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Analyzing the Feldstein-Horioka puzzle in continuous wavelet transform

Olaolu R Olayeni
Department of Economics, Obafemi Awolowo University, Ile-Ife, Nigeria

Abstract

We use a novel time-frequency approach to estimate wavelet-based correlation and retention coefficients of the well-known Feldstein-Horioka (1980) puzzle for a cross-section of 39 countries over the period 1960-2012. Our approach reveals many interesting issues in the connection between investment and saving rates among these countries and distinguishes between the short and the long run fluctuations in correlation and retention coefficients.

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Contact: Olaolu R Olayeni - rolayeni@oauife.edu.ng.
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1. Introduction

In a seminal paper, Feldstein and Horioka (1980) investigated a long-term relationship between investment and saving rates and found that these variables are positively and highly correlated. The subsequent interpretation that the high correlation reflects limited capital mobility is regarded as a puzzle because this interpretation is contrary to the theoretical thrust in an open economy inbuilt with capital mobility. More importantly, the study by Sachs (1981, 1982) examined the validity of the Feldstein-Horioka proposition by investigating the relationship between investment and current account deficits. High negative coefficient in the regression of current account deficits on investment indicates that investment booms are partly financed by capital inflows. The Sachs (1981) finding that the coefficient is indeed negative and high casts doubts on the Feldstein-Horioka findings. The contrasting results are what has become a puzzle. Following these studies, the literature on the puzzle has gradually snowballed. The vast literature is surveyed in Apergis and Tsoumas (2009). Early studies evaluating the results include Obstfeld (1986), Murphy (1984), Penati and Dooley (1984), Tesar (1991), Coakley et al. (1994, 1995). Methods of analysis also range from simple correlation coefficients to show co-movements between the variables to sophisticated analytical and econometric models to understand forces driving the co-movement. For instance, Felmingham and Cooray (2006), Sinha and Sinha (2004) and Pelgrin and Schich (2008) employed (panel) error correction models while Hoffmann (2004) employed bivariate VAR model. Methodological divide also runs through the use of time-series and cross-sectional studies. A partial list in this respect is given in Coakley et al. (1998).

The primary objective of this paper is to analyze the Feldstein-Horioka puzzle from time-scale perspective. Few issues in the literature motivate the choice of this analyzing technique. First, we would like to differentiate between high and low frequency in investment-saving relationship as this has been one area of concern in the literature. For instance, Tesar (1991) was interested in understanding the role of data frequency. She undertook averaging the time-series and concluded that her estimates are robust in high and low frequencies. While her simple approach may be quite appealing, a superior and sophisticated alternative is to employ time-scale analysis that is capable of zeroing in on the highly evolving details without skipping the slowly manifesting features. Apart from this and from policy perspective, short and long horizons may portend different policy implications and goals. This objective leads us to the choice of method. One of the time-scale workhorses capable of unfolding time-series into layers of constituent horizons is wavelet analysis. As wavelet analysis provides a good resolution in time and scale, transient behaviour and structural breaks can be nicely captured. Wavelet analysis comes basically in two flavours, namely, the discrete and the continuous wavelet transforms. Both the discrete and the continuous wavelet transforms are very resourceful, but the continuous wavelet transform additionally allows us to compute the statistics of interest in three dimensions all of which may be useful. In our case, the benefit of rendering the correlation and retention coefficients in frequency and over time will assist in making our points.

To a good extent, we are able to resolve the puzzle insomuch as we establish that the relationship between investment and current account deficits on the one hand and between investment and saving on the other may well be consistent in the time-frequency space. This result shows the superiority of the time-frequency domain analysis over either the time-domain analysis or the frequency-domain analysis.

The rest of the paper is organized as follows. In section 2, we provide the background to the study. The results in this section are fundamental to section 3, which deals with the translation from the time domain statistics to the time-frequency domain statistics. Section 4 presents the empirical results, while section 5 concludes.

2. Preliminary background

Feldstein and Horioka (1980) estimated the following regression

$$\left(\frac{I}{Y}\right)_t = a + b\left(\frac{S}{Y}\right)_t + \varepsilon_t\,, \qquad (1)$$

where $(I/Y)_t$ and $(S/Y)_t$ are the investment and saving rates respectively and ε_t is the error term encapsulating relevant omitted variables. b denotes what is termed the retention coefficient representing the fraction of domestic saving retained in mobilizing domestic investment. A retention coefficient close to unit reflects perfect capital immobility as most of the saving is used to mobilize domestic investment, while a low value tending to zero indicates perfect capital mobility. The intermediate range suggests various degrees of capital mobility. Defined this way, the OLS estimate of b is given by

$$\hat{b} = \frac{\operatorname{cov}((I/Y)_t,(S/Y)_t)}{\operatorname{var}((S/Y)_t)} = \rho((I/Y)_t,(S/Y)_t)\cdot\frac{\sigma((I/Y)_t)}{\sigma((S/Y)_t)}. \qquad (2)$$

The second equality in the above expression makes the analysis quite transparent. Indeed, in a number of studies analyzing the Feldstein-Horioka puzzle, the correlation between these two rates $\rho((I/Y)_t,(S/Y)_t)$ has been a subject-matter of analysis. An example in this respect is Obstfeld (1986), who computes correlations for seven OECD countries using quarterly data between changes in saving and investment rates. A high correlation coefficient approaching unit indicates capital immobility while a low correlation coefficient approaching zero indicates high capital mobility. A key assumption underlying correlation-based analysis is that variabilities in the two series are quantitatively the same, i.e. $\sigma((I/Y)_t) \approx \sigma((S/Y)_t)$. This assumption cannot however be justified empirically given the stylized facts that investment variability is very high and that consumption smoothing prevents saving rate from being as volatile as investment. A related strand of literature pioneered by Sachs (1981) stresses the relationship between current account deficits and investment. We can simplify this relationship thus,

$$\left(\frac{CA}{Y}\right)_t = \alpha + \beta\left(\frac{I}{Y}\right)_t + \upsilon_t\,, \qquad (3)$$

where $CA = S - I$. It is generally found that the OLS estimate $\hat{\beta}$ is absolutely high. This means that international capital inflows are evocative about the investment booms, the finding that appears to rebuff the implications of the high saving retention coefficient. In fact, this is the soul of the puzzle. Realizing that these two estimates can be linked provides a comforting platform for the resolution of this antithesis. It can be shown that[1]

[1] See Appendix A

$$\hat{\beta} = \hat{b} \cdot \left(\frac{\sigma((S/Y)_t)}{\sigma((I/Y)_t)} \right)^2 - 1 . \tag{4}$$

This result shows that the relation between the two estimates is determined by the variability of saving rate relative to that of investment. Given that the variabilities are quantitatively the same, then $\hat{\beta} = \hat{b} - 1$. It then follows that the case of perfect capital mobility, i.e., $\hat{b} = 1$, would imply no correlation between current account deficit and investment booms. On the other hand, if there is no capital mobility such as the case under the autarchy, such that $\hat{b} = 0$, then $\hat{\beta} = -1$. Of course, a continuum of possibilities lies within these extremes.

Now theory and stylized facts suggest that the variabilities in investment and saving rates might well not be equal; indeed, the variability in investment is widely acknowledged to be higher than the variability in consumption because of consumption smoothing by the households. Thus, by implication, it is higher than the variability in saving. It then follows that the following relation holds,

$$0 \le \frac{\sigma((S/Y)_t)}{\sigma((I/Y)_t)} < 1 .$$

But then if the lower bound in the relation above is activated, we can still observe that $\hat{\beta} = -1$ even though $\hat{b}$ is non-zero. Even if this quantity is not zero, the fact that investment is more volatile indicates that the impact of $\hat{b}$ will more rapidly wear out the closer the quantity is to zero. This channel provides a possible explanation for a high saving retention coefficient that is nevertheless consistent with a strong correlation between current account deficit and investment booms, and hence the fear that the mother of all puzzles would not go away (Sinha and Sinha, 2004).

We now beam the wavelet searchlight on this relationship to capture the fluctuations at different scales and over time. In the next section, we proceed to capture the retention coefficient and other measures in time-frequency space.

3. Measuring the retention coefficient in time-frequency space

In spectral analysis, which is a close cousin of wavelet analysis in vocabulary, regression coefficient is often obtained by computing the gain. However, a well-known problem with this measure of regression coefficient is that it ignores the sign in the relationship between the dependent and independent variables. It is thus of limited utility in empirical analysis. We circumvent this problem by constructing a new measure based on the decomposition in Eq. (2). From this equation, we see that the retention coefficient is made up of three components all of which can be conveniently rendered in time-frequency space. We now show how this can be done. First, we need to introduce some basic concepts necessary to adequately render a wavelet-based retention coefficient. The first primary concept is the continuous wavelet transform (CWT) that decomposes a time series $x(t)$ into some elementary functions through the process of dilation and translation of the mother wavelet function $\psi(t)$. Specifically, the dilated and translated daughter wavelet is given by $\psi_{\tau,s}(t) = \psi\left((t-\tau)/s\right)$ while the wavelet coefficient is given by

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t) \circ \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s} \right) dt . \tag{5}$$

If the mother wavelet is complex-valued, the wavelet coefficient will be complex-valued as well. The complex-valued wavelet coefficient is of great utility in wavelet analysis and can be factored into real and imaginary parts as $W_x(s,\tau)=\Re\{W_x(s,\tau)\}+i\Im\{W_x(s,\tau)\}$. The symbol $\circ$ is the convolution operator. In view of the terms in Eq. (2), $W_{xx}(s,\tau)=\left|W_x(s,\tau)\right|^2$ is the power spectrum capturing the variability of the variable under investigation. Given the wavelet coefficient for variable *y*, the power spectrum can be similarly defined for *y* as $W_{yy}(s,\tau)=\left|W_y(s,\tau)\right|^2$. In the same vein, the cross-spectrum, an equivalent of covariance in the time-domain analysis between *x* and *y*, is given by $W_{xy}(s,\tau)=W_x(s,\tau)\cdot\widetilde{W}_y(s,\tau)$, where $\widetilde{W}_y(s,\tau)$ is the complex conjugate of $W_y(s,\tau)$. Taking the square root of the power spectrum, we obtain an analogue of the standard deviation given in Eq. (2). The last requirement to define the wavelet-based retention coefficient is the measure of correlation in time-frequency. In this respect, Rua (2013) proposed wavelet-based correlation measure given by

$$\rho_{xy}(s,\tau)=\frac{\zeta\left(s^{-1}\left|\Re(W_{xy}(s,\tau))\right|\right)}{\zeta\left(s^{-1}\sqrt{W_{xx}(s,\tau)}\right)\cdot\zeta\left(s^{-1}\sqrt{W_{yy}(s,\tau)}\right)}, \tag{6}$$

where s and τ are the scale and location parameters, $\zeta(Z)=\zeta_{scale}(\zeta_{time}(Z))$ with ζ_{scale} is the smoothing operator along scale axis while ζ_{time} is the smoothing operator along the time axis. As a correlation measure, $\rho_{xy}(s,\tau)$ is bounded between -1 and +1, that is, $-1\le\rho_{xy}(s,\tau)\le 1$. Given these measures, we arrive at the following retention coefficient in time-frequency space, which compares with the retention coefficient in Eq. (2):

$$b(s,\tau)=\rho_{xy}(s,\tau)\cdot\frac{\zeta\left(s^{-1}\sqrt{W_{yy}(s,\tau)}\right)}{\zeta\left(s^{-1}\sqrt{W_{xx}(s,\tau)}\right)}. \tag{7}$$

Since we are interested in a cross-section of countries, we develop weighted averages of $\rho_{xy}(s,\tau)$ and $b(s,\tau)$. For instance, we define

$$\overline{\rho}_{xy}(s,\tau)=\sum_j\left(\frac{\varpi_j}{\sum_i\varpi_i}\right)\rho_{xy}^j(s,\tau), \tag{8}$$

where ϖ_j is some quantity say GDP while the term in bracket denotes the significance of country *j* in the cross-section. Thus closer this term is to unit, the more important is country *j* in the cross-section. We similarly define $\overline{b}(s,\tau)$ as

$$\overline{b}(s,\tau)=\sum_j\left(\frac{\varpi_j}{\sum_i\varpi_i}\right)b^j(s,\tau). \tag{9}$$

Using the relationship in Eq. (4), we can likewise define the Sachs coefficient in time-frequency domain. This is given by

$$\overline{\beta}(s,\tau)=\overline{b}(s,\tau)\cdot\frac{\zeta\left(s^{-1}W_{yy}(s,\tau)\right)}{\zeta\left(s^{-1}W_{xx}(s,\tau)\right)}-1. \tag{10}$$

Fundamental to our analysis is the choice of the wavelet function. We stated earlier that the complex-valued wavelet function is of great utility. One such function having this property is the Morlet wavelet function. Besides, the Morlet wavelet function can be shown to achieve an optimal localization between the resolution in time and in frequency due to its Gaussian envelop. This property is guaranteed by Heisenberg's uncertainty theorem stating that there is a lower limit to the product of time and frequency resolution. Also implying a trade-off between the resolution in time and in frequency, the theorem ensures that any improvement in time degrades the frequency resolution and any improvement in frequency degrades the time resolution. Thus, to achieve optimal balance, we employ the Morlet wavelet function given by

$$\psi(\eta) = \pi^{-1/4} e^{i\omega\eta} e^{-\frac{1}{2}\eta^2}, \tag{11}$$

where ω is dimensionless frequency and η is dimensionless time. For optimal balance, we set $\omega = 6$ as suggested by Torrence and Compo (1998). Since the idea behind the CWT is to apply the wavelet as a band pass filter to the time series, the wavelet is stretched in time by varying its scale s, so that $\eta = s \cdot t$ and normalizing it to have unit energy. For the Morlet wavelet, the Fourier period (λ_{wt}) is almost equal to the scale $(\lambda_{wt} = 1.03)$. The wavelet transform also inherits this property.

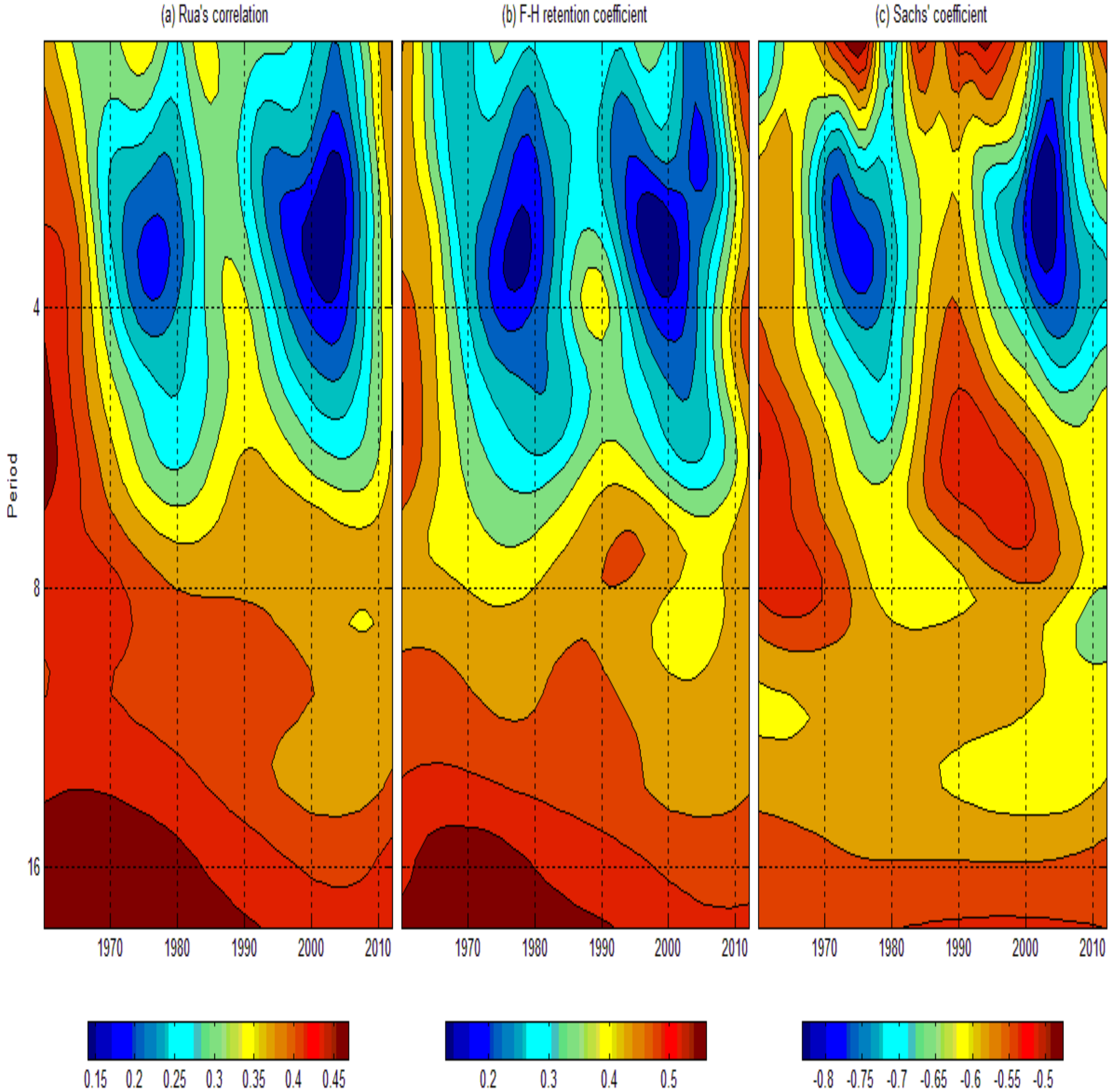
4. Empirical results

We study the Feldstein-Horioka puzzle for a cross-section of 39 countries over the period 1960-2012. The data for the study are obtained from the World Bank's World Development Indicators (WDI) online database. Data availability for this period is a major criterion for the inclusion of any country in the cross-section. Thus, we include Argentina, Australia, Bangladesh, Botswana, Brazil, Chile, Colombia, Democratic Republic of Congo, Congo, Costa Rica, Dominican Republic, Ecuador, Egypt, Ghana, Guatemala, Honduras, Hong Kong SAR, India, Indonesia, Kenya, Lesotho, Malaysia, Mauritania, Mexico, Morocco, Nicaragua, Peru, Philippines, Puerto Rico, Rwanda, Singapore, South Africa, Swaziland, Thailand, Turkey, Uganda, Uruguay, Venezuela, and Zambia.

In Figure-1, we present the estimates of wavelet-based weighted correlation proposed by Rua (2013) in Panel (a), the F-H retention coefficient between investment and saving rates in Panel (b) and the Sachs coefficient between investment and current account in Panel (c). Note that the application of Rua's (2013) correlation here is slightly different from the application in his synchronization study. Being a level curve showing relationship in three dimensions, the points in these panels record the undulating heights. The first two plots in Panels (a) and (b) look quite similar. This similarity might be an offshoot of quantitatively similar variances. As expected, the correlation coefficient is positive indicating that saving and investment rates move in the same direction for the countries under investigation. The strength of co-movement however varies across the time-frequency space. More precisely, co-movements are stronger in the long run than they are in the short run most especially before the 1990s. However, this finding might not be indicative of the idea that these countries are on average satisfying their intertemporal budget constraints courtesy of no-Ponzi-game condition because correlation coefficient is still in the neighbourhood of 0.5 and not high enough to justify such a position.

Looking at the colorbar, we see that the values of the estimated wavelet-based correlation coefficient range between 0.1 and 0.5. We also find that the bulk of the contour plot shows that the range covered is between 0.2 and 0.4. Thus, capitals appear to be more

Figure-1: A time-frequency view of the Feldstein-Horioka puzzle



mobile for the countries under investigation than suggested by Feldstein and Horioka (1980) who reported a very high point estimate of 0.887. Furthermore, two islands of extremely low correlation can be identified both bounded on 1~8 year frequency cycle. The first occurs between 1975 and 1985 and the second between 1990 and 2010. Can we then conclude that capital mobility during these periods is high? To be sure, the Sachs coefficient in Panel (c) comes in handy. This result gives a clean reconciliation of the puzzle: a high negative Sachs coefficient that indicates increases in investment are financed by capital inflows is consistent with a low F-H retention coefficient that indicates high capital mobility during these periods *and* on the indicated scale. Correspondingly, the areas of low negative Sachs coefficient square very well with the areas of high F-H retention coefficient, which again resolves the

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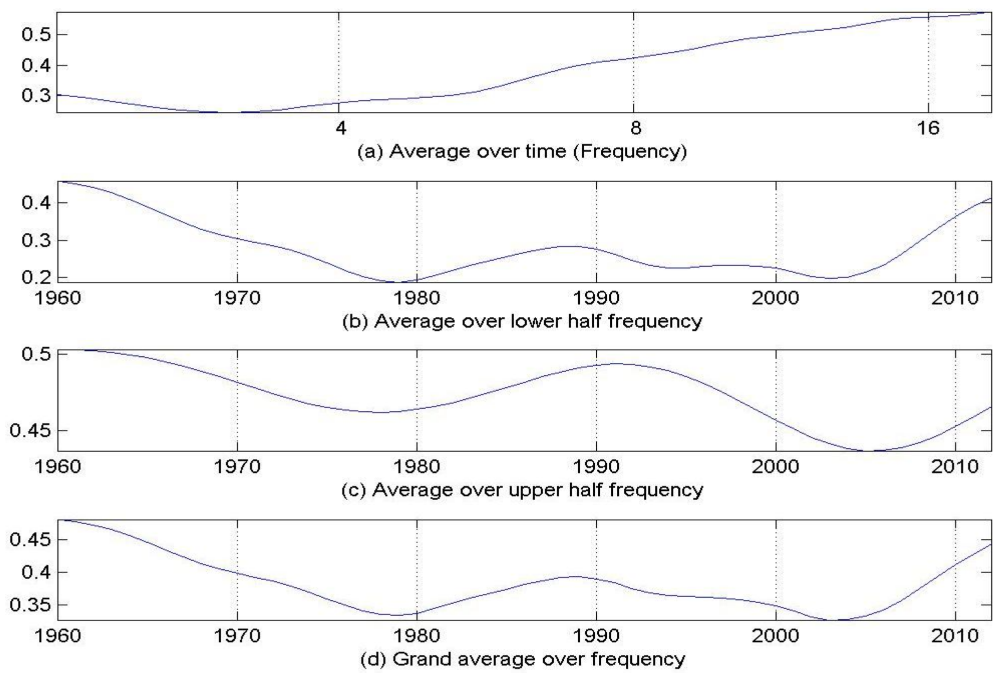
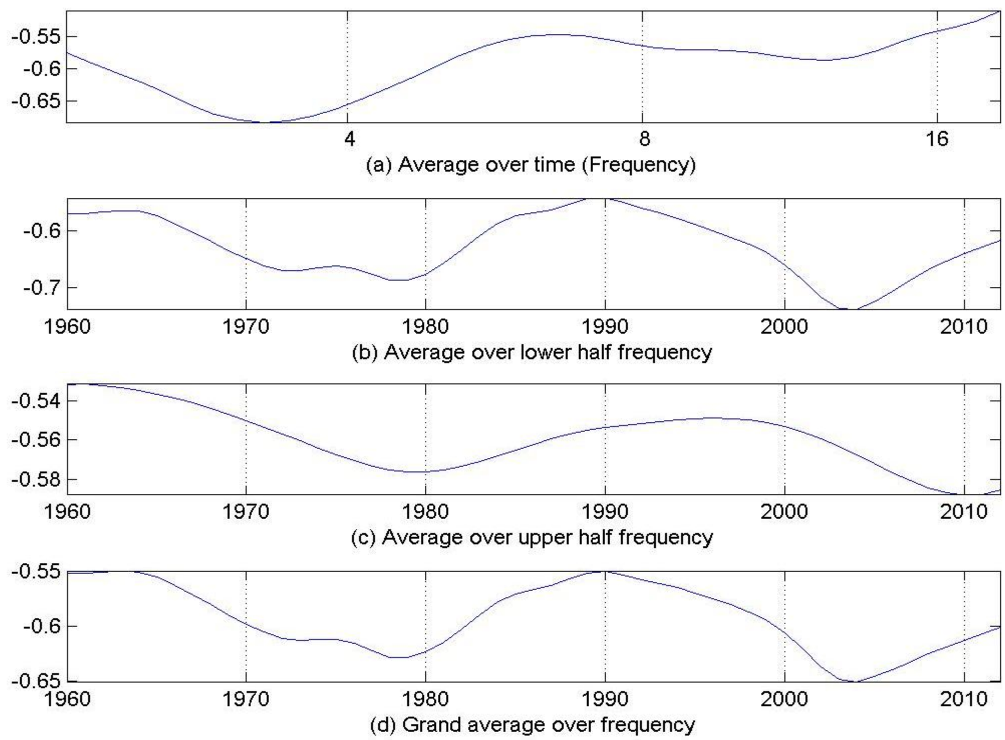
puzzle. Hence, we answer the question posed by Baxter and Crucini (1993) on *how* to resolve the puzzle. Furthermore, on the empirical front, the findings in this paper explain capital mobility as a consistent outcome of competitive position of a country that undertakes investment in association with the balance of payment deficits service[2]. This study thus reveals that the proposition of Feldstein and Horioka and that of Sachs are two sides of the same coin if perused from time-frequency perspective. Thus, the confusion in the literature stems from frequency of the data as envisaged by Tesar (1991) and others who averaged their data before computation.

Figure-2: The retention coefficient in the Feldstein-Horioka puzzle



[2] I thank an anonymous reviewer for this point.

Figure-3: The Sachs coefficient of investment on current account



To gain further insight into this relationship, we average the level curves for retention and the Sachs coefficients both over frequency and over time. The results are reported in Figures-2 and -3 respectively. In Panel (a) of Figures-2 and -3, we observe that the retention and the Sachs coefficients are quite low in high frequency region (that is, with low number of years per cycle) and trends upward in low frequency region (that is, with high number of years per cycle). In Panels (b)-(d) in both figures, we focus on the average retention and the average Sachs coefficients in the short run, the long run and over the entire horizons respectively. We find that the retention and the Sachs coefficients share similar oscillations over time and are generally lower in the short than in the long run. For the entire horizons, these coefficients are in the intermediate range. One further finding is that the estimated retention and Sachs coefficients are time-varying. The time-variations in these estimates clearly point out again that the system has not been stable. Hence, focusing on different time periods might well result in different estimates.

5. Conclusion

In this paper, we have studied the well-known Feldstein-Horioka puzzle for a cross-section of 39 countries over the period 1960-2012 using wavelet time-frequency analysis. The novel application of this approach reveals many aspects of the relationship previously omitted but which may well help in resolving the puzzle. As an offshoot of this resolution, we also undertake the analysis of the Sachs coefficient relating investment to current account deficits. At the end, a truce is provided. Although this may not have resolved most of the entangled propositions that have emerged over the years, it provides a synthesis with respect to the position of a country that undertakes investment in association with balance of payment deficits service in the presence of capital mobility. We are optimistic that although the method applied here finds nice application in this puzzle, it promises to be a universal method that could supplement the time-domain analysis.

Appendix A

Recall the OLS estimate of the Feldstein-Horioka puzzle given in Eq. (2):

$$\hat{b} = \frac{\operatorname{cov}((I/Y)_t,(S/Y)_t)}{\operatorname{var}((S/Y)_t)}.$$

The OLS estimate of β in Eq. (3) can be likewise derived. This is given by

$$\hat{\beta} = \frac{\operatorname{cov}((I/Y)_t,(CA/Y)_t)}{\operatorname{var}((I/Y)_t)}.$$

Recall also that the current account is defined as $CA = S - I$. Using this definition we have:

$$\hat{\beta} = \frac{\operatorname{cov}((I/Y)_t,(\{S-I\}/Y)_t)}{\operatorname{var}((I/Y)_t)} = \frac{\operatorname{cov}((I/Y)_t,(S/Y)_t)}{\operatorname{var}((I/Y)_t)} - 1.$$

It now follows that the above can be decomposed as

$$\hat{\beta} = \frac{\operatorname{cov}((I/Y)_t,(S/Y)_t)}{\operatorname{var}((S/Y)_t)} \cdot \frac{\operatorname{var}((S/Y)_t)}{\operatorname{var}((I/Y)_t)} - 1.$$

Finally, using the result in Eq. (2), we have

$$\hat{\beta} = \hat{b} \cdot \frac{\operatorname{var}((S/Y)_t)}{\operatorname{var}((I/Y)_t)} - 1 = \hat{b} \cdot \left(\frac{\sigma((S/Y)_t)}{\sigma((I/Y)_t)} \right)^2 - 1,$$

which is Eq. (4) given in the text.

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