\phi)t} \cdot K(t), e^{(g_Y - g_L)t} \cdot L(t), \mathbf{A}_F(0)\right].$$

  For the same reason as above, there exists a linear homogeneous function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ such that

$$Y(t) = F\left[e^{g_Y(1-\phi)t} \cdot K(t), e^{(g_Y - g_L)t} \cdot L(t)\right] = Y\left[B(t)K(t), A(t)L(t)\right]$$

  with $B(t) = e^{g_Y(1-\phi)t} \in \mathbb{R}_{++}$ and $A(t) = e^{(g_Y - g_L)t} \in \mathbb{R}_{++}$.

From Part II it is immediate that capital per worker grows at rate $\phi g_Y - g_L$. In addition, Part I and constant returns to scale imply that all $Y(t)/L(t)$, $C(t)/L(t)$, and $I(t)/L(t)$ grow at rate $g = g_Y - g_L$. ■

6.2 Proof of Proposition 1

Let $\tau = 0$ and suppose to the contrary that a steady state exists and $g_Y < 0$. Then, from Part I of Theorem 1 I have $I(t) = I(0)e^{g_Y t} > 0$. From Part II, $\Phi\left(I(t)\right) = c \cdot I(t)^{\phi}$, with $c > 0$ and $0 < \phi < 1$. Then, condition (2.4) requires[7]

$$I(t) > c^{\frac{1}{1-\phi}} \quad \text{for all } t \geq 0. \tag{6.2}$$

However, since $I(t)$ declines at a constant rate, there is, for any level $I(0) > c^{1/(1-\phi)}$, a critical period $\tilde{t} \in (0, \infty)$ such that $I(t) \leq c^{1/(1-\phi)}$ for all $t \geq \tilde{t}$. In fact, $\tilde{t}$ satisfies

$$\infty > \tilde{t} = \frac{(-1)}{g_Y}\left[\ln I(0) - \frac{\ln c}{1-\phi}\right] > 0.$$

If $g_Y = 0$ then $\Phi\left(I\left(t\right)\right)$ need not be a power function so that condition (6.2) is replaced by $I\left(0\right) > \Phi\left(I\left(0\right)\right)$ for any permissible installation function of capital. ■

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[7]Observe that any level $I(t) > 0$ that satisfies (6.2) will also satisfy (2.6) requiring $1 > d\Phi\left(I(t)\right)/dI(t) = c \cdot \phi \cdot I(t)^{\phi-1}$ for all $t \geq 0$.

6.3 Proof of Proposition 2

1. If $g_Y > 0$, then using (4.1), Part I and II of Theorem 1 delivers for all $t \geq \tau$

$$g_{AC}(t) = g_Y \left(\frac{I(t)^{1-\phi} - \phi c}{I(t)^{1-\phi} - c} \right)$$

   where the term in parenthesis is $\varepsilon_{AC}(t) > 1$. As $\phi \in (0,1)$, $\dot{g}_{AC}(t) < 0$. Moreover, since $I(t)$ grows without bound an application of l'Hôpital's rule delivers that $\lim_{t \to \infty} \varepsilon_{AC}(t) = 1$. Hence, $\lim_{t \to \infty} g_{AC}(t) = g_Y$ as claimed.

2. If $g_Y = 0$, then $g_I = 0$ and the result is immediate from (4.1).

■

6.4 Proof of Proposition 3

To be found in the main text. ■

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