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Optimal partial privatization in mixed oligopoly: a geometric approach

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Abstract

In homogenous mixed oligopoly neither full nationalization nor full privatization is optimal under moderate conditions. This paper provides an intuitive geometric explanation to the result by means of best-response functions and iso-social-surplus curves. It is also shown that the optimal degree of partial privatization can easily be derived from the explanation.

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1. Introduction

Geometric explanations are often useful and easy to understand in game theory although development of mathematical expressions is required in rigorous proofs. Although the optimal degree of privatization in mixed oligopoly has attracted growing interest, no simple geometric explanation has appeared in the literature. This paper presents simple graphs that explain these results not only for researchers in the field but also for the broader audience.

Mixed oligopoly is considered as an important remedy for market failures. This, however, is not always the case. A seminal work by De Fraja and Delbono (1989) points out the possibility that the social welfare in a market can be improved by the full privatization of a government-owned firm, comparing the two polar cases: full privatization and full nationalization. Another seminal work by Matsumura (1998) shows that neither full privatization nor full nationalization is optimal in mixed duopoly under moderate conditions. The graphs presented in this paper give the intuitive explanations to these results.

The geometric explanations are also useful in finding the optimal degree of partial privatization and in understanding its implications. Suppose that a fully-nationalized firm can be a leader. Then, the Stackelberg equilibrium with the fully-nationalized firm moving first gives the optimal allocation in mixed oligopoly. The outcome is demonstrated by using the best-response functions and iso-total-surplus curves because the fully-nationalized firm's objective is to maximize the total surplus in the market. This is parallel to the canonical explanations in the Cournot and Stackelberg oligopoly. Employing the Stackelberg equilibrium as a reference, we can easily find the optimal degree of partial privatization.

2. The model

Consider a mixed duopoly of homogeneous products. Firm 0 is a firm that is owned by either the public sector or the private sector, or both while firm 1 is a profit-maximizing private firm. Firm 0 becomes fully-nationalized when private ownership is zero while it becomes purely-private when it is 100 percent. Also, it is called a "privatized" firm when it is jointly owned by the two sectors.

2-1. The standard Cournot-Nash equilibrium

The reverse demand function is given by

$$p = p(y) = p(y_0 + y_1) \quad \text{with} \quad p'(y) < 0, \tag{1}$$

where p is the price, y is the total output in the market, y_0 is Firm 0's output, and y_1 is Firm 1's output. Suppose that $c_i(y_i)$ is firm i's cost function. Then the profit of firm i, π_i, is expressed as:

$$\pi_i = p(y)y_i - c_i(y_i) \quad \text{with} \quad c_i'(y_i) > 0 \quad \text{and} \quad c_i''(y_i) > 0. \tag{2}$$

Suppose that firm 0 is purely private. Then, from the first-order conditions for profit-maximum, the following two best-response functions are derived:

$$p(y_0 + y_1) + p'(y_0 + y_1)y_0 = c_0'(y_0), \tag{3a}$$

$$p(y_0 + y_1) + p'(y_0 + y_1)y_1 = c_1'(y_1), \tag{3b}$$

where $p(y_0 + y_1) + p'(y_0 + y_1)y_0 = MR_0(y_0, y_1)$ is the marginal revenue of firm 0 and $p(y_0 + y_1) + p'(y_0 + y_1)y_1 = MR_1(y_0, y_1)$ is the marginal revenue of firm 1. Hence, the above equations show that the marginal revenue is equal to the marginal cost at optimum. Suppose that the marginal revenues are downward-sloping, i.e., $\partial MR_0(y_0, y_1)/\partial y_0 < 0$ and $\partial MR_1(y_0, y_1)/\partial y_1 < 0$. Then, the Cournot-Nash Equilibrium can be shown by a standard graphical presentation.

2-2. The first-best output allocation and iso-total-surplus curves

The consumers' surplus in the market, CS, is given by

$$CS(y_0, y_1) = \int_0^{y_0+y_1} [p(y) - p(y_0 + y_1)]dy = \int_0^{y_0+y_1} p(y)dy - p(y_0 + y_1)\cdot(y_0 + y_1).$$

Hence, the total surplus in the market, TS, which is the sum of the consumers' surplus and the two firms' profits, becomes

$$TS(y_0, y_1) = CS(y_0, y_1) + \pi_0(y_0, y_1) + \pi_1(y_0, y_1) = \int_0^{y_0+y_1} p(y)dy - c_0(y_0) - c_1(y_1). \tag{4}$$

The first-best allocation is determined by

$$\partial TS/\partial y_0 = 0 \text{ i.e., } p(y_0 + y_1) = c_0'(y_0), \tag{5a}$$

$$\partial TS/\partial y_1 = 0 \text{ i.e., } p(y_0 + y_1) = c_1'(y_1). \tag{5b}$$

The above equations are of course the standard optimal allocation conditions, i.e., the price is equal to the marginal costs at social optimum. Putting it differently, the two conditions are derived assuming that each firm behaves as a competitive firm.

The above two functions can be drawn in the (y_0, y_1) plane together with the previously-derived best-response curves in Figure-1. The two solid lines CE_0 and CE_1 indicate (5a) and (5b) while the two dashed lines BR_0 and BR_1 stand for (3a) and (3b).[1] By definition, the intersection of BR_0 and BR_1, point C, is the Cournot-Nash equilibrium. Also, the maximum total surplus is attained at the intersection of CE_0 an CE_1, point B. Around point B, iso-total-surplus curves can be drawn as contour lines in the figure. Since the total surplus is the largest at point B, i.e., (y_0^B, y_1^B), the closer to the point a contour line is, the higher the total surplus is.

The slope of an iso-total-surplus curve is obtained from the total differentiation of (4):

$$dTS = (\partial TS/\partial y_0)\,dy_0 + (\partial TS/\partial y_1)\,dy_1 = 0$$

as follows:

[1] See Appendix for the relative position of the four curves.

$$\frac{dy_1}{dy_0} = -\frac{\partial TS/\partial y_0}{\partial TS/\partial y_1} = -\frac{p(y_0 + y_1) - c_0'(y_0)}{p(y_0 + y_1) - c_1'(y_1)}. \tag{6}$$

The slope of an iso-total contour is zero when it intersects with CE_0 and infinite when it cuts CE_1, as is also shown in Fig.1.

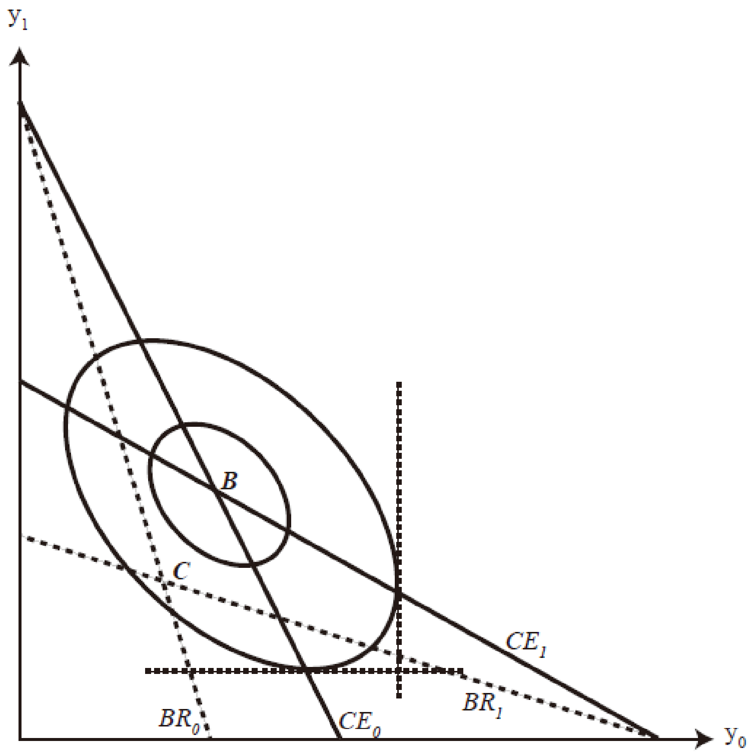

Fig.1 First-best allocation, Cournot-Nash equilibrium, and iso-total-surplus curves

3. Optimality of partial privatization

Following the existing literature, it is assumed that the privatized firm's maximand is a weighted sum of its profits and the total surplus. Usually, the weights are chosen to reflect the ownership shares of the privatized firm. Suppose that $\lambda \times 100\,\%$ of firm 0's shares are held by the private sector and $(1-\lambda)\times 100\,\%$ by the public sector. Then the objective function of the privatized firm V_0 becomes:

$$V_0 = \lambda \pi_0 + (1-\lambda)TS\ . \tag{7}$$

From the first-order condition, the following best-response function is derived:

$$(1-\lambda)[p(y_0 + y_1) - c_0'(y_0)] + \lambda[p(y_0 + y_1) + p'(y_0 + y_1)y_0 - c_0'(y_0)] = 0\,, \tag{8a}$$

or

$$p(y_0 + y_1) + \lambda p'(y_0 + y_1)y_0 = c_0'(y_0)\,. \tag{8b}$$

Eq.(8a) shows that the best-response function under partial privatization is basically equal to the weighted sum of (3a) and (5a). It becomes (3a) when $\lambda = 1$ while it becomes (5a) when

$\lambda = 0$. For $0 < \lambda < 1$, (8a) can be drawn as the BR_P curve in Fig.2. As λ becomes larger, curve BR_P gets away from curve CE_0 and becomes closer to curve BR_0.[2] Under partial privatization, the Cournot-Nash equilibrium is determined by the intersection of curves BR_P and BR_1, i.e., point P in Fig.2.

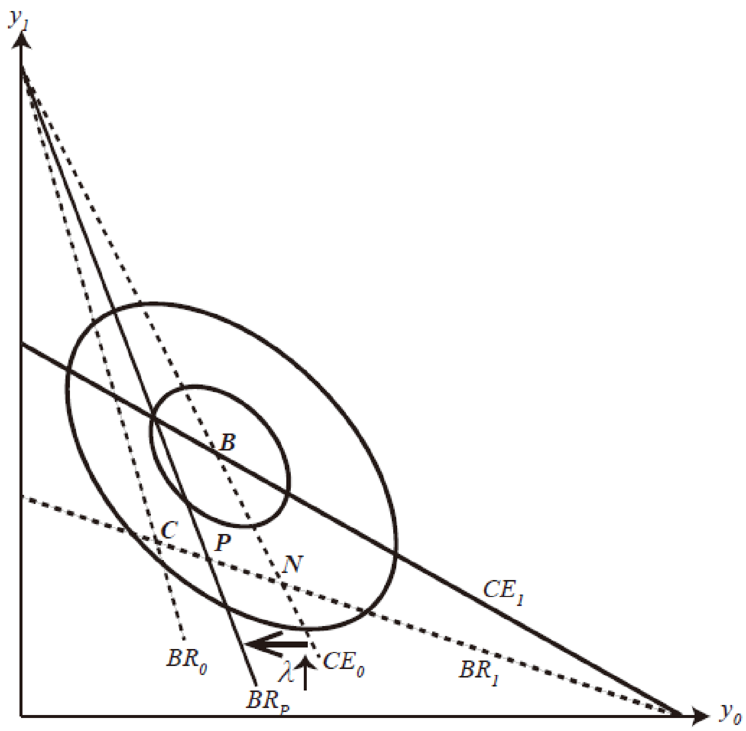

Fig.2 Cournot-Nash equilibrium under partial privatization

3-1. Non-optimality of full-nationalization

As is shown in the above, curve BR_P becomes curve CE_0 when $\lambda = 0$. Hence, the Cournot-Nash equilibrium under full nationalization is determined by the intersection of CE_0 and BR_1, i.e., point N in Fig.3. An iso-total-surplus curve that goes through point N is also drawn in the figure. As (6) shows, each iso-total-surplus contour is horizontal where it cuts curve CE_0 while vertical where it intersects with CE_1. This is of course true for the iso-total-surplus contour passing through point N in the figure. At N, the contour is horizontal while the best-response curve of firm 1, BR_1, is downward-sloping.

If λ increases, then the equilibrium moves to the left on curve BR_1 or the best-response curve of the private firm. Suppose, for instance, the equilibrium moves to N' from N in the figure. Then, the total surplus increases because the point is inside the underlying iso-total-surplus contour. As long as the best response curve of the private firm is

[2] Again, when $y_0 = 0$, then (3a), (6a) and (8) have the same solution for y_1, i.e., the y_1-intercept is the same for the three curves.

downward-sloping, the total surplus can be increased by raising λ above zero. This clearly shows the non-optimality of full-nationalization.

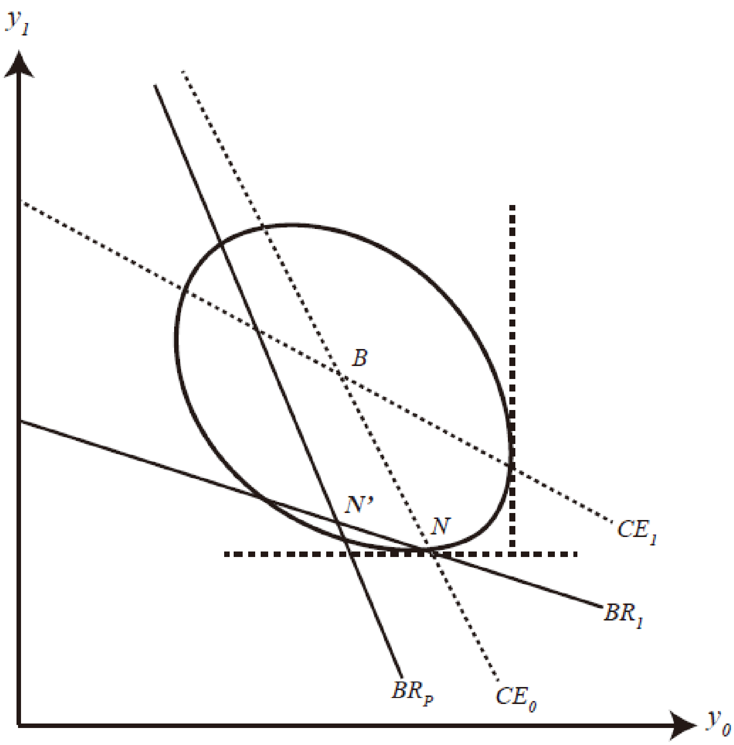

Fig.3 Non-optimality of full-nationalization

Proposition 1 (Proposition 1 in Matsumura 1998)

As long as the best response function of a private firm is downward-sloping, as usually assumed, full nationalization cannot be optimal.

3-2. Non-optimality of full-privatization

The full-privatization case appears when $\lambda = 1$, in which the standard Cournot-Nash equilibrium emerges. An iso-total-surplus curve passing through C in Fig.4 is surely downward-sloping at C. There are two cases: one is that the slope is larger than that of the best-response curve of the private firm BR_1 in absolute value, and the other is the slope is smaller than that of the best-response curve. The figure depicts the former case. In this case, if firm 0 becomes partially nationalized, i.e., λ decreases, then the total surplus increases becomes the privatized firm's best response function moves to the right, which in turn moves the equilibrium inside the contour curve. Hence, full-privatization is not optimal.

If, in contrast, the contour's slope at point C is smaller than that of the private firm's best-response curve, then full privatization is optimal. Since the two cases are possible, as Matsumura correctly states, it is difficult to derive any general conclusions. The observation, however, brings us to the following proposition.

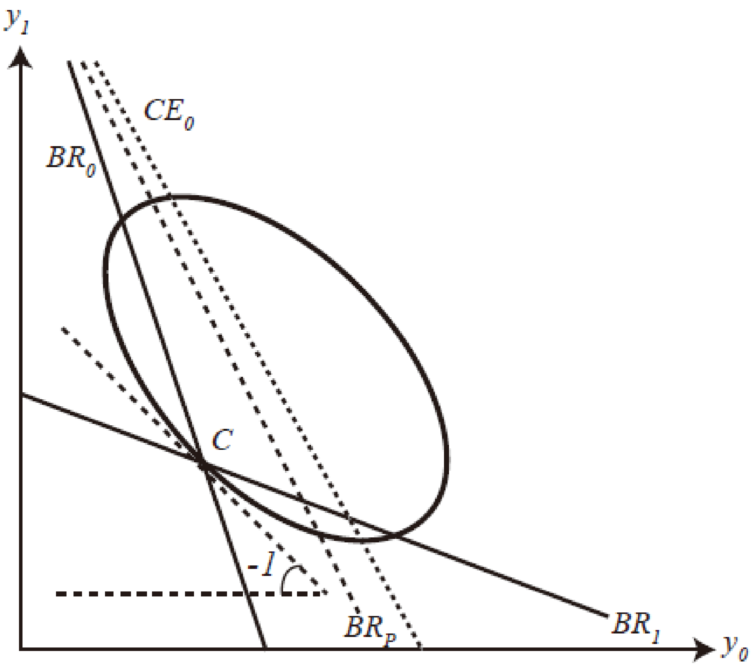

Fig.4 Non-optimality of full-nationalization

Proposition 2 (A condition for non-optimality of full privatization)
If a contour passing through the Cournot-Nash equilibrium is steeper than that of the private firm's best-response function at the equilibrium, then full privatization cannot be optimal.

Matsumura considers a special case in which the two firms have the same cost function that has a clear result.[3] Under the same cost function, i.e., $c_0(\cdot)=c_1(\cdot)=c(\cdot)$, the output of each firm is also the same at the Cournot-Nash equilibrium, i.e., $y_0 = y_1 = \tilde{y}$. From (6), the slope of the contour becomes

$$\frac{dy_1}{dy_0} = -\frac{\partial TS/\partial y_0}{\partial TS/\partial y_1} = -\frac{p(y_0+y_1)-c_0'(y_0)}{p(y_0+y_1)-c_1'(y_1)} = -\frac{p(\tilde{y}+\tilde{y})-c'(\tilde{y})}{p(\tilde{y}+\tilde{y})-c'(\tilde{y})} = -1.$$

For the Cournot-Nash equilibrium to be stable, the slope of the best-response must be smaller than unity in absolute value. Hence we have the following corollary.

Corollary (Proposition 2 in Matsumura 1998)
Suppose that the two firms have the same cost function. Then full-privatization cannot be optimal.

4. The optimal degree of partial privatization

The privatized firm's best-response function is (8) while that of the private firm is (3b). As Figure-2 shows, the Cournot-Nash equilibrium under partial privatization is always on BR_1.

[3] To be precise, the same "marginal" cost function is sufficient.

Hence, the possible maximum total surplus can be attained at the allocation at which an iso-total-surplus contour is tangent to the best-response curve, as Fig.5 shows. Not surprisingly, it is the Stackelberg equilibrium with the fully-nationalized firm moving first and the private firm moving second. In general, the fully-nationalized firm cannot be a leader in the product market. Hence, instead of gripping full control of the firm, the government can achieve the "second-best" allocation by giving up some degree of control.

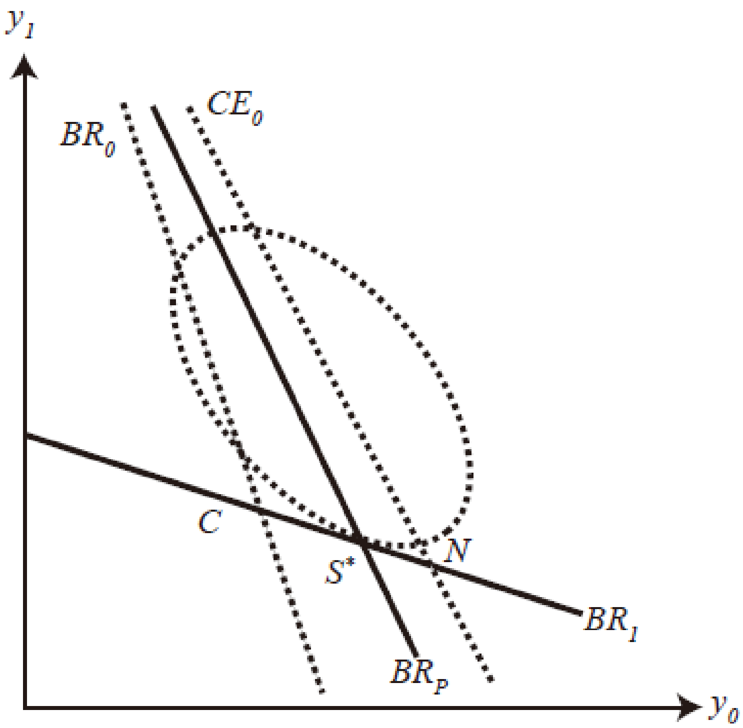

Fig.5 Optimal degree of partial privatization

In general, to obtain the optimal degree of partial privatization, one more stage must be introduced into the model, in which the government determines the degree before the two firms choose the outputs simultaneously. In other words, setting up a sequential-move game with the government moving first and the two firms simultaneously moving second, the optimal degree is obtained as one of the strategies characterizing the possible subgame-perfect equilibria. Once, however, we realize that the second best allocation, (y_0^*, y_1^*) is obtained as the Stackelberg equilibrium, a different way to pin down the optimal degree of partial privatization is possible. First, we find the second best allocation (y_0^*, y_1^*), point S^* in Fig.5, considering the Stackelberg equilibrium with the fully-nationalized firm moving first. Second, substituting y_0^* and y_1^* into (8b), we can find the optimal degree of partial privatization, λ^*. Namely, the following equation gives λ^*:

$$p(y_0^* + y_1^*) + \lambda^* p'(y_0^* + y_1^*) y_0^* = c_0'(y_0^*), \tag{9a}$$

or,

$$\lambda^* = -\frac{p(y_0^* + y_1^*) - c_0'(y_0^*)}{p'(y_0^* + y_1^*)y_0^*}. \tag{9b}$$

The second-best allocation (y_0^*, y_1^*) is between BR_0 and CE_0. On one hand, in the region right to BR_0, since $\partial\pi(y_0, y_1)/\partial y_0 = p(y_0 + y_1) + p'(y_0 + y_1)y_0 - c_0'(y_0) < 0$, $p'(y_0^* + y_1^*)y_0^* < p(y_0^* + y_1^*) - c_0'(y_0^*)$. On the other hand, in the left to CE_0, since $\partial TS(y_0, y_1)/\partial y_0 = p(y_0 + y_1) - c_0'(y_0) < 0$, $p(y_0^* + y_1^*) - c_0'(y_0^*) < 0$. Hence, $p'(y_0^* + y_1^*)y_0^* < p(y_0^* + y_1^*) - c_0'(y_0^*) < 0$ and $|p'(y_0^* + y_1^*)y_0^*| > |p(y_0^* + y_1^*) - c_0'(y_0^*)|$. This proves $0 < \lambda^* < 1$.

Proposition 3 (Optimal degree of partial privatization)
Suppose that (y_0^*, y_1^*) is the output attained at the Stackelberg equilibrium with the fully-nationalized firm moving first and the private firm moving second. Then, the optimal degree of partial privatization is given by (9b).

5. Concluding Remarks

Best-response functions, and the associated iso-profit and indifference curves are very useful to understand the equilibria and their implications in game theory, especially at the early stage of research. Over past three decades research on the optimal privatization in mixed oligopoly has been developed. However, little effort has been done to present the results in a way that allows those who are interested but not specialized in the field to easily follow them. The aim of this paper is to present the existing important results using simple and easily-understandable graphs with the familiar best-response and contour curves. As a result, it is clearly demonstrated that the Cournot-Nash equilibrium under the optimal partial privatization is identical to the Stackelberg equilibrium with a fully-nationalized firm moving first and the private firm moving second. This observation leads to the optimal degree of partial privatization.

References

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Matsumura, T. (1998), "Partial Privatization in Mixed Duopoly," *Journal of Public Economics*, 70, 473–483.

Appendix: Relative position of the four curves

Substituting $y_0 = 0$ into (3a) and (5a), we have the same equation: $p(0 + y_1) = c_0'(0)$. Hence, when $y_0 = 0$, then (3a) and (5a) have the same solution for y_1. Namely, the y_1-axis intercept for BR_0 is the same as that for CE_0 in the (y_0, y_1) plane. Since the same argument applies to (3b) and (5b), the y_0-axis intercept for BR_1 is the same as that for CE_1 in the (y_0, y_1) plane.

We obtain the slope of (5a) as follows:

$$\frac{dy_1}{dy_0} = \frac{c_0''(y_0)}{p'(\cdot)} - 1 \leq -1,$$

and similarly the slope of (5b)

$$\frac{dy_0}{dy_1} = \frac{c_1''(y_1)}{p'(\cdot)} - 1 \leq -1.$$

Hence, the both curves are downward-sloping. Also, CE_0 is always above BR_0 expect $y_0 = 0$. Denote the solution of (3a) by $\hat{y}_1$ and that of (5a) by $\tilde{y}_1$. Suppose that $\hat{y}_1 \geq \tilde{y}_1$. Then, for some $y_0 > 0$, $p(y_0 + \hat{y}_1) \leq p(y_0 + \tilde{y}_1)$ since $p'(y_0 + \hat{y}_1) < 0$. Comparing (3a) with (5a), $p(y_0 + \hat{y}_1) + p'(y_0 + \hat{y}_1)y_0 = p(y_0 + \tilde{y}_1) = c_0'(y_0)$. Since $p'(y_0 + \hat{y}_1)y_0 < 0$, $p(y_0 + \hat{y}_1) > p(y_0 + \tilde{y}_1)$. This leads to contradiction. Therefore, $\hat{y}_1 < \tilde{y}_1$. Applying the same argument to (3b) and (5b), CE_1 is always above BR_1 expect $y_1 = 0$.

In sum, CE_0 and BR_0 are both downward-sloping and have the same y_1-axis intercept although CE_0 is always above BR_0 expect the intercept. Similarly, CE_1 and BR_1 are both downward-sloping and have the same y_0-axis intercept although CE_1 is always above BR_1 expect the intercept.