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A note on quasi-hyperbolic discounting, risk aversion, and the demand for insurance

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We show that quasi-hyperbolic discounting not only affects savings decisions but also the demand for insurance. In general, insurance demand by quasi-hyperbolic discounters is time-inconsistent. Without liquidity constraints they buy more insurance than initially planned. In the presence of binding liquidity constraints, hyperbolic discounters tend to revise their insurance demand downwards.

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1 Introduction

When making intertemporal decisions, individuals often suffer from self-control problems. For example, individuals make plans to save in the future but when they have to implement these plans, they rather prefer to consume more of their income instead of putting it aside. This behavior can be modeled with hyperbolic discount functions (see Frederick *et al.*, 2002 for an overview). Hyperbolic discounting agents have a higher discount rate over short horizons than over long horizons (Strotz, 1956). Up to now, the main focus of the literature was on analyzing hyperbolic discounting and its implications for saving behavior (Laibson 1997; 1998).

Insurance decisions also imply a cost now and yield a benefit later in life. Hyperbolic discounting can therefore be expected to change insurance demand as well. As this paper demonstrates, hyperbolic discounting does not necessarily reduce the demand for insurance. The interplay between present bias and risk aversion needs to be considered. Another important factor is whether liquidity constraints are binding or not. Importantly, insurance demand by hyperbolic discounters is time-inconsistent. The direction of the time inconsistency depends crucially on the extent of the present bias.

In this paper, we focus on naive consumers who do not understand their self-control problem and act as if they are not exposed to a present bias in the future. In the literature, also sophisticated consumers are considered. These foresee their time inconsistency and undertake steps to manage it. As O'Donoghue and Rabin (1999) state, most individuals seem to lack sophistication as they consistently do not have the willpower to stick to their prior intentions, *e.g.*, to quit smoking or to start exercising, but they predict that tomorrow they will have this willpower. Noting the high demands on rationality required by sophisticated behavior, we concentrate on naive individuals. However, we also consider possible commitment devices which could help individuals to overcome their time-inconsistent behavior.

2 The Model Economy

Individuals live for three periods, $t = 1, 2, 3$. In each period they receive a net income y_t. Individuals are liquidity constrained implying that savings s_t can not be negative, *i.e.*, $s_t \geq 0$. In period 3, individuals suffer a monetary loss L (*e.g.*, expenses for health care services) with probability π.[1] In the first two periods, individuals decide how much insurance coverage I they want to buy. The price per unit of coverage is $(1+\lambda)\pi$ where λ is the loading factor. If $\lambda = 0$ insurance is actuarially fair. More realistic is a premium that exhibits a mark-up above expected benefits implying $\lambda > 0$.[2]

In each period individuals derive utility $u(c)$ from consumption ($u'(c) > 0, u''(c) < 0$). Following Laibson (1997) we assume quasi-hyperbolic discounting agents (QHDs) with (β, δ)-preferences. The traditional discount factor is $\delta \in (0, 1]$ whereas $\beta \in (0, 1]$ is called the *present-bias factor*. Expected utility in period 1 and 2 is

$$EU_1 = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 \left[\pi u(c_{3b}) + (1-\pi)u(c_{3g})\right], \tag{1}$$

$$EU_2 = u(c_2) + \beta\delta \left[\pi u(c_{3b}) + (1-\pi)u(c_{3g})\right] \tag{2}$$

[1] We do not consider losses in period 2 for which insurance would need to be purchased in period 1 because no self-control problem arises in this case.

[2] Kunreuther *et al.* (2013) regard a load of thirty to forty percent as typical in insurance markets.

where c_1, c_2, c_{3b} and c_{3g} denote consumption at ages 1, 2 and 3. The bad and good state of nature are indicated with the subscript 'b' and 'g' respectively. For $\beta = 1$ the standard exponential discounting model with time-consistent preferences arises. For $\beta \in (0, 1)$ the discount factor between period 2 and period 3 is $\beta\delta$ if it is computed at period 2 while it is δ if it is computed at period 1 implying that preferences are time-inconsistent. Naive individuals make their choices and plans in period 1 under the false belief that they will implement these plans in period 2.

3 Savings and Insurance Decisions

3.1 Decisions in Period 1

In period 1, QHDs maximize

$$\begin{aligned} \max_{c_1^1, c_2^1, c_{3g}^1, c_{3b}^1, s_1^1, s_2^1, I_2^1} \quad & EU_1 = u(c_1^1) + \beta\delta u(c_2^1) + \beta\delta^2 \left[\pi u(c_{3b}^1) + (1-\pi)u(c_{3g}^1)\right] \\ \text{s.t.} \quad & c_1^1 = y_1 - s_1^1, \quad s_1^1 \geq 0, \\ & c_2^1 = y_2 + s_1^1 - (1+\lambda)\pi I_2^1 - s_2^1, \quad I_2^1, s_2^1 \geq 0, \\ & c_{3g}^1 = y_3 + s_2^1, \quad c_{3b}^1 = y_3 + s_2^1 - L + I_2^1. \end{aligned} \tag{3}$$

The superscript indicates the age in which the variable is chosen and the subscript indicates the time. Whenever the superscript coincides with the subscript, the variable is actually implemented. Note that buying insurance in period 2 weakly dominates buying insurance in period 1 since the individual may be liquidity constrained. Without loss of generality, we therefore set $I_1^1 = 0$. We concentrate on interior solutions with respect to I_2^1; a necessary condition is $(1+\lambda)\pi < 1$.[3]

Maximization of (3) leads to the following first-order conditions (FOCs)

$$\frac{\partial EU_1}{\partial s_1^1} = -u'(c_1^1) + \beta\delta u'(c_2^1) \leq 0, \quad s_1^1 \geq 0, \tag{4}$$

$$\frac{\partial EU_1}{\partial s_2^1} = -\beta\delta u'(c_2^1) + \beta\delta^2 \left[\pi u'(c_{3b}^1) + (1-\pi)u'(c_{3g}^1)\right] \leq 0, \quad s_2^1 \geq 0, \tag{5}$$

$$\frac{\partial EU_1}{\partial I_2^1} = -\beta\delta u'(c_2^1)(1+\lambda)\pi + \beta\delta^2\pi u'(c_{3b}^1) = 0. \tag{6}$$

Equations (4)–(6) yield implemented savings in period 1 s_1^1, planned savings s_2^1 and insurance coverage I_2^1 for period 2. In part 1 of the Appendix, we examine the solution to problem (3) and its comparative statics in detail. One result is that individuals who have a lower present bias (a higher β), save more. These individuals are less likely to be liquidity constrained. We discuss this case first.

3.1.1 Non-Binding Liquidity Constraints

If liquidity constraints are not binding, comparative statics of the above FOCs with respect to β show:

[3] When $(1+\lambda)\pi \geq 1$, the return of savings is higher than of insurance implying that private insurance is dominated by savings.

Proposition 1 *If individuals are not liquidity-constrained in both periods, those with a higher present-bias (a lower β) save less in period 1 and plan to save less in period 2. For actuarially unfair insurance premiums, their planned insurance demand is higher, equal, lower if preferences reflect decreasing, constant, increasing absolute risk aversion.*

The result of lower savings is well-known in the literature (Laibson, 1997; Diamond and Köszegi, 2003). The higher preference for the present induces individuals to consume more now and, hence, save less for the future. However, our analysis also reveals that planned insurance demand differs unless constant absolute risk aversion (CARA) is present or $\lambda = 0$. This result may come as a surprise. As insurance implies a cost now but yields a benefit later in life, one might be tempted to conclude that more present-biased agents demand less insurance coverage independent of their risk preferences. Section 3.2 reveals that this intuition is true only for a binding liquidity constraint in period 2, that is, when agents are equal wealthy in period 3. If no liquidity constraints are binding, more present-biased agents save less in period 1 which makes them less wealthy later in life. Empirical evidence supports decreasing absolute risk aversion (DARA) preferences (Meyer and Meyer, 2006). Then, being less wealthy goes hand in hand with more insurance demand.[4]

3.1.2 Binding Liquidity Constraints

A high present bias (low β) but also an increasing earnings profile or mandatory savings can cause individuals to be liquidity constrained. First, we consider the effects of a binding liquidity constraint in period 1. Savings and insurance demand in period 2 are then determined solely by equations (5) and (6). Denoting $A(c) = -u''(c)/u'(c)$ the Arrow-Pratt coefficient of absolute risk aversion, comparative statics wrt s_1^1 show that

$$\frac{\partial s_2^1}{\partial s_1^1} = \frac{\delta^3 \pi (1-(1+\lambda)\pi) u''(c_2^1) u''(c_{3b}^1)}{|\mathcal{H}_{LC1}|} > 0 \tag{7}$$

$$\frac{\partial I_2^1}{\partial s_1^1} = \frac{\delta^3 \pi (1-(1+\lambda)\pi) u''(c_2^1) u'(c_{3b}^1) \left[A(c_{3b}^1) - A(c_{3g}^1)\right]}{|\mathcal{H}_{LC1}|} \begin{cases} > 0 & \text{IARA} \\ = 0 & \text{CARA} \\ < 0 & \text{DARA} \end{cases} \tag{8}$$

as $|\mathcal{H}_{LC1}| > 0$ and $c_{3g}^2 > c_{3b}^2$ if $\lambda > 0$.[5] By ruling out negative savings, the binding liquidity constraint effectively forces individuals to save more than they want to and thus exogenously increases s_1^1. They then have more resources available for periods 2 and 3. A share of these resources is transferred to period 3 which explains why planned savings in period 2 increase (Eq. (7)), implying a higher level of wealth in period 3. Insurance demand then changes according to how absolute risk aversion depends on wealth (Eq. (8)). For DARA preferences, individuals cut back their planned insurance demand as their higher wealth makes them less risk averse.

When the liquidity constraint is binding in period 1, planned savings and insurance demand are independent of β; in equations (5) and (6) β cancels out. A binding liquidity

[4]This result can be contrasted with the effect of an independent background risk with zero mean in period 3. In this case DARA is not sufficient to yield an increase in insurance demand (Schlesinger 2000). The reduction of wealth due to the present bias has therefore a different effect than a wealth shock in the final period.

[5]The latter simply states that individuals are less than fully insured when insurance premiums comprise a positive loading.

constraint in period 1 implies that individuals are equally wealthy in period 2. And, when it comes to decisions further in the future, all individuals are equally patient. This result holds *a fortiori* if they are additionally liquidity constrained in period 2.

When agents only face a binding liquidity constraint in period 2, insurance demand in period 2 is determined by equations (4) and (6). I_2^1 is lower for those who are more present biased

$$\frac{\partial I_2^1}{\partial \beta} = \frac{-\beta\delta^2(1+\lambda)\pi u'(c_2^1)u''(c_2^1)}{|\mathcal{H}_{LC2}|} > 0, \tag{9}$$

since $|\mathcal{H}_{LC2}| > 0$. Individuals with a lower β save less in period 1 implying that they are less wealthy in period 2. A binding liquidity constraint in period 2 hits these agents stronger. To uphold period-2 consumption they cut back more on their insurance expenses.

Proposition 2 *Whenever the liquidity constraint in period 1 is binding, planned savings and insurance coverage are independent of β. If only the liquidity constraint in period 2 is binding, individuals with a lower β plan to buy less insurance coverage.*

Propositions 1 and 2 compare two individuals with different values of β as long as the same liquidity constraints are (not) binding. If both values of β are high enough such that no liquidity constraint is binding, the individual with the lower β will plan to buy more insurance given DARA. If both values of β are low enough such that the liquidity constraint in period 1 or 2 is binding, individuals will have the same planned demand or the individual with the lower β will plan to buy less insurance. For the remaining case in which the individual with the higher β is not liquidity constrained while the other individual is constrained, the one with the lower β will plan to buy more *or* less insurance. From equation (9), we can infer that the second case is more likely the lower the β of this individual.

3.2 Decisions in Period 2

In period 2, QHDs maximize

$$\begin{aligned} \max_{c_2^2, c_{3g}^2, c_{3b}^2, s_2^2, I_2^2} \quad & EU_2 = u(c_2^2) + \beta\delta\left[\pi u(c_{3b}^2) + (1-\pi)u(c_{3g}^2)\right] \\ \text{s.t} \quad & c_2^2 = y_2 + s_1^1 - (1+\lambda)\pi I_2^2 - s_2^2, \quad s_2^2, I_2^2 \geq 0, \\ & c_{3g}^2 = y_3 + s_2^2, \quad c_{3b}^2 = y_3 + s_2^2 - L + I_2^2. \end{aligned} \tag{10}$$

which leads to the following FOCs

$$\frac{\partial EU_2}{\partial s_2^2} = -u'(c_2^2) + \beta\delta\left[\pi u'(c_{3b}^2) + (1-\pi)u'(c_{3g}^2)\right] \leq 0, \quad s_2^2 \geq 0, \tag{11}$$

$$\frac{\partial EU_2}{\partial I_2^2} = -u'(c_2^2)(1+\lambda)\pi + \beta\delta\pi u'(c_{3b}^2) = 0. \tag{12}$$

Equations (11) and (12) determine actually implemented savings s_2^2 and insurance coverage I_2^2 in period 2. In part 2 of the Appendix, we analyze the solution to problem (10) in detail. Again, individuals who have a lower present bias (a higher β), save more and are less likely to be liquidity constrained.

3.2.1 Non-Binding Liquidity Constraint

Comparative statics wrt β show that

$$\frac{\partial s_2^2}{\partial \beta} = -\frac{\delta\pi(1-(1+\lambda)\pi)u''(c_{3b}^2)u'(c_2^2)}{|\mathcal{H}_{P2}|} > 0, \tag{13}$$

$$\frac{\partial I_2^2}{\partial \beta} = \frac{\delta(1+\lambda)\pi(1-\pi)u'(c_2^2)u'(c_{3g}^2)[A(c_{3g}^2)-A(c_{3b}^2)]}{|\mathcal{H}_{P2}|} \begin{cases} >0 & \text{IARA} \\ =0 & \text{CARA} \\ <0 & \text{DARA} \end{cases} \tag{14}$$

as $|\mathcal{H}_{P2}| > 0$ and $c_{3g}^2 > c_{3b}^2$ if $\lambda > 0$. That is, agents with a lower β demand more insurance if preferences reflect DARA. This result corresponds to Proposition 1, which analyzed planned insurance demand in period 1. It holds for *given* first-period savings. From Proposition 1 we know that individuals with a lower β also save less which additionally increases their insurance demand as

$$\frac{\partial I_2^2}{\partial s_1^1} = \frac{\beta\delta\pi(1-(1+\lambda)\pi)u''(c_2^2)u'(c_{3b})[A(c_{3b}^2)-A(c_{3g}^2)]}{|\mathcal{H}_{P2}|} \begin{cases} >0 & \text{IARA} \\ =0 & \text{CARA} \\ <0 & \text{DARA} \end{cases} \tag{15}$$

The excessive consumption in period 1 and 2 thus causes a reduction in wealth in period 3 which makes individuals more risk averse and induces them to demand more insurance.

Proposition 3 *If insurance is unfair, individuals with a higher present bias who save in period 2 choose a higher, equal, lower long-term care insurance coverage if preferences are characterized by DARA, CARA, IARA.*

3.2.2 Binding Liquidity Constraint

When the liquidity constraint is binding in period 2, optimal insurance demand is determined by equation (12). The implicit function theorem yields

$$\frac{\partial I_2^2}{\partial s_2^2} = -\frac{(1+\lambda)u''(c_2^2)+\beta\delta u''(c_{3b}^2)}{(1+\lambda)^2\pi^2 u''(c_2^2)+\beta\delta u''(c_{3b}^2)} < 0. \tag{16}$$

If individuals want to borrow against their future income, but the liquidity constraint prevents them from doing so, they reduce their insurance demand to uphold their consumption in period 2. The insurance demand on the private insurance market will thus be smaller than in the absence of a liquidity constraint.

Taking the derivative of (12) with respect to β yields

$$\frac{\partial I_2^2}{\partial \beta} = -\frac{\delta u'(c_{3b}^2)}{u''(c_2^2)(1+\lambda)^2\pi+\beta\delta u''(c_{3b}^2)} > 0. \tag{17}$$

Holding first-period savings constant, individuals with a lower β demand less insurance. To satisfy their higher consumption needs, they reduce more their insurance coverage.

Again, their lower period-1 savings only supplement the negative effect on insurance

$$\frac{\partial I_2^2}{\partial s_1^1} = \frac{u''(c_2^2)(1+\lambda)}{u''(c_2^2)(1+\lambda)^2\pi + \beta\delta u''(c_{3b}^2)} > 0. \tag{18}$$

This result is in contrast to the outcome with a non-binding liquidity constraint where individuals with a lower β demand more insurance (for DARA preferences).

Proposition 4 *If agents are liquidity constrained in period 2, those with a higher present bias demand less insurance coverage independent of their risk preferences.*

Again, we can compare two individuals with different values of β. Propositions 3 and 4 give opposite results depending on whether the liquidity constraint in period 2 is binding or not. As above, the demand for insurance may be larger or smaller for the individual with the lower β if only this individual is liquidity constrained. According to equation (17), the second case is more likely the lower the β of this individual.

3.3 Time-Inconsistent Behavior

A well-known result is that QHD's savings behavior is time inconsistent. In the following, we show that this also holds true for the insurance decision, both with and without binding liquidity constraints.

3.3.1 Non-Binding Liquidity Constraint

To analyze how the initially planned variables s_2^1 and I_2^1 differ from the actual implemented variables s_2^2 and I_2^2, we simply compare the FOCs (5) and (6) of period 1 with those of period 2, equations (11) and (12). For given first-period savings, the latter differ from the former solely by the additional discount factor β. In period 2, QHDs discount period 3 utility with $\beta\delta$ instead of δ as in period 1. To determine how planned savings and insurance demand differ from their implementation in period 2, we thus have to determine the sign of $\partial s_2^2/\partial\beta$ and $\partial I_2^2/\partial\beta$ which we already derived in equations (13) and (14): $\partial s_2^2/\partial\beta > 0$ and $\partial I_2^2/\partial\beta < 0$ for DARA. In period 2 QHDs value immediate consumption more than from the perspective of period 1 and thus save less. This reduction in period-2 savings also affects the demand for insurance coverage which again depends on risk preferences. As agents are less wealthy due to their downward revision in savings, they revise their insurance demand upwards if preferences are characterized by DARA.

Proposition 5 *Quasi-hyperbolic discounters who are not liquidity constrained in period 2 save less than initially planned. They choose a higher, equal, lower long-term care insurance coverage than planned in period 1 if preferences are characterized by DARA, CARA, IARA.*

3.3.2 Binding Liquidity Constraint

When individuals in period 1 anticipate the binding liquidity constraint, I_2^1 is solely characterized by (6). Comparing equation (6) with (12), again shows that for given period-1 savings the difference between these two equations lies in the additional discount factor β in condition (12). From equation (17) we know that $\partial I_2^2/\partial\beta > 0$. That is, QHDs buy less insurance than initially planned. In period 2, they want to consume more than from the perspective of period 1. If they are not allowed to borrow against their future income the only way to satisfy their increased consumption needs is by a downward revision in insurance coverage.

Proposition 6 *If quasi-hyperbolic discounters are liquidity constrained in period 2 and anticipate the liquidity constraint in period 1, they buy less insurance coverage in period 2 than actually planned in period 1.*

If the liquidity constraint is not anticipated in period 1, it is *a priori* not clear whether I_2^1 is smaller or larger than I_2^2. From the perspective of period 1, savings prior to period 3 are then positive. This, in turn, increases planned insurance coverage compared to the case where the individual is aware of his future liquidity constraint; equation (16). It may thus well be that $I_2^1 > I_2^2$.

Overall, the direction of the time inconsistency depends crucially on the extent of the present bias. With a small present bias (high β), the individual is unlikely to be liquidity constrained and we find an upward revision of insurance demand for DARA preferences (Prop. 5). For low values of β, Proposition 6 predicts that planned insurance demand is reduced. For intermediary cases, the direction of the time inconsistency depends on β.

3.4 Commitment Devices

Sophisticated individuals understand their present bias and have an interest in commitment devices. To avoid their time-inconsistent savings behavior, these individuals would join a savings plan in period 1 that forces them to save in period 2 the amount they consider to be optimal from the perspective of period 1, that is s_1^1 and s_2^1. For such a savings plan to act as a perfect commitment device, latter (downward) adjustments must be sufficiently expensive. However, commitment solely to a savings plan is not sufficient. Comparing the first-order conditions (6) and (12) for fixed savings implies $I_2^2 < I_2^1$, i.e. insurance coverage will be too low in period 2 from the perspective of period 1. With "enforced" higher savings, only a reduction in insurance coverage opens up the possibility to increase consumption in period 2. Commitment to an insurance plan in period 1 is thus also necessary. Again, such a plan would need to make downward adjustments of insurance coverage in period 2 sufficiently costly to act as a perfect commitment device.

4 Conclusion

We introduced the demand for insurance in the framework of quasi-hyperbolic discounting. For DARA preferences individuals with a higher present bias demand more insurance when liquidity constraints are not binding. For a binding liquidity constraint their insurance demand is lower. Additionally, we found that QHDs' insurance decisions are time-inconsistent. QHDs with DARA preferences save less than initially planned, and revise their insurance demand upwards. In the presence of a binding liquidity constraint, however, their insurance demand tends to be lower than intended in early lifetime. The latter finding can have potential policy implications. Quasi-hyperbolic discounting not only provides a rationale for public policy measures to increase savings (Laibson, 1998 and İmrohoroğlu *et al.* 2003) but also to increase insurance coverage.

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A Appendix

A.1 Decisions in Period 1

A.1.1 Non-Binding Liquidity Constraints

Total differentiation of the FOCs (4) to (6) yields

$$u''(c_1^1)\mathrm{d}s_1^1 + \beta\delta u''(c_2^1)\mathrm{d}s_1^1 - \beta\delta u''(c_2^1)\mathrm{d}s_2^1 - \beta\delta(1+\lambda)\pi u''(c_2^1)\mathrm{d}I_2^1 = \triangle^{s1}$$
$$-\beta\delta u''(c_2^1)\mathrm{d}s_1^1 + \beta\delta u''(c_2^1)\mathrm{d}s_2^1 + \beta\delta^2[\pi u''(c_{3b}) + (1-\pi)u''(c_{3g}^1)]\mathrm{d}s_2^1$$
$$+\beta\delta(1+\lambda)\pi u''(c_2^1)\mathrm{d}I_2^1 + \beta\delta^2\pi u''(c_{3b}^1)\mathrm{d}I_2^1 = \triangle^{s2}$$
$$-\beta\delta(1+\lambda)\pi u''(c_2^1)\mathrm{d}s_1^1 + \beta\delta(1+\lambda)\pi u''(c_2^1)\mathrm{d}s_2^1 + \beta\delta^2\pi u''(c_{3b}^1)\mathrm{d}s_2^1$$
$$+\beta\delta(1+\lambda)^2\pi^2 u''(c_2^1)\mathrm{d}I_2^1 + \beta\delta^2\pi u''(c_{3b}^1)\mathrm{d}I_2^1 = \triangle^{I1}$$

with $\triangle^{s1} = -\delta u'(c_2^1)\mathrm{d}\beta, \triangle^{s2} = 0$ and $\triangle^{I1} = 0$. These equations can be written as the linear system

$$\begin{bmatrix} u''(c_1^1) + \beta\delta u''(c_2^1) & -\beta\delta u''(c_2) & -\beta\delta(1+\lambda)\pi u''(c_2^1) \\ -\beta\delta u''(c_2^1) & \beta\delta u''(c_2^1) + \beta\delta^2\left[\pi u''(c_{3b}^1) + (1-\pi)u''(c_{3g}^1)\right] & \beta\delta(1+\lambda)\pi u''(c_2^1) + \beta\delta^2\pi u''(c_{3b}^1) \\ -\beta\delta(1+\lambda)\pi u''(c_2^1) & \beta\delta(1+\lambda)\pi u''(c_2^1) + \beta\delta^2\pi u''(c_{3b}^1) & \beta\delta(1+\lambda)^2\pi^2 u''(c_2^1) + \beta\delta^2\pi u''(c_{3b}^1) \end{bmatrix}$$
$$\times \begin{bmatrix} \mathrm{d}s_1^1 \\ \mathrm{d}s_2^1 \\ \mathrm{d}I_2^1 \end{bmatrix} = \begin{bmatrix} \triangle^{s1} \\ \triangle^{s2} \\ \triangle^{I1} \end{bmatrix}.$$

The determinant of the Hessian is given by

$$|\mathcal{H}_{P1}| = (u''(c_1^1) + \beta\delta u''(c_2^1))\beta^2\delta^4(1-\pi)u''(c_{3g}^1)\pi u''(c_{3b}^1)$$
$$+ u''(c_2^1)u''(c_1^1)\beta^2\delta^3 u''(c_{3g}^1)(1+\lambda)^2\pi^2(1-\pi) + u''(c_2^1)u''(c_1^1)\beta^2\delta^3 u''(c_{3b}^1)(1-\pi(1+\lambda))^2\pi,$$

which is negative as we assume $(1+\lambda)\pi < 1$. Using Cramer's rule and denoting the coefficient of absolute risk aversion $A(c) = -u''(c)/u'(c)$, we obtain after some simplifications (and by noting that $c_{3g}^1 > c_{3b}^1$ if $\lambda > 0$)

$$\frac{\partial s_1^1}{\partial\beta} = \frac{-\beta^2\delta^4\pi u'(c_2^1)}{|\mathcal{H}_{P1}|}\left[(1-\pi)u''(c_{3g}^1)(\pi(1+\lambda)^2 u''(c_2^1) + \delta u''(c_{3b}^1)) + (1-(1+\lambda)\pi)^2 u''(c_2^1)u''(c_{3b}^1)\right] > 0,$$

$$\frac{\partial s_2^1}{\partial\beta} = \frac{-\beta^2\delta^4\pi(1-(1+\lambda)\pi)u''(c_2^1)u''(c_{3b}^1)u'(c_2^1)}{|\mathcal{H}_{P1}|} > 0,$$

$$\frac{\partial I_2^1}{\partial\beta} = \frac{\beta^3\delta^4\pi(1-\pi)u''(c_2^1)u'(c_{3b}^1)u'(c_{3g}^1)\left[A(c_{3g}^1) - A(c_{3b}^1)\right]}{|\mathcal{H}_{P1}|}\begin{cases} > 0 & \text{IARA} \\ = 0 & \text{CARA} \\ < 0 & \text{DARA.} \end{cases}$$

A.1.2 Liquidity Constraint in Period 1 is Binding

Total differentiation of (5) and (6) amounts to the following linear system of equations

$$\begin{bmatrix} \delta u''(c_2^1) + \delta^2[\pi u''(c_{3b}^1) + (1-\pi)u''(c_{3g}^1)] & \delta(1+\lambda)\pi u''(c_2^1) + \delta^2\pi u''(c_{3b}^1) \\ \delta(1+\lambda)\pi u''(c_2^1) + \delta^2\pi u''(c_{3b}^1) & \delta(1+\lambda)^2\pi^2 u''(c_2^1) + \delta^2\pi u''(c_{3b}^1) \end{bmatrix} \times \begin{bmatrix} \mathrm{d}s_2^1 \\ \mathrm{d}I_2^1 \end{bmatrix} = \begin{bmatrix} \triangle_{LC1}^{s2} \\ \triangle_{LC1}^{I1} \end{bmatrix},$$

where $\triangle_{LC1}^{s2} = \delta u''(c_2^1)\mathrm{d}s_1^1$ and $\triangle_{LC1}^{I1} = \delta(1+\lambda)\pi u''(c_2^1)\mathrm{d}s_1^1$. The determinant of the Hessian is

$$|\mathcal{H}_{LC1}| = \delta^3(1-(1+\lambda)\pi)^2\pi u''(c_2^1)u''(c_{3b}^1) + \delta^3\pi(1-\pi)u''(c_{3g}^1)((1+\lambda)^2\pi u''(c_2^1) + \delta u''(c_{3b}^1)) > 0.$$

Using Cramer's rule and noting that $c_{3g}^1 > c_{3b}^1 \Leftrightarrow \lambda > 0$, we obtain after some simplifications equations (7) and (8).

A.1.3 Liquidity Constraint in Period 2 is Binding

Total differentiation of (4) and (6) amounts to the following linear system of equations

$$\begin{bmatrix} u''(c_1^1)+\beta\delta u''(c_2^1) & -\beta\delta(1+\lambda)\pi u''(c_2^1) \\ -\beta\delta(1+\lambda)\pi u''(c_2^1) & \beta\delta(1+\lambda)^2\pi^2 u''(c_2^1)+\beta\delta^2\pi u''(c_{3b}^1) \end{bmatrix} \times \begin{bmatrix} \mathrm{d}s_1^1 \\ \mathrm{d}I_2^1 \end{bmatrix} = \begin{bmatrix} \triangle_{LC2}^{s1} \\ \triangle_{LC2}^{I1} \end{bmatrix},$$

where $\triangle_{LC2}^{s1} = \beta\delta u''(c_2^1)\mathrm{d}s_2^1 - \delta u'(c_2^1)\mathrm{d}\beta$ and $\triangle_{LC2}^{I1} = -\beta\delta\pi\left((1+\lambda)u''(c_2^1)+\delta u''(c_{3b}^1)\right)\mathrm{d}s_2^1$.
The determinant of the Hessian is given by

$$|\mathcal{H}_{LC2}| = \beta\delta\pi u''(c_1^1)\left((1+\lambda)^2\pi u''(c_2^1)+\delta u''(c_{3b}^1)\right)+\beta^2\delta^3\pi u''(c_2^1)u''(c_{3b}^1) > 0.$$

Using Cramer's rule, we obtain after some simplifications equation (9).

A.2 Decisions in Period 2

A.2.1 Non-Binding Liquidity Constraints

Total differentiation of the first-order conditions (11) and (12) yields

$$\begin{aligned} u''(c_2^2)\mathrm{d}s_2^2+\beta\delta[\pi u''(c_{3b}^2)+(1-\pi)u''(c_{3g}^2)]\mathrm{d}s_2^2+(1+\lambda)\pi u''(c_2^2)\mathrm{d}I_2^2+\beta\delta\pi u''(c_{3b}^2)\mathrm{d}I_2^2 &= \triangle^s, \\ (1+\lambda)\pi u''(c_2^2)\mathrm{d}s_2^2+\beta\delta\pi u''(c_{3b}^2)\mathrm{d}s_2^2+(1+\lambda)^2\pi^2 u''(c_2^2)\mathrm{d}I_2^2+\beta\delta\pi u''(c_{3b}^2)\mathrm{d}I_2^2 &= \triangle^I \end{aligned}$$

with

$$\begin{aligned} \triangle^s &= u''(c_2^2)\mathrm{d}s_1^1-\delta\left[\pi u'(c_{3b}^2)+(1-\pi)u'(c_{3g}^2)\right]\mathrm{d}\beta, \\ \triangle^I &= (1+\lambda)\pi u''(c_2^2)\mathrm{d}s_1^1-\delta\pi u'(c_{3b}^2)\mathrm{d}\beta. \end{aligned}$$

These equations can be written as the following linear system

$$\begin{bmatrix} u''(c_2^2)+\beta\delta[\pi u''(c_{3b}^2)+(1-\pi)u''(c_{3g}^2)] & (1+\lambda)\pi u''(c_2^2)+\beta\delta\pi u''(c_{3b}^2), \\ (1+\lambda)\pi u''(c_2^2)+\beta\delta\pi u''(c_{3b}^2) & (1+\lambda)^2\pi^2 u''(c_2^2)+\beta\delta\pi u''(c_{3b}^2) \end{bmatrix} \begin{bmatrix} \mathrm{d}s_2^2 \\ \mathrm{d}I_2^2 \end{bmatrix} = \begin{bmatrix} \triangle^s \\ \triangle^I \end{bmatrix}.$$

The determinant of the Hessian is given by

$$|\mathcal{H}_{P2}| = \beta\delta u''(c_2^2)u''(c_{3b}^2)\pi(1-\pi(1+\lambda))^2+\beta\delta(1-\pi)u''(c_{3g}^2)\left[(1+\lambda)^2\pi^2 u''(c_2^2)+\beta\delta\pi u''(c_{3b})\right] > 0$$

Noting that $c_{3g}^2 > c_{3b}^2 \Leftrightarrow \lambda > 0$ and using Cramer's rule, we obtain after some simplifications equations (14) and (15).

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