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Optimal contracts for central bankers: a note

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Abstract

Walsh (1995) was the first author to find a full solution to the problem of time inconsistency in monetary policy, namely, a contract that eliminates the inflation bias without incurring any output stabilization costs. We provide an alternative method for obtaining such an optimal contract. Its components are shown explicitly to be derived from a constrained optimization problem which is solved by applying the Kuhn-Tucker conditions to a muti-stage game. We also conclude that there are more socially optimal contracts apart from the one considered by Walsh (1995).

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1 Introduction

One of the most important papers in monetary policy is the article "Optimal Contracts for Central Bankers" by Carl Walsh published in 1995 in the American Economic Review. This has been widely acknowledged by academics and practitioners. For instance, in 2004 the former Chairman of the Federal Reserve Ben S. Bernanke in the Conference on *Reflections on Monetary Policy 25 Years after October 1979* described the article as "one of the three most influential papers in macroeconomics over the past 25 years". The theory proposed by Walsh underpins the widespread practice of delegating monetary policy to independent central banks and is commonly referred to as the "Walsh Contract".

In a nutshell, Walsh (1995) was the first author to find a full solution to the problem of time inconsistency in monetary policy first raised in the pathbreaking articles of Kydland and Prescott (1977) and Barro and Gordon (1983). These two papers showed that when the monetary authority faces an incentive to expand output above its natural level, discretionary monetary policy gives rise to an inefficiently high level of inflation: the so-called "inflation bias". As a way out of this problem, Walsh (1995) showed that this bias can be eliminated without incurring any output stabilization costs if the government offers the central bank a well designed incentive scheme (an inflation contract) which penalizes the latter for creating inflation.[1]

However, in the Walsh paper the two parameters that determine the central bank contract are not explicitly derived but rather implicitly. They are obtained so that they fulfill the following two requirements: (a) the inflation bias be eliminated and (b) the contract be accepted by the central banker without having a surplus in excess of the outside option. At this respect, we provide an innovation in the procedure through which Walsh (1995)'s conclusions can be arrived at. That is, we show that the optimal contract can be derived when the government (the principal) minimizes an objective function (the social loss function) subject to a constraint (the participation constraint of the central bank). More precisely, we characterize the optimal

[1] Rogoff (1985) has proposed appointing a central bank which puts a higher weight on inflation than society. Other authors have focused on the central banker's incentives which can be influenced by a linear inflation contract (Walsh, 1995) or an explicit inflation target (Svensson, 1997). See Walsh (2010, chapter 7) for a detailed analysis of this literature and the references therein.

contract by applying the Kuhn-Tucker conditions. Furthermore, in order to show more explicitly the sequential character of the decisions taken by the economic agents involved, we make use of a multi-stage game. This helps us to show that even though the resulting monetary institution does eradicate the inflation bias, it does not require the central bank not to earn a surplus since, in contrast to Walsh (1995), the equilibrium is not unique.

The rest of the paper is organized as follows. Section 2 briefly presents the model used by Walsh (1995). Section 3 is devoted to our alternative approach. Finally, Section 4 concludes.

2 The Walsh's (1995) approach

As in Walsh (1995), and following his notation for ease of comparison, the working of the economy is summarized by the following equations:

$$y = y^c + \alpha(\pi - \pi^e) + \varepsilon \quad (1)$$

$$V = (y - y^*)^2 + \beta\pi^2 \quad (2)$$

where y^c, α, $\beta > 0$. Equation (1) shows that the economy possesses a Lucas supply function, so that the difference between output (y) and the natural level (y^c) depends on the deviations of inflation (π) from its expected value (π^e) and on a supply shock (ε) with zero mean and finite variance (σ_ε^2). Expectations are rational, that is, $\pi^e = E\{\pi\}$, where $E\,\{.\}$ is the expectations operator. Expression (2) represent the loss function of the society (or the government) which cares about deviations of output and inflation from some desired levels ($(y^*, 0)$ respectively). Besides, as it is standard in the literature on credibility in monetary policy, it is assumed that the output objective (y^*) is higher than the natural level, i.e., $y^* > y^c$, and define $k \equiv y^* - y^c > 0$. This discrepancy gives rise to the classical time inconsistency problem to discretionary monetary policy which causes an "inflation bias". More precisely, dating back to the articles of Kydland and Presscott (1977) and Barro and Gordon (1982) a conclusion of this literature is that a central banker with society's preferences is unable to implement the optimal monetary policy. That is, the resulting discretional (subgame perfect) equilibrium inflation is:

$$\pi^D = \frac{\alpha k}{\beta} - \frac{\alpha}{\beta + \alpha^2}\varepsilon \quad (3)$$

while the optimal monetary policy implies that inflation is:

$$\pi^O = -\frac{\alpha}{\beta + \alpha^2}\varepsilon \tag{4}$$

Therefore, subtracting (4) minus (3) we get that inflation in the discretional scenario exceeds the optimal rate by the amount $\frac{\alpha k}{\beta}$, i.e., the so-called inflation bias.

With the aim of dealing with this problem, Walsh (1995) proposed an institution that consist of the delegation of monetary policy to an independent central bank with the right incentives to implement the socially optimal outcome. In his model, the utility function of the central banker is:

$$U = (t_0 - t\pi) - V \tag{5}$$

where V is defined in (2).The government's chooses the parameters t_0 and t that shape the contract to be offered to the central banks so that the inflation bias is eliminated and the central bank accepts the proposal without earning any surplus in excess of it the reservation utility.

Implicit in the Walsh's setup is a multi-stage game that models the interactions between the government, the central bank and the private sector. The sequence of events can be described as follows:

1) The government offers the central bank a contract, namely, it chooses the values of t_o and t.

2) The private sector observes the incentive scheme and then forms its expectations on inflation (π^e).

3) The realization of the output shock (ε) becomes common knowledge.

4) The central bank sets the inflation rate (π)

Applying backward induction, in the last stage of the game, the central banker selects the value for π that solves the following program:

$$\begin{array}{ll} \underset{\{\pi\}}{Max} & (t_0 - t\pi) - \left[(y - y^*)^2 + \beta\pi^2\right] \\ s.t. & y = y^c + \alpha(\pi - \pi^e) + \varepsilon, \end{array}$$

The solution yields the following reaction function of the monetary authorities:

$$\pi = \frac{\alpha^2}{\alpha^2+\beta}\pi^e + \frac{\alpha}{\alpha^2+\beta}k - \frac{\alpha}{\alpha^2+\beta}\varepsilon - \frac{1}{2\left(\alpha^2+\beta\right)}t. \tag{6}$$

Anticipating the central bank's behavior, the private sector forms its rational expectations on inflation:

$$\pi^e = \frac{\alpha}{\beta}k - \frac{1}{2\beta}t. \tag{7}$$

Now, plugging this value for the expected inflation into equation (6) one obtains:

$$\pi = \frac{\alpha}{\beta}k - \frac{1}{2\beta}t - \left(\frac{\alpha}{\alpha^2+\beta}\right)\varepsilon. \tag{8}$$

Finally, Walsh's approach is equivalent to choosing, in the first stage in our game-theoretic setup, the parameters that shape the contract, i.e., t_0 and t, so that the following requirements are met:

(i) The inflation bias is eliminated. That is equating (8) and (4) (see Walsh (1995), pp.156-157) we have that:

$$t = 2\alpha k \tag{9}$$

(ii) The contract is accepted by the central banker who does not obtain any surplus in excess of the reservation payoff. Thus, t_0 is obtained so that $E\left[(t_0 - t\pi) - V\right] = 0$ (see Walsh, p.157).

3 An alternative approach

We address the issue studied in Walsh's (1995) but, by making use of the Kuhn-Tucker conditions, we show more explicitly that (the parameters of) the contract are obtained through a fully maximizing process of the government (principal) subject to a participation constraint of the central banker (agent).

Our focus is on the first stage. To solve it, first we need to express the expected objective functions of the government and the central bank in terms of the variables which define the contract, namely, t_0 and t. With this aim, first we substitute (1) into (2) and (5). Then, we plug the values for π^e and π (appearing in equations (7) and (8)) into the resulting two expressions for V, and U. After doing so, taking expectations yields:

$$E(V) = \frac{1}{4\beta}t^2 - \frac{\alpha k}{\beta}t + \left(\alpha^2 + \beta\right)\left(\frac{k^2}{\beta}\right) + \frac{\beta}{\alpha^2 + \beta}\sigma_\varepsilon^2, \tag{10}$$

$$E(U) = t_0 + \frac{1}{4\beta}t^2 - \left(\alpha^2 + \beta\right)\left(\frac{k^2}{\beta}\right) - \frac{\beta}{\alpha^2 + \beta}\sigma_\varepsilon^2. \tag{11}$$

In the initial stage, the principal chooses the value of its strategic variables, t_0 and t. It does so bearing in mind that the monetary authorities must accept the incentive scheme being offered. This "participation constraint" states that the expected utility obtained by the central bank when signing the contract must be higher or equal to a given reservation level, normalized to zero. Therefore, the government solves:

$$\underset{\{t_0,t\}}{Min} \quad E(V)$$
$$s.t. \quad E(U) \geq 0,$$

which results in the following Lagrangian function:

$$\pounds = E(V) + \mu E(U).$$

Because $E(V)$ is convex, $E(U)$ quasi-convex and both functions are continuously differentiable, the Kuhn-Tucker conditions are necessary and sufficient to solve the problem. These first-order conditions are:

$$\frac{\partial \pounds}{\partial t_0} = \frac{\partial \pounds}{\partial t} = 0, \tag{12}$$

$$\frac{\partial \pounds}{\partial \mu} \geq 0 \quad \text{and} \quad \left(\frac{\partial \pounds}{\partial \mu} = 0 \quad if \quad \mu \geq 0; \quad \frac{\partial \pounds}{\partial \mu} \geq 0 \quad if \quad \mu = 0\right). \tag{13}$$

Solving the two equations appearing in (12) for the Lagrangian multiplier, μ, and equating yields:

$$\mu = -\frac{\frac{\partial E(V)}{\partial t_0}}{\frac{\partial E(U)}{\partial t_0}} = -\frac{\frac{\partial E(V)}{\partial t}}{\frac{\partial E(U)}{\partial t}},$$

$$\mu = -\frac{0}{\frac{\partial E(U)}{\partial t_0}} = -\frac{\frac{\partial E(V)}{\partial t}}{\frac{\partial E(U)}{\partial t}}.$$

Therefore, we have that:

$$\mu = 0 \tag{14}$$

$$\frac{\partial E(V)}{\partial t} = 0 \tag{15}$$

Solving (15) for t yields:

$$t^* = 2\alpha k. \tag{16}$$

Therefore, the Walsh contract not only eliminates the inflation bias but also minimizes the government's loss. Besides, our approach helps address more explicitly the question of whether or not the participation constraint needs to be binding. In this respect, out of all the two possibilities contemplated in (13) , we are in the case where $\frac{\partial \pounds}{\partial \mu} \geq 0 \quad (\text{because } \mu = 0)$. That is, $E(U) \geq 0$. Therefore, the term t_0 is chosen to insure that $E[(t_0 - t\pi) - V] \geq 0$. More precisely, solving this inequation for t_0, one obtains the expression for the multiple equilibrium values of the fixed part of the contract:

$$t_0 \geq k^2 + \frac{\beta}{(\alpha^2 + \beta)} \sigma_\varepsilon^2. \tag{17}$$

In other words, we have confirmed that there exist an optimal contract where the participation constraint of the central bank is binding. Walsh (1995) refers to this no-surplus-rent-for-the-central-bank contract when he states "with the constant t_0 set to ensure $E[(t_0 - t\pi) - V] = 0$" (p. 157 first paragraph). However, our Kuhn-Tucker approach makes it explicit (see (17)) that there are more socially optimal contracts that have not been considered in Walsh (1995) which imply that the participation constraint is not binding.

4 Conclusions

Walsh (1995) was the first author to find a full solution to the problem of time inconsistency in monetary policy. He proposed a central bank contract that eliminates the inflation bias without incurring any output stabilization costs. In the paper by Walsh (1995), the central bank contract is not shown (explicitly) to be derived from a constrained optimization problem. Instead, it is obtained after imposing two conditions, namely, that the inflationary bias be eliminated and that it be accepted by the central banker without having any surplus. We provide an alternative method for deriving the components of such a monetary institution. Our approach applies the Kuhn-Tucker conditions in a multi-stage game so that it makes explicit that the contract proposed by Walsh (1995) is the outcome of a fully optimization process. Moreover, in this setting we show that there are more socially optimal contracts that have not been considered in Walsh (1995). All of them share the same variable part (the one that changes with inflation) but differ in the fixed component (the one that does not vary with inflation).

5 References

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