Monetary Policy Committees and DeGrootian Consensus

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Abstract
Reaching a decision on the policy interest-rate often requires members of a monetary policy committee (MPC) to form a consensus. To model this phenomenon, a formal deliberation mechanism that captures how committee members achieve consensus à la Morris DeGroot (1974) is employed. Numerical simulations demonstrate how DeGroot’s framework informs how so-called autocratically collegial, genuinely collegial, and individualistic MPCs (Blinder, 2007) reach agreement.
1 Introduction

Groups of individuals are responsible for making important decisions in an array of bodies and organizations in society; and, for many such groups, the significance of its members reaching a consensus on an issue cannot be overstated. In criminal trials, juries have the task of determining the guilt or innocence of a defendant. Crucially, the ability of the jury members to deliver a verdict hinges on their ability to reach a consensus, which may or may not emerge over the course of the deliberation process (Neilson and Winter, 2008). By way of analogy, in the same way that juries are a prominent feature of judicial systems across the world, so too are monetary policy committees (MPCs), albeit in the sphere of economic policy making.

This note applies Morris DeGroot’s (1974) model of group consensus formation to how MPCs reach interest-rate decisions. Given the role of consensus formation in real-world MPCs,\(^1\) this approach has potential relevance for the modelling of these decisions. To the best knowledge of the author, whilst there exists a growing literature on what drives MPC voting behavior,\(^2\) scant attention has been paid to the deliberation process, a notable exception being Maurin and Vidal (2012). These authors discuss the role of committee deliberation in a formal model grounded in Condorcet’s jury theorem, with a view to analyzing the relationship between the optimal committee size and different voting rules. Our model introduces a mechanism which captures how MPC members align their views through the deliberation process, which we propose informs how so-called autocratically collegial, genuinely collegial, and individualistic MPCs (Blinder, 2007) reach agreement. In terms of the typology of committees introduced above, an autocratically-collegial committee is characterized by the Chairman being a monetary policy ‘dictator’: policy decisions are effectively the Chairman’s choice. He may take a decision prior to the meeting, and merely notify his colleagues at its outset. Alternatively, he might take on board the views of other committee members during the meeting, then announce his decision and expect everyone to close ranks. The United States Federal Open Market Committee (FOMC) is classed as such a committee.\(^3\) Members of a genuinely-collegial committee, such as the European Central Bank Governing Council (GC), may openly disagree about the appropriate policy stance in the course of deliberations, but ultimately all members compromise on a decision.\(^4\) Finally, on an individualistic committee, members not only openly disagree about the most appropriate policy stance, but cast votes that reflect their position. Unanimous decisions are neither expected nor sought, as is the case of the Bank of England MPC.\(^5\)

\(^1\)Fry et al. (2000) find that most MPCs reach agreement by arriving at a consensus, without taking a formal vote. This finding does not, however, preclude the possibility that where a formal vote does take place, members reach a consensus prior to it being taken.

\(^2\)See Belden 1989; Havrilesky and Schweitzer 1990; Chappell et al. 2002; Gerlach-Kristen 2006; Riboni and Ruge-Murcia 2010.

\(^3\)Maisel (1973) argues that although the FOMC Chairman may be influenced by other committee members, any policy preferred by him is likely to be adopted, with very few dissents.

\(^4\)Consider statements made by ECB President Wim Duisenberg to questions fielded at routine ECB press conferences following monetary policy decisions made by the GC: “First, there was no formal vote. Again....it was a consensus decision.” (February 3rd 2000); “We had an intensive discussion, a prolonged discussion, which was very useful, and, in the end, resulted in a consensus on what we had to do” (June 8th 2000).

\(^5\)Harris et al. (2011) report that Bank of England MPC members dissent twice as often as FOMC
2 Model

Using DeGroot’s (1974) model of group consensus formation, we envisage an $m$ member MPC with interest-rate setting responsibility. At the start of a meeting, each MPC member weights the opinions of other members, including himself. Weighting allocations can be viewed as being an increasing function of career concerns and the perceived impact of other committee members on an individual’s future career path, the relative seniority of members, and the perceived expertise of other members. Let $p_{j,k}$ denote the weight placed on member $k$’s opinion by the $j^{th}$ member. For each member let $\sum_{k=1}^{m} p_{j,k} = 1$, where $0 \leq p_{j,k} \leq 1$ for all $j, k \in \{1, 2, ..., m\}$. This determines the elements of an $m \times m$ transition matrix where each row corresponds to respective members’ weight allocations

$$P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,m} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,m} \end{bmatrix}$$

(1)

Call the configuration of $P$ the influence structure of the committee. $P$ captures the extent to which all members of the committee are influenced by each other. Define $p_{j,k}$, the weight placed on member $k$’s opinion by the $j^{th}$ member, as the direct influence of $k$ on $j$. If $p_{j,k} = 0$, $k$ has no direct influence on $j$. However, even if $p_{j,k} = 0$, $k$ can still influence $j$: when member $j$ is not directly influenced by $k$, $k$ influences $j$ if $k$ influences a member $l$ who directly influences $j$. Call this indirect influence of $j$ on $k$. Moreover, $k$ exerts indirect influence on $j$ if $k$ influences a member $l$ who indirectly influences $j$ via another member, and so on. Further if a member $j$ neither directly nor indirectly influences member $k$, then that member is not influenced by $j$; and, if $j$ and $k$ do not influence each other, they do not communicate with each other. Finally, in the same way that $m$ individuals do not communicate with each other, if there are two groups within the committee, $J$ and $K$, where no member of either group communicates with each other, then group $J$ does not communicate with group $K$.\(^6\)

Corresponding to $P$ is a vector containing members’ interest-rate preferences, $I$, prior to the deliberation process.\(^7\) This vector contains the interest-rates members would choose were they given individual responsibility for monetary policy.\(^8\) Denote the transpose of this

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\(^6\)An excellent treatment of how members of a group exert direct and indirect influence on each other is also provided in Neilson and Winter (2008), who develop a sophisticated model of jury decision making with a basis in DeGroot’s (1974) framework.

\(^7\)Although not explored here, we note that DeGroot’s framework can be extended to encompass a model in which each MPC member is defined by a vector of characteristics, which in addition to his or her interest-rate preference, might include a member’s preferences for the rate of inflation and the unemployment level. I thank an anonymous referee for pointing this out.

\(^8\)Endogenizing the elements of $I$ and $P$ is the topic of ongoing work by the author: this entails linking the elements in $P$ to the degree of ‘uncertainty’ corresponding to MPC members’ optimal interest-rates. Such uncertainty in turn determines the willingness of members to be influenced by others, and determines the makeup of $P$. This paper presents preliminary findings based on the elements of $P$ being exogenous: we merely assume that MPC members have different preferred interest-rates due to their holding competing views of how the economy works, different levels of economic expertise, and weighting information differently when forming opinions. These are not an unrealistic set of assumptions. As Goodhart (1999) states, “What...is
vector as \( \mathbf{v}^{(0)} = [i_{1,0}, \ldots, i_{m,0}] \), where numbers given in subscripts \( M, N \) correspond to the respective interest-rate preferences for members \( M = \{1, \ldots, m\} \) and the stage of the deliberation process \( N = \{1, 2, \ldots, n\} \) which is also denoted in the square bracket \( \mathbf{v}^{(N)} \). Members’ revised views after the first period of deliberation are given by \( \mathbf{v}^{(1)} = \mathbf{P}\mathbf{v}^{(0)} \), the transpose of which is \( \mathbf{v}^{(1)\prime} = [i_{1,1}, \ldots, i_{m,1}] \). Consensus is reached by a discrete iterative process: following the first deliberative round, members’ original interest-rate preferences change from \( [i_{1,0}, \ldots, i_{m,0}] \) to \( [i_{1,1}, \ldots, i_{m,1}] \). If a majority of members’ revised rates have not converged to the same interest-rate in the first period, then the process of revision continues until it does. Revised opinions are calculated up to the \( n^{th} \) period as

\[
\mathbf{v}^{(1)} = \mathbf{P}\mathbf{v}^{(0)} = \mathbf{P}\mathbf{v}^{(1)} = \mathbf{P}^2\mathbf{v}^{(0)}; \ldots; \mathbf{v}^{(n)} = \mathbf{P}^n\mathbf{v}^{(0)}
\]

(2)

where \( \mathbf{P}^n \) is the matrix \( \mathbf{P} \) raised to the \( n^{th} \) power, \( n = 1, 2, \ldots, n \). We note here that under simple majority rule, it is only necessary for over half of MPC members to reach a consensus amongst themselves for a majority decision to be reached.\(^9\)

Figure 1 depicts possible influence structures using directed graphs, comprising nodes, which represent members of a committee, and directed edges, which have the appearance of arrows. \((i)\) is a special case of a one-member MPC where the policymaker listens only to himself; the unidirectional edge running from the node to itself implies that \( j \) directly influences himself, such that \( p_{jj} = 1. \((ii)\) depicts a two-member committee comprised of \( j \) and \( k \), who do not communicate with each other - both members only weight their own opinion, so \( p_{jk}, p_{kj} = 0 \) and \( p_{jj}, p_{kk} = 1 \). In \((iii)\) member \( j \) is directly influenced by \( k \) - as captured by the unidirectional edge running from \( j \) to \( k \) - whereas \( k \) chooses to listen to himself only. Therefore \( p_{jj}, p_{kk} = 0 \) and \( p_{jk}, p_{kj} = 1 \). In \((iv)\) both committee members listen to themselves and are directly influenced by each other. Therefore \( p_{jj}, p_{kk}, p_{jk}, p_{kj} \in (0, 1) \). \((v)\) depicts the nature of indirect influence in a three-member group, demonstrating that even with a small committee, the nature of influence between members may be complex.\(^10\)

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\(^9\) Using a similar iterative model, DeMarzo et al. (2003) argue that newspapers sway readers toward their views over time, even when the political affiliation of a newspaper is common knowledge. This tendency is termed persuasion bias. Their model represents a simple boundedly rational heuristic for dealing with a very complicated inference problem - that is, correctly adjusting for repetitions of information at each stage of the deliberation process. As the authors state, “Correctly adjusting for repetitions would require individuals to recount not only the source of all the information that has played a role in forming their beliefs, but also the source of the information that led to the beliefs of those they listen to, of those who they listen to listen to, and so on. This would become extremely complicated with just a few individuals and a few rounds of updating…” (p.911). Analogously, some members of a MPC may sway other members to their views, even when their views are well known. In this regard, the author is indebted to David Miles, member of the Bank of England MPC, for an illuminating discussion of MPC procedures, and in particular, insights into the way that MPC members’ opinions, prior to a vote being taken, are shaped in practice via the deliberation process at meetings.

\(^10\) The notions of direct and indirect influence can be mapped to well-known concepts in Markov chain theory (see for instance: Feller 1968; Karlin 1966; Kemeny and Snell 1960; Theil 1972). In Figure 1, \((iii)\), \( j \) is equivalent to a transient state, which is characterized by a steady-state probability of zero: once \( j \) reaches \( k \) it is impossible to get back. A transient set contains a group of states all of which have steady-state values of zero. A recurrent set contains a set of states such that once the system enters it, it always makes transitions within the set and never leaves it. This is depicted in \((v)\), such that \( l \) and \( k \) form a recurrent set.
3 Unanimous and Majority Consensus

A unanimous consensus (UC) is reached by all MPC members if all elements in the belief vector converge to the same value in the limit as \( n \to \infty \), such that \( \lim_{n \to \infty} i_{j,n} = i^* \) for all \( j = 1, 2, ..., m \). This occurs where there is a \( (1 \times m) \) row vector \( \pi = [\pi_1, ..., \pi_m] \) such that for \( j = (1, 2, ..., m) \) and \( l = (1, 2, ..., m) \), \( \lim_{n \to \infty} p_{j,l}^{(n)} = \pi_j \), where \( p_{j,l}^{(n)} \) is an element belonging to the transition matrix \( P^n \) from row \( j \) and column \( l \). UC is thus achieved when the elements of \( P^{n \to \infty} \) converge on a distribution characterized by \( m \) identical rows. We propose that UC is applicable to autocratically-collegial and genuinely-collegial committees, a deliberative outcome characterized by all members being in agreement with each other.

A majority consensus (MC) is reached if at least half of all elements in the belief vector converge to the same value as \( n \to \infty \). Formally, MC is achieved if \( \lim_{n \to \infty} i_{j,n} = i^* \) for all \( j \neq k, k = \{1, 2, ..., \frac{m-1}{2}\} \), where \( \lim_{n \to \infty} i_{k,n} = i^* \) for all \( k \). This will only emerge when there is a vector \( \pi_j = [\pi_1, ..., \pi_m] \) such that for \( j, l = (1, 2, ..., m) \), \( \lim_{n \to \infty} p_{j,l}^{(n)} = \pi_j \), for \( j, l \neq k \) where \( p_{j,l}^{(n)} \) is an element belonging to the transition matrix \( P^n \) from row \( j \) and column \( l \). In other words, for an \( m \) member committee, MC is achieved when the elements of \( P^{n \to \infty} \) converge on a distribution characterized by \( j \geq \frac{m+1}{2} \) identical rows. Unlike UC, all that is required is the presence of identical elements corresponding to a majority of columns.

The conditions under which UC and MC are achieved are now explored using graph theoretic representation. Figure 2 depicts various influence structures for a five-member MPC under alternative P-matrix parameterizations. In line with the institutional arrangements at a number of major central banks, we assume that there are two types of committee set - once \( j \) enters it, it is impossible to return. An absorbing state is a special case of a recurrent set that contains only one state. This is the case in \((iii)\), such that \( k \) is an absorbing state. If the entire system is a recurrent set, then it is called ergodic. \((iv)\) is therefore an ergodic system. If a system is not ergodic, then there may be more than one recurrent set in the system. \((v)\) is therefore not ergodic - it contains one recurrent set \((k \text{ and } l)\) and a transient state \((j)\), and is characteristic of an absorbing chain.

11 To keep the analysis simple, odd \( m \) is assumed.

12 Using five-member examples keeps the analysis sufficiently simple whilst capturing realistic MPC features.
Figure 2: Directed graphs corresponding to alternative weighting matrices for a five-member MPC
member, referred to here as insiders and outsiders. Further, the committee comprises three of the former (Governor, Insider 1, Insider 2) and two of the latter type (Outsider 1, Outsider 2), where the Governor assumes the role of the Chairman. As shown at the top of Figure 2, MPC members’ initial ideal interest-rates are captured by the belief vector $I^0$ and opinion weights in the influence matrix $P$ are allocated such that for instance, element $p_{24}$ captures the weight Insider 1 places on the opinion of Outsider 1, and so on. This parameterization of $I^0$ and the general configuration of $P$ applies to all numerical examples.

In (a), UC is achieved as all members listen to each other: this implies that all MPC members are strongly connected, a term which defines the case where every member is either directly or indirectly influenced by each other. This property also applies to example (d), and leads to the following Proposition:

**Proposition 1:** UC will be reached by a MPC if $P$ is irreducible and aperiodic. 

**Proof:** See Theil (1972), Chapter 5. This is a corollary of DeGroot’s Theorem 2.

$P$ is irreducible if and only if for every $(j, k)$ there exists a natural number $q$ such that $p_{j,k}^q \in (0, 1)$. If all elements in the influence matrix are positive for some power $q$, it has a unique long-run stationary distribution. Monetary policy committees whose members are strongly connected will thus necessarily achieve UC. We propose that such an influence structure may correspond to genuinely collegial monetary policy committees, such as the ECB Governing Council, recalling Wim Duisenberg’s earlier assertions (see footnote 4) about all members reaching a consensus amongst themselves.

In (b), the consequence of members not weighting each others’ views is depicted: here, each member listens only to himself. Assuming diverse initial interest-rate preferences, and because no MPC member is directly or indirectly influenced by any other member other than himself, a consensus will never be reached. This leads to the following proposition:

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13We use the terminology of Gerlach-Kristen (2009) and Harris and Spencer (2009), where insiders and outsiders refer, respectively, to MPC members appointed from within the ranks of Bank of England staff, and outside such ranks (academia, the financial sector, and so on). The empirical literature finds that different member types are associated with markedly different voting patterns. Depending on the parameterization of $P$, one might alternatively consider the distinction made above as corresponding to members of the Board of Governors of the Federal Reserve System and Federal Reserve Bank Presidents who vote at regular FOMC meetings; or, the National Central Bank Governors and Executive Board members who sit on the ECB’s Governing Council.

14As is the case with the Bank of England, we suppose that insiders comprise the majority of members.

15It is also possible for matrix $P$ to be simultaneously irreducible and periodic (and hence not ergodic). However, periodicity is intuitively unappealing: it implies that the deliberation process is cyclical, leading members’ preferred rates to exhibit considerable amplitude over the deliberation process. This possibility is therefore not considered. I thank Parimal Bag for discussions pertaining to this issue.

16Even if $P$ initially contains some zero elements as in (d), all members are sufficiently connected such that when $P$ is raised to some power $q$, all of the elements in the listening matrix become strictly positive but less than unity. It is crucial to note here that no element $p_{j,k}^q = 1$, which would imply periodicity or an absorbing class. Example (d) demonstrates that if two members – one from each group - both communicate with any member of the the other group, UC is reached. This is because $P$ is both aperiodic and irreducible.

17I am grateful to Marco Catenaro and Nick Vidalis at the European Central Bank for helpful discussions relating to how decisions are reached by the ECB Governing Council.
Proposition 2: If no MPC member is directly or indirectly influenced by any other member, UC or MC will never be reached.

Proof: If no MPC member is directly or indirectly influenced by any other member $P$ reduces to an identity matrix, which implies that $P^n = P \quad \forall \quad n = 1, 2, \ldots, n$. $P$ remains unchanged irrespective of the stage of the deliberation process, and thus no consensus is achieved. ■

It is notable that the group would achieve MC or UC if a majority of or all respective committee members were endowed with identical initial interest-rate preferences. In such a case, no deliberation would even be required for an agreement to be reached.\(^{18}\)

In (c), four members weight only the opinion of a single member - in this case the Governor - in addition to their own opinions. As the Governor only weights his own opinion ($p_{11} = 1$), the interest-rate, over the course of the deliberation process, converges to his preferred rate. A UC is thus reached. This is an example of an absorbing chain. We thus introduce the following proposition:

Proposition 3: UC will be reached by the committee if any member $j$ is influenced only by himself, and influences all other members, either directly or indirectly. Members’ beliefs will necessarily converge to those of member $j$.

Proof: See Kemeny and Snell (1960), Theorem 1.11.1.

We propose that such a member $j$ is akin to monetary policy dictator of the type assumed to yield influence in an autocratically-collegial MPC, such as the FOMC. Here, all members weight the opinion of the Governor, but he does not reciprocate, opting to weight his own opinion only.

Finally, (e) and (f) depict possible states of affairs for an individualistic MPC, drawing on the notion of MC. Both examples are geared towards the institutional nuances of the Bank of England MPC, and reproduce the stylized facts of voting behavior associated with its members. Here, we note that if insiders and outsiders listen only to members of their own type (thereby assigning zero weight to the views of those members not of their type) insiders will be neither directly nor indirectly influenced by outsiders, and vice versa. As such, in the presence of initially diverse interest-rates, insiders and outsiders will each form a separate aperiodic recurrent class and reach a consensus amongst themselves only. This result is formally shown in the Appendix, and builds on Degroot’s Example 3 and his Section 6.1. In both (e) and (f) our highly stylized assumption is that outsiders tend to prefer lower interest-rates than insiders. This assumption is reinforced by empirical studies which have shown that: (i) insiders on average vote for higher interest-rates than outsiders (Gerlach-Kristen 2003, 2009); (ii) are more likely to vote as a “cohesive homogeneous group” (Edmonds 1999, p.12); and (iii) dominate decisions due to their institutional majority over outsiders (Harris and Spencer 2009). With respect to points (ii) and (iii), and through the lens of the model employed here, one explanation for insiders achieving MC is because they restrict themselves

\(^{18}\)This type of scenario is formalized by Berger (1976), who demonstrates that by failing to take into account this possibility, DeGroot (1974) provides sufficient but not necessary conditions for consensus to be reached. Like DeGroot, this paper restricts itself to situations where no two individuals are in agreement prior to the deliberation process. This is reflected in our simulations, which focus only on MPC members having initially diverse interest-rate preferences.
to only weighting the opinions of members of their own type; this results in a policy outcome characterized by an interest-rate higher than what would be chosen by outsiders.\textsuperscript{19}

Examples (e) and (f) also illustrate the importance of \textit{listening to others} in the course of MPC deliberations, and its relation to the \textit{speed} at which consensus will be reached.\textsuperscript{20} In any $P$ matrix which is irreducible and aperiodic (therefore entailing that $UC$ is reachable) the absolute value of the largest eigenvalue will equal unity, with the corresponding moduli of all other eigenvalues being smaller than one. The \textit{rate} at which convergence is achieved will be related to the \textit{second largest eigenvalue in absolute value}. Define this as $\delta(P) = \max\{|\lambda| : \lambda \in \sigma(P), \lambda \neq 1\}$ where $\sigma(P)$ is the set of all eigenvalues corresponding to $P$.\textsuperscript{21} However, the presence of more than one eigenvalue of modulus 1 does not necessarily imply that a consensus of some form has not been reached. For instance, in the examples of MPC shown in (e) and (f), there are two eigenvalues with moduli equal to 1, with each eigenvalue of unity corresponding to each group of members: moreover, for each diagram, the corresponding $P$ matrix is characterized by two disjoint communicating classes. Parameter values have been chosen such that in both examples, insiders and outsiders \textit{still} each converge on the same interest-rate, albeit it takes longer for members in (f) to arrive at a consensus than (e). This would imply that $\delta(P)_{(e)} < \delta(P)_{(f)}$ for both groups of cohorts. In the context of these examples, this is attributable to the fact that whilst the symmetry of $P$ has been maintained, members of committee (f) weight their own opinions more heavily than members of committee (e).\textsuperscript{22}

4 Conclusion

This paper has sought to account for the MPC deliberation process by demonstrating how members’ views align when interest-rate preferences are initially diverse. We utilize Morris DeGroot’s (1974) iterative framework, which is exploited to show how and why policy decisions associated with different MPC types are reached. A number of conclusions emerge from our analysis. First, it is possible to populate MPCs with people who hold very different views about the ideal interest-rate and \textit{still} reach an agreement. However, the type of consensus achieved will be a function of how members weight each others’ opinions, which is determined by the parameterization of the $P$ matrix: specifically, what we suggest is that different weighting allocations in $P$ may be viewed as corresponding to consensus outcomes associated with autocratically collegial, genuinely collegial, and individualistic MPCs. By explicitly modeling the deliberation process, DeGroot’s framework thus serves as a fruitful

\textsuperscript{19}In the presence of career concerns, it is not implausible that insiders may weight the opinions of fellow insiders more heavily than outsiders, as they may have a significant bearing on their future career path at the Bank of England. The views of outsiders may even be discounted altogether.

\textsuperscript{20}DeGroot (1974) does not consider the speed of convergence to a consensus. This neglected aspect of DeGroot’s paper may be important, insofar as monetary policy committees are time-constrained, and only have a limited amount of time to reach a decision.

\textsuperscript{21}For some $P$ matrix parameterizations some roots may be complex. If the second-largest eigenvalue $\lambda_2$ is complex, convergence towards the stationary distribution will be of the \textit{damped oscillatory} type. As we rule out periodicity, this possibility is not explored further.

\textsuperscript{22}In this sense, (e) can be viewed as comprising a committee of relative ‘pragmatists’, whereas (f) comprises a committee of ‘egoists’.

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starting point for analyzing MPC decision making. In relation to this assertion, whilst a diversity of views are considered desirable on MPCs (Bernanke 2007), unless there are mechanisms by which members can align their views over the course of the deliberation process, UC or even MC may not be achievable. The model applied in this paper represents one such mechanism.

Appendix

In this section we formally demonstrate that if there exists two distinct groups of members within a MPC who do not communicate with each other, and each group forms an aperiodic recurrent class, then for an \( m \) member committee, a MC will be reached by the group with the largest number of members. To prove this, we draw on Proposition 1. For an \( m \) member committee, begin by introducing the block matrix

\[
P = \begin{bmatrix}
\Upsilon & 0 \\
0 & \Delta
\end{bmatrix}
\]

(3)

where \( \Upsilon \) and \( \Delta \) comprise two disjoint communicating classes. Specifically, let \( \Upsilon \) denote a \( (m - s) \times (m - s) \) bloc of opinion weights for insiders and \( \Delta \) denote a corresponding \( s \times s \) bloc for outsiders. In the limit it necessarily holds that

\[
\lim_{n \to \infty} P^n = \begin{bmatrix}
\Upsilon^n & 0 \\
0 & \Delta^n
\end{bmatrix}
\]

(4)

Given (4) it follows that each bloc can be treated as a matrix in its own right (as the elements of \( \Upsilon \) and \( \Delta \) do not influence each other). Because both matrices each comprise a single aperiodic recurrent class (i.e. all members within each group are strongly connected) the results for Proposition 1 apply to the \( (m - s) \times (m - s) \) matrix \( \Upsilon \) and the \( s \times s \) matrix \( \Delta \). Thus the limiting distribution of \( P \) takes the form

\[
\lim_{n \to \infty} P^n = \begin{bmatrix}
\pi_1 & \cdots & \pi_{m-s} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\pi_1 & \cdots & \pi_{m-s} & 0 & \cdots & 0 \\
0 & \cdots & 0 & \pi_{m-s+1} & \cdots & \pi_m \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \pi_{m-s+1} & \cdots & \pi_m
\end{bmatrix} = \begin{bmatrix}
\pi'_{\text{Insiders}} \\
\vdots \\
\pi'_{\text{Insiders}} \\
\pi'_{\text{Outsiders}} \\
\vdots \\
\pi'_{\text{Outsiders}}
\end{bmatrix}
\]

(5)

The first \( (m - s) \) rows will converge to a stationary distribution characterized by the first \( (m - s) \) columns containing strictly positive elements and the latter \( s \) columns comprising zeros. The last \( s \) rows will converge to a limit characterized by the elements in the first \( (m - s) \) columns comprising zeros, with the remaining \( s \) columns containing strictly positive elements. Assuming \( (m - s) > s \) implies that insiders will reach a MC. \( \blacksquare \)
References


