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# Two-Outcome dictatorial mechanisms in constrained combinatorial auctions

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## Abstract

We study the limitations of the general space of deterministic, dominant-strategy incentive compatible, individually rational, and Pareto-optimal combinatorial auctions in a model with two players and two nonidentical items. Our model has multidimensional types, private values, nonnegative prices, quasilinear preferences for players with one relaxation - one of the players is subject to publicly-known budget constraints. The study concludes that the space described above includes dictatorial mechanisms even if only a single player is subject to publicly-known budget constraints. Moreover we show that if it is publicly known that the player's budget restricts his ability to pay then for mechanisms with two possible outcomes there are two families of dictatorial mechanisms that uniquely fulfill the properties of deterministic, dominant-strategy incentive compatible, individually rational, and Pareto optimal.

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# 1. Introduction

We study the limitations of the general space of deterministic, dominant-strategy incentive compatible, individually rational, and Pareto-optimal combinatorial auctions in a model with two players and two nonidentical items (four outcomes). Our study concludes that even if one of the players is subject to publicly-known budget constraints then the space includes dictatorial mechanisms. Moreover, it appears that the dictatorial aspect depends on the introduction of budgets. We draw the above conclusion as the research community has indication to believe that the general space of dominant-strategy incentive compatible, individually rational and Pareto-optimal combinatorial auctions without budgets includes only VCG mechanisms[1]. Therefore, the discovery of dictatorial mechanisms in our study is most likely brought about by our inclusion of a budgeted player.

When attempting to create auctions for use in industry, theory and practice immediately uncouple with the introduction of budgets. Consider that a central element of auction theory is the set of players' valuations, how much value each player assigns to each of the auction's possible outcomes. However, in practice players' ability to pay is often less than their valuation for the goods or services they desire. Even in the simple case of two people bidding to acquire a new home it is not uncommon for one bidder to desire a home outside of his or her means or for one of the parties to have significantly greater financial resources. As such, it is important for designers to understand which parts of the budget-constrained combinatorial auction domain are capable of supporting desirable mechanisms. Our result takes a step in this direction by characterizing a portion of the dictatorial space.

Formally our model has multidimensional types[2], private values, nonnegative prices, quasilinear preferences for the players with one relaxation: one of the players is subject to publicly-known budget constraints. More specifically, we show that if it is publicly known that a player's budget restricts his ability to pay, i.e., the restricted player values the possible bundles more than his budget, then for mechanisms with two possible outcomes there are two families of dictatorial mechanisms. In one family the unrestricted player is the dictator and in the other family the restricted player is the dictator. The families of dictatorial mechanisms are unique for all possible dictatorial mechanisms in the space. The mechanisms in the dictatorial families are identical except for one or two price parameters that differentiate them.

Budgets are central to most economic theory but relatively little attention has been given to them in auction theory as the theory mainly focuses on models with quasilinear preferences without budgets and income effects. (For relatively recent works that study auctions with budgets see Che and Gale (1998), Benoit and Krishna (2001), Laffont and Robert (1996), Pai and Vohra (Unpublished results 2008), Maskin (2000)). Similar to models in recent work dealing with budgeted auctions, our model integrates quasilinear preferences with budgets and captures players' utilities in situations such as home buying where a buyer's preferences may exceed his or her available budget.

[1]See Lavi et al. (2003). Lavi et al. (2003) do not require Pareto optimality but requires other properties that imply Pareto optimality

[2]Multidimensional types, meaning that a player may have a separate arbitrary value for each of the four possible outcomes.

There are classic as well as recent results showing that dictatorship (or sequential dictatorship) is the only mechanism that is not subject to individual manipulations and is Pareto optimal in mechanism design models without the quasilinearity assumption (See Arrow (1950), Budish and Cantillon (2012), Gibbard (1973), Hatfield (2009), Satterthwaite (1975)). Arrow's seminal impossibility, Arrow (1950), shows that for unrestricted domains under determinism and transitivity axioms, independence of irrelevant alternatives (IIA), and Pareto-optimality conditions, every social choice function must be a dictatorship or imposed. However, the conditions of Arrow's theorem as well as the conditions of Gibbard (1973) and Satterthwaite (1975) can be satisfied when the requirement for unrestricted domains is relaxed, as was shown for one-dimensional domains such as single peaked. While the possibility/impossibility of maintaining Arrow's desired properties is known for the space of the nonmonetary domain of preferences, when restricting attention to the assumption of side payments and transferable currency much is yet left to be understood.

In recent years several papers studied budget-constrained combinatorial auctions. Dobzinski et al. (2012) showed that there does not exist a deterministic auction that is individually rational, dominant-strategy incentive compatible, and Pareto optimal with potentially negative prices and privately known budgets, even when players are one-dimensional types. Fiat et al. (2011) showed that the same impossibility holds for one-dimensional types with different items and publicly known multi-item demand. Lavi and May (2012) also showed the same impossibility with publicly known budgets if multidimensional types (two identical items with three outcomes) are considered.

Dobzinski et al. (2012), Fiat et al. (2011), Lavi and May (2012) allow negative prices to exist, i.e., some players are paid for participation in the auction either by the mechanism or by the other players. Practical auction implementations usually can not afford or are unwilling to consider paying bidders for their participation nor are they interested in encouraging side payments among the participants. Therefore, similar to Maskin (2000) 's model, we chose to assume that all prices are nonnegative. The assumption that all prices are nonnegative narrows down the domain of possible allocations in comparison to the potential negative prices model with multidimensional types. Nevertheless some of the the mechanisms which fulfill the three properties of dominant strategy incentive compatible, individually rational, and Pareto optimal in the nonnegative price model are not included in the mechanism space that fulfills the same properties in the negative price model. The reason for the above is the property of Pareto optimality. Since the model with nonnegative prices has a smaller set of possible allocations there exist situations where a mechanism does not fulfill the Pareto optimal property in the model with negative prices but does fulfill the Pareto optimal property in the nonnegative price model.

Dobzinski et al. (2012) also characterizes the possibility space of dominant-strategy incentive compatibility and Pareto optimal budget-constrained combinatorial auction mechanisms. Dobzinski et al. (2012)'s characterization is restricted to one-dimensional types and therefore their possibility space characterization does not imply the possibility space in our model with multidimensional types. More specifically, Dobzinski et al. (2012) showed that for multi-unit demand and identical items Ausubel's clinching auction, which assumes public budgets and additive valuations, uniquely satisfies the properties described above. Similarly, Ausubel's clinching auction was concluded by Fiat et al. (2011) for one-dimensional types with different items and publicly known multi-item demand. The one-dimensional types model for unit-demand players with private values and budget constraints yields several deterministic mechanisms that fulfill the properties of incentive compatible and Pareto optimality (see Ashlagi et al. (2010)). In nondeterministic mechanisms with a one-dimensional types model (one indivisible unit) Maskin (2000) characterizes constrained-efficiency mechanisms, which are mechanisms that maximize the expected social welfare under Bayesian incentive compatibility and budget constraints in a nonnegative price model.

## 2. Notation and Definitions

We consider combinatorial auction mechanisms with 2 different types of items and 2 players. Let $N = \{1, 2\}$ be the set of players and $C = \{c_1, c_2\}$ be the set of items. Let $\mathcal{B}$ be the set of all subsets of items, i.e., $\mathcal{B} = 2^C = \{\emptyset, \{c_1\}, \{c_2\}, \{c_1, c_2\}\}$.

Players are multi-minded[3] such that each player $i$ has a private value $v_i(B)$ for every bundle $B \in \mathcal{B}$ drawn from a valid valuation space $\mathcal{V}_i$ [4]. We denote player $i$'s private values by the triple $V_i = (v_i(c_1), v_i(c_2), v_i(c_1, c_2)) \in \mathcal{V}_i$ and assume that $v_i(\emptyset) = 0$, i.e., the valuation of the empty bundle is zero for both players.

We assume that player 1 has a limited budget, b1, while player 2 has an unlimited budget for acquiring the items. We also assume that player 1's budget is publicly known information.

We denote the auction mechanism $F(V_1, V_2, b_1) = (B_1, B_2, p(B_1), p(B_2))$ where $B_i$ is the bundle allocated to player $i$ and $p(B_i)$ is bundle $B_i$'s price. We assume that all prices are nonnegative, i.e., $p(B_i) \geq 0$ for $i = \{1, 2\}$.

Denote by $v_{min} = \min_i v_i(B)$, $B \in \mathcal{B}$, $B \neq \emptyset$.
Throughout the paper we assume $v_{min} > b_1$ and "$v_{min}$ valuation space" refers to any valuation space such that $v_i(B) > b_1$ for every $i$ and every $B \in \mathcal{B}$, $B \neq \emptyset$.

Definition 2.1. *Player 1, player 2, and the auctioneer's utilities are defined as follows: player* 1*'s utility is*

$$u_1(F(V_1, V_2, b_1)) = \begin{cases} v_1(B_1) - p(B_1) & if\ p(B_1) \leq b_1 \\ -\infty & otherwise \end{cases}$$

*player* 2*'s utility is*

$$u_2(F(V_2, V_2, b_1)) = v_2(B_2) - p(B_2)$$

*the auctioneer's utility is*

$$u_a(F(V_1, V_2, b_1)) = p(B_1) + p(B_2)$$

For notational simplicity whenever $F, V_1, V_2$, and $b_1$ are clear from the context we denote $u_i(F(V_1, V_2, b_1))$ by $u_i$.

[3] A multi-minded player is a player that may have a separate arbitrary value for each of the four possible outcomes.

[4] Through the paper we consider valuation spaces where not all valuations are included in the valuation space.

Definition 2.2. **Determinism**
*An auction mechanism* $F(V_1, V_2, b_1)$ *is called* **deterministic** *if for every given input it outputs a* single *outcome.*

Definition 2.3. **Dictatorship**
*An auction mechanism* $F(V_1, V_2, b_1)$ *is called* **dictatorial** *if there exists a player,* $i \in \{1, 2\}$ (**the dictator**)*, such that for every* $V_i$, $V_{\hat{i}}$, $V'_{\hat{i}}$, $u_i(F(V_i, V_{\hat{i}}, b_1)) = u_i(F(V_i, V'_{\hat{i}}, b_1))$.

Intuitively, a mechanism is a dictatorship if there is a player $i$ such that the other player's valuations, $\hat{i}$, can not affect his utility. Note that if dictator $i$ is indifferent to two alternative allocations then player $\hat{i}$'s valuations can affect the output.

Our definition of a dictatorial mechanism is a natural extension of Arrow's dictatorial social welfare function to a monetary domain. Consider Arrow (1950)'s definition: "A social welfare function is said to be "dictatorial" if there exists an individual $i$ such that for all $x$ and $y$, $xP_iy$ implies $xPy$ regardless of the ordering of all individuals other than $i$, where P is the social preference relation corresponding to those orderings."

As in Arrow's setting the outcome is identical for all players, Arrow's definition can be interpreted to define bossy dictatorship and non-bossy dictatorship. A bossy dictator determines the outcome as a whole regardless of the other players' preferences and a non-bossy dictator determines *his own* utility regardless of the other players' preferences.

In a monetary domain a dictator that determines the outcome as a whole (i.e., bossy dictator) determines the other players' payments and therefore will determine negative prices for himself (meaning that he will determine that the other players will pay him). Such are the Groves dictatorial mechanisms where there is zero social welfare without the dictator's participation and therefore society should pay the dictator the amount of social welfare that the other players benefit from when the dictator participates, i.e., negative prices for the dictator. Our model and results are based on the requirement for nonnegative prices and therefore a bossy dictatorial mechanism is not suitable. Moreover Groves's dictatorial mechanism is more loosely related to Arrow's dictatorial social welfare function than our definition as Groves's dictatorial mechanism does not necessarily imply that when the dictator is present no other player can influence the outcome, otherwise it would have meant that the dictator has a constant price.

Next we define three properties: Individual Rationality (IR), Pareto Optimality and Truthfulness.

Definition 2.4. **Property 1: Individual Rationality (IR)**
*An auction mechanism* $F(V_1, V_2, b_1)$ *is called* **individually rational** *if for every player* $i$, $u_i(F(V_1, V_2, b_1)) \geq 0$. *Specifically the following must hold:*

- $v_1(B_1) - p(B_1) \geq 0$ *and* $p(B_1) \leq b_1$ *(player 1's IR)*
- $v_2(B_2) - p(B_2) \geq 0$ *(player 2's IR)*

Note that the auctioneer's utility is nonnegative from our assumption that all the prices are nonnegative.

DEFINITION 2.5. **Property 2: Pareto Optimality**
*An auction mechanism $F$ is called* **Pareto optimal** *if for every input $V_1, V_2, b_1$, such that $(V_1, V_2)$ in $\mathcal{V}_1 \times \mathcal{V}_2$ and output allocation $(B_1, B_2, p(B_1), p(B_2))$, there is no allocation $(B_1', B_2', p(B_1'), p(B_2')) \neq (B_1, B_2, p(B_1), p(B_2))$ such that all the following inequalities hold, with at least one strong inequality:*

- $u_1(B_1', B_2', p(B_1'), p(B_2')) \geq u_1(F(V_1, V_2, b_1))$
- $u_2(B_1', B_2', p(B_1'), p(B_2')) \geq u_2(F(V_1, V_2, b_1))$
- $u_a(B_1', B_2', p(B_1'), p(B_2')) \geq u_a(F(V_1, V_2, b_1))$

DEFINITION 2.6. **Property 3: Truthfulness**
*An auction mechanism $F(V_1, V_2, b_1)$ is called* **truthful** *if neither of the two players can increase his own utility by reporting false valuations. That is, given the true valuations $V_1 \in \mathcal{V}_1$ and $V_2 \in \mathcal{V}_2$, for every $V_1' \in \mathcal{V}_1$ and $V_2' \in \mathcal{V}_2$ the following hold:*

- $u_1(F(V_1, V_2', b_1)) \geq u_1(F(V_1', V_2', b_1))$
- $u_2(F(V_1', V_2, b_1)) \geq u_2(F(V_1', V_2', b_1))$

## 3. Dictatorial Mechanisms

In this section we study all of the two-outcome dictatorial mechanisms that satisfy the three properties in the $v_{min}$ valuation space.

LEMMA 3.1. *Let $F$ be a deterministic mechanism that satisfies the properties of IR, truthfulness, and Pareto optimality.*

*Then for every $b_1 > 0$ the following two statements hold:*

- *There exists $V_1$ and $V_2$ in the $v_{min}$ valuation space such that $F(V_1, V_2, b_1) = (B_1, B_2, p(B_1), p(B_2))$ and $B_1 = \{c_1\}$ and $B_2 = \{c_2\}$.*
- *There exists $V_1'$ and $V_2'$ in the $v_{min}$ valuation space such that $F(V_1', V_2', b_1) = (B_1', B_2', p(B_1'), p(B_2'))$ and $B_1' = \{c_2\}$ and $B_2' = \{c_1\}$.*

PROOF OF LEMMA 3.1. *Suppose to the contrary w.l.o.g. that the allocation $B_1 = \{c_1\}$ is not feasible. Assume the following valuations:*

- $v_1(c_1, c_2) = v_1(c_1) = 3 \cdot b_1$, $v_1(c_2) = 2 \cdot b_1$
- $v_2(c_1, c_2) = v_2(c_2) = v_2(c_1) = 3 \cdot b_1$

*The allocation $B_1 = \{c_1, c_2\}$, with any $p(B_1) = x$, can not be Pareto as the allocation $B_1' = \{c_1\}$ with $p(B_1') = x$ and $p(B_2) = \varepsilon < b_1$ is strictly better for player 2 and the auctioneer while player 1 is indifferent. The allocation $B_2'' = \{c_1, c_2\}$ cannot be Pareto for similar arguments. Any allocation $\bar{B}_1 = \{c_2\}$ cannot be Pareto as switching the singletons such that $\bar{B}'_1 = \{c_1\}$, $\bar{B}'_2 = \{c_2\}$, $p(\bar{B}'_1) = p(\bar{B}_1)$ and $p(\bar{B}'_2) = p(\bar{B}_2)$ is strictly better for* *player 1 while player 2 and the auctioneer are indifferent. Therefore the allocation* $B_1 = \{c_1\}$ *must be feasible as it is the only option for these valuations.*

*A similar argument can show that allocation* $B_1' = \{c_2\}$ *and* $B_2' = \{c_1\}$ *must also be feasible.*

Conclusion 3.1. *We therefore conclude that any mechanism that satisfies the three properties and has exactly two outcomes must have outcomes* $B_2' = \{c_1\}$ *or* $B_2'' = \{c_2\}$.

Our main theorem follows.

Theorem 3.1. *There are two unique families of dictatorial mechanisms with two outcomes that satisfy the properties of IR, truthfulness, and Pareto optimality.*

**Family I - player 1 is the dictator:**
$p(B_1) = 0$ *and* $p(B_2) = x$ *such that* $0 \leq x \leq b_1$. *Then* $x$ *can either be a constant or a function* $g$ *of* $V_1$ *as long as for every feasible* $V_1$, $0 \leq g(V_1) \leq b_1$.
*If* $v_1(c_1) > v_1(c_2)$ *or* $(v_1(c_1) = v_1(c_2)$ *and* $v_2(c_2) \geq v_2(c_1))$ *then* $B_1 = \{c_1\}$ *and* $B_2 = \{c_2\}$ *else* $B_1 = \{c_2\}$, $B_2 = \{c_1\}$.

**Family II - player 2 is the dictator:**
$p(B_1) = x'$ *and* $p(B_2) = y$ *such that* $0 \leq x', y \leq b_1$ *and* $(x' = b_1$ *or* $y = 0)$. *Then* $x'$ *can either be a constant or a function* $g'$ *of* $V_2$ *as long as for every feasible* $V_2$, $0 \leq g'(V_2) \leq b_1$.
*If* $v_2(c_1) > v_2(c_2)$ *or* $(v_2(c_1) = v_2(c_2)$ *and* $v_1(c_2) \geq v_1(c_1))$ *then* $B_1 = \{c_2\}$, $B_2 = \{c_1\}$ *else* $B_1 = \{c_1\}$ *and* $B_2 = \{c_2\}$.

Proof of Theorem 3.1. *From Lemma 3.1 it follows that if there are two outcomes then the outcomes are the two singletons. We first show that for any dictatorial mechanism with two possible outcomes that satisfies the three properties the prices are as follows:*

1. *If player 1 is the dictator then* $p(B_1) = 0$ *and* $p(B_2) = x$ *such that* $0 \leq x \leq b_1$.
   *Suppose that player 1 is the dictator. We start by proving that* $p(B_2) = x$ *for any* $B_2$ *of the two possible outcomes. Suppose to the contrary that when* $B_2 = \{c_1\}$ *then* $p(B_2) = x$, *and when* $B_2' = \{c_2\}$ *then* $p(B_2') = y$. *From the truthfulness of player 2 it follows that* $x = y$. *Otherwise, whenever player 1 is indifferent to the two singleton alternatives, player 2 might be better off lying and changing the allocation to the less preferred item for a lower price. If* $x > b_1$ *then the allocation is not IR whenever* $b_1 < v_2(c_1), v_2(c_2) < x$. *Therefore we conclude that* $p(B_2) \leq b_1$.

   *We continue by proving that* $p(B_1) = 0$. *Suppose to the contrary that when* $B_1 = \{c_1\}$ *then* $p(B_1) = x'$ *and when* $B_1' = \{c_2\}$ *then* $p(B_1') = y'$. *Suppose w.l.o.g. that* $x' < y'$. *Consider the valuations* $v_2(c_1) > v_2(c_2)$, $v_1(c_2) > v_1(c_1)$, $v_1(c_1) - x' > v_1(c_2) - y'$. *Then from player 1's truthfulness the mechanism must allocate* $B_1 = \{c_1\}$ *for* $p(B_1) = x'$. *This allocation is not Pareto as the allocation* $B_1' = \{c_2\}$, *for the same prices, is better for the two players while the auctioneer is indifferent. Therefore we conclude that* $x' = y'$. *Suppose to the contrary that* $p(B_1) = x' \neq 0$. *Let* $0 < \varepsilon < 2 \cdot x'$ *and* $x = p(B_2)$. *Consider the valuations* $v_1(c_1) = v_1(c_2) + \varepsilon$ *and* $v_2(c_1) = v_2(c_2) + 3\varepsilon$. *As the dictator is* *player 1 the outcome must be* $B_1 = \{c_1\}$. *However, this allocation is not Pareto as the allocation* $B_1' = \{c_2\}$, $B_2' = \{c_1\}$, $p(B_1') = x' - 2\varepsilon > 0$ *and* $p(B_2') = x + 2\varepsilon$ *is strictly better for the two players while the auctioneer is indifferent.*

2. *If player 2 is the dictator then there are two options:*

   - $p(B_2) = 0$ *and* $p(B_1) = x'$ *such that* $0 \leq x' \leq b_1$. *The proof of this case is similar to the previous case where player 1 is the dictator.*
   - $p(B_2) = y$ *such that* $0 \leq y \leq b_1$ *and* $p(B_1) = b_1$. *In addition to the previous case where* $p(B_2) = 0$, *it might be that player 2 pays a nonzero price. The proof that player 2 pays a price* $y$ *that is the same in both possible outcomes is identical to the equivalent case where player 1 is the dictator. Player 2's price* $y$ *can be nonzero as any such allocation is Pareto following from the fact that player 1's budget is saturated, i.e.* $p(B_1) = b_1$, *and therefore he cannot pay a higher price for a better item.*

*We now show that the two families above satisfy the three properties.*

- *IR - no player pays more than* $b_1$ *and all values are assumed to be higher than* $b_1$.
- *Truthful - the dictator gets his best choice for a fixed price, which is obviously better than the other item for the same price. The other player either can not change the allocation or (if the dictator is indifferent) is allocated the best choice for a fixed price.*
- *Pareto optimal - switching the items can not benefit both players. If the dictator is allocated the preferred item for free then in both cases the dictator is worse off from switching the items. If on the other hand player 2 is the dictator and he pays a nonzero price then it must be the case that* $p(B_1) = b_1$. *In order for the dictator to benefit the price must be lower. The auctioneer will be indifferent only if player 1 pays the gap but as player 1 payed* $b_1$, *so he can not pay any more. Allocating both items to one player is strictly worse for the other player as all the utilities are strictly positive.*

## 4. Concluding Comment

Though our possibility space analyses only two players, two items, and two outcome mechanisms it is easy to see that the mechanisms proved in the space can be extended to deal with more items, more players, and more possible outcomes. Such an extension is the subject of our future work.


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