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Average consumer decisions in an economy with heterogeneous subjective discount rates and risk aversion coefficients: the finite horizon case

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Abstract

This paper aims to study the behavior of the average rational consumer of an economy populated by heterogeneous agents in a finite horizon framework. Heterogeneity takes into account both the subjective discount rate and risk aversion coefficient. Closed-form solutions for the optimal paths of consumption and capital, of the average consumer, are derived. Moreover, a closed form solution for the economic welfare of the average consumer is obtained.

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1. Introduction

Even though the concept of heterogeneous preferences has been widely studied, the diversity of the notion continues growing, for example: Kuplow (2008) analyzes optimal policies when the preferences for commodities, public goods and externalities are heterogeneous; Shapiro (2008) studies the overvaluation of a risky asset in a framework with heterogeneous agents with non-rational expectations; Chen *et al.* (2008) consider heterogeneity in preferences over a local public good, human capital formation, and residential locations through an overlapping generation model; Fethke and Jagannathan (1996) examine the dynamics of consumption in a setting where imperfectly competitive producers face consumers with various intensities of rational habit persistence; Boswijk *et al.* (2007) estimate a dynamic asset pricing model characterized by heterogeneous boundedly rational agents; Andersen (2007) analyzes an intertemporal general equilibrium model with heterogeneous labor markets; and Xiouros and Zapatero (2010) study economies populated with agents with heterogeneous risk aversion.

This research focuses on the decision making process, in a finite horizon, of the average consumer of an economy populated by individuals differing in their preferences. Specifically, heterogeneity is introduced via a joint distribution function of the subjective discount rate and the risk aversion coefficient; both parameters being driven by the exponential distribution. We also suppose that individuals are endowed with a negative exponential utility function. This functional form is appropriate to be conjugated with the exponential density so that the discounted total utility of the average consumer can be analytically treated. One distinguishing feature of this research is that closed-form solutions for the optimal paths of consumption and capital, of the average consumer, are obtained in a finite horizon framework. Furthermore, a closed form solution for the economic welfare of the average consumer is derived. Finally, several analytical and graphical experiments of comparative statics are accomplished.

This paper is organized as follows; section 2 describes the heterogeneity of the population; section 3, briefly, states the resource allocation in the economy; section 4 defines the firms' behavior; section 5 solves the utility maximization problem of the rational average consumer; and section 6 provides the conclusions and acknowledges the limitations of the proposed model.

2. Preference Heterogeneity

Consider an economy where individuals are rational consumers endowed with the negative exponential function. This economy consumes and produces a single perishable good, and is populated by heterogeneous agents. Heterogeneity is represented by two distribution functions. The first distribution $F = F(\rho), \quad \rho > 0,$ is associated with the subjective discount rate, ρ, The second distribution $G = G(\alpha), \quad \alpha > 0,$ is related to the parameter α appearing in the negative exponential utility function $u(c_t;\alpha) = -e^{-\alpha c_t}$. It is reasonable to assume stochastic independence between ρ and α since anxiety for present consumption is not related to risk aversion. In what follows, it will be assumed that ρ and α are both driven by the exponential distribution, that is to say, the parameters α and ρ have, respectively, densities $g(\alpha) = -\mu e^{-\alpha\mu}$, $\mu > 0$, and $f(\rho) = -\lambda e^{-\lambda\rho}$, $\lambda > 0$, where both μ and λ are known parameters.

3. Resource Allocation

Next, it is assumed that resource allocation in the economy is given by the national income identity and not by a price system. For the sake of simplicity, it will be assumed a closed economy without government, *i.e.*, a closed autarky. Suppose also that the rate of depreciation of capital is zero, thus the per capita national income identity satisfies

$$f(k_t) = c_t + \dot{k}_t$$

where k_t is capital, $f(k_t)$ is the production function, and c_t is consumption; all of them in per capita terms.

4. Firms' Behavior

It is assumed that production is carried out by a representative firm using an "*Ak*" technology, *i.e.*, $y_t = f(k_t) = Ak_t$. The present value, *PV*, of the representative firm is given by:

$$PV = \int_0^T \left(Ak_t - rk_t\right) e^{-rt} \mathrm{d}t$$

where the difference in the integral is nothing more than the income of the firm less the payment to factor; in this case there is only payment to capital. It is worth noting that the above expression represents the benefits of the firm discounted with the real interest rate. The first order condition of the maximization problem of the representative firm leads to $r = A$. Thus, the marginal product of capital satisfies that the technological level is constant and equal to the real interest rate. Thus, after discounting and taking the present value of both sides of the per capita national income identity, and considering a finite transversality condition, it follows that

$$0 = \int_0^T c_s e^{-rs} \mathrm{d}s + \lim_{t \to T} k_t e^{-rt} - k_0,$$

or $k_0 = \int_0^T c_s e^{-rs} \mathrm{d}s$, where k_0 is given.

5. Central planner's problem

It is assumed that a central planner wishes to maximize the consumption satisfaction of the average agent. Specifically, the central planner wishes to solve

$$\begin{aligned}&\text{Maximize} \quad \int_0^\infty \left(\int_0^\infty \left(\int_0^T -e^{-\alpha c_t} e^{-\rho t} \mathrm{d}t \right) \mu e^{-\mu\alpha} \mathrm{d}\alpha \right) \lambda e^{-\lambda\rho} \mathrm{d}\rho \\ &\text{subject to} \quad k_0 = \int_0^T c_t e^{-rt} \mathrm{d}t.\end{aligned} \tag{1}$$

By assuming the underlying conditions in Fubini's theorem, the average agent utility function may be rewritten as follows:

$$\int_0^T \frac{-\mu\lambda}{(t+\lambda)(\mu+c_t)} \mathrm{d}t.$$

The Lagrangian for this problem is given by:

$$\mathfrak{L}(c_t,\lambda) = \frac{-\mu\lambda}{(t+\lambda)(c_t+\mu)} + \beta\left(rk_0 - c_t\right)e^{-rt}.$$

Differentiating with respect to c_t, it follows that $\dfrac{\mu\lambda}{(t+\lambda)(c_t+\mu)^2} - \beta e^{-rt} = 0,$

and after solving for c_t, it is obtained that

$$c_t = \sqrt{\frac{\mu\lambda}{\beta}}\sqrt{\frac{e^{rt}}{(t+\lambda)}} - \mu. \tag{2}$$

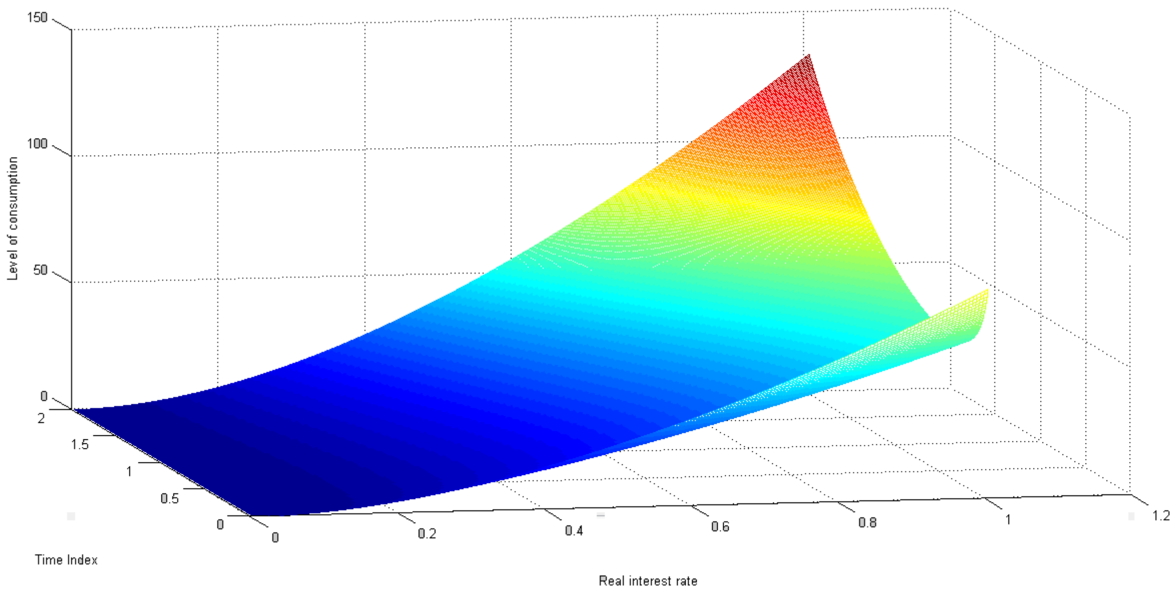
In the above equation, as usual, the Lagrange multiplier, β, is unknown. In order to find it, equation (2) is substituted into the constraint in (1). The optimal consumption path satisfies

$$c_t = \frac{e^{\frac{r}{2}(t-\lambda)}}{\sqrt{t+\lambda}}\left[\frac{\left(Ak_0 + \mu\left(1-e^{-rT}\right)\right)}{\sqrt{8A\pi}\left(\Phi\left(\sqrt{r(T+\lambda)}\right) - \Phi\left(\sqrt{r\lambda}\right)\right)}\right] - \mu. \tag{3}$$

where Φ represents the cumulative distribution function of the standard normal random variable, and, as before, $r = A$. Notice now that at time $t = 0$,

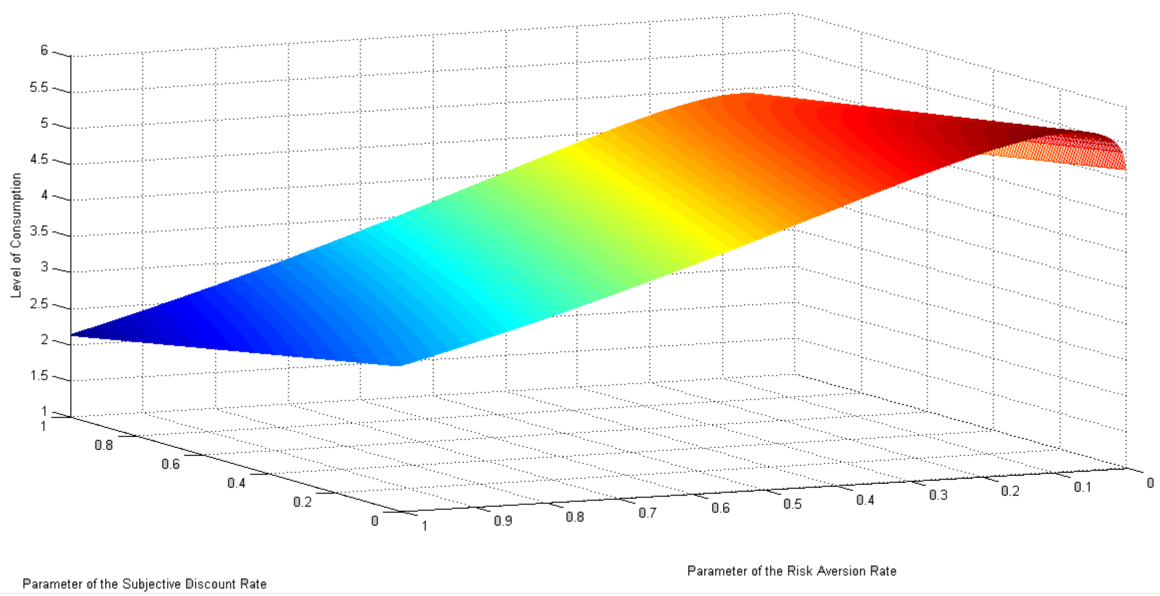
$$\frac{e^{-\frac{r\lambda}{2}}}{\sqrt{\lambda}}\left[\frac{\left(Ak_0 + \mu\left(1-e^{-rT}\right)\right)}{\sqrt{8A\pi\left(\Phi\left(\sqrt{r(T+\lambda)}\right) - \Phi\left(\sqrt{r\lambda}\right)\right)}}\right] > \mu. \tag{4}$$

The left side of (4) is always positive, so c_t will also be positive for all $t \in [0,T]$. In order to carry out a graphic comparative statics exercise it is illustrated in Graph 1 the path of optimal consumption, for the average agent, as a function of A and t, with all other parameters remaining constant. In this case is supposed that $k_0 = 100$, $\lambda = 0.1$, $\alpha = 0.05$, and $r \in (0,1]$. It is worth pointing out that consumption increases when both *r* and *t* rise.



Graph 1. The level of consumption as a function of *r* and *t* (Source: own elaboration).

Moreover, Graph 2 shows the behavior of the optimal path of consumption as a function of the risk aversion parameter and the subjective discount rate. In this case $\lambda \in (0,1]$ and $\mu \in (0,1]$.



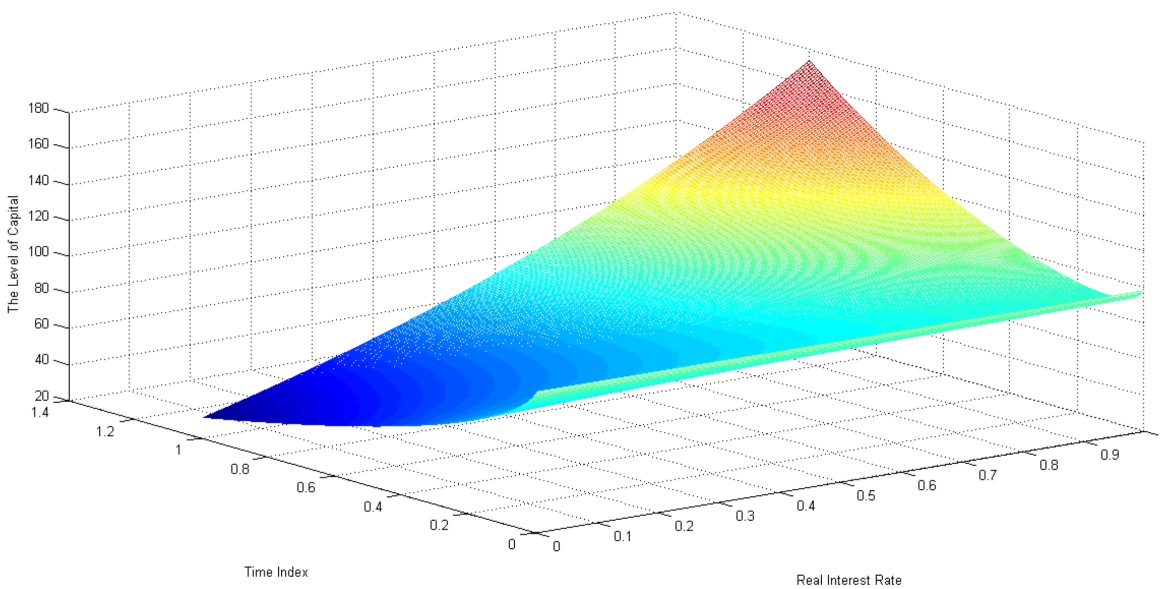
Graph 2. The level of consumption as a function of the risk aversion parameter and the subjective discount rate (Source: own elaboration).

Notice also that $\partial c_t/\partial k_0 > 0$, that is, the level of consumption increases when the initial level of stock increases. Moreover, $\partial c_t/\partial \mu > 0$, that is, an increase in the risk aversion rate parameter positively affects the level of consumption. It is important to point out that the sign of $\partial c_t/\partial t$ depends on the sign of the difference $\sqrt{r(t+\lambda)}-\sqrt{8\pi}$. Unfortunately, $\partial c_t/\partial \lambda$ and $\partial c_t/\partial T$ have ambiguous signs. Finally, $\partial c_t/\partial r > 0$, hence if the real interest rises, the level of consumption increases.

On the other hand, by substituting optimal consumption of the average individual in the national income identity, it follows that

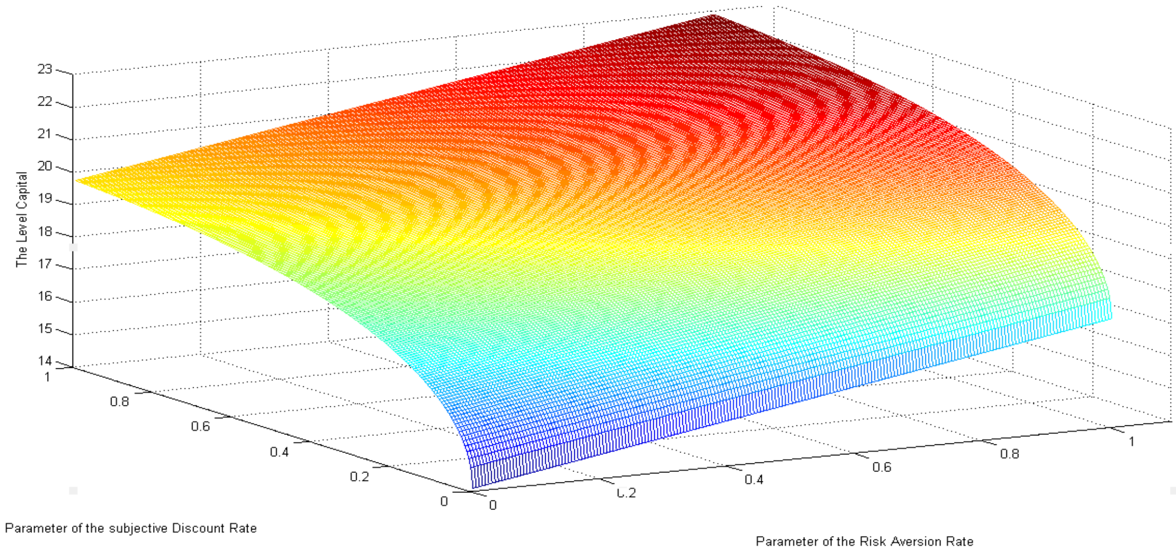
$$k_t = e^{rt}\left[k_0+\frac{\mu}{r}\right]-\frac{\mu}{r}-\frac{1}{r}e^{rt}\left[\frac{\left(rk_0+\mu\left(1-e^{-rT}\right)\right)}{\left(\Phi\left(\sqrt{r(T+\lambda)}\right)-\Phi\left(\sqrt{r\lambda}\right)\right)}\right]\left[\Phi\left(\sqrt{r(t+\lambda)}\right)-\Phi\left(\sqrt{r\lambda}\right)\right].$$

After computing the partial derivatives of k_t with respect to other variables, it is observed that: $\partial k_t/\partial k_0 > 0$, just as in the case of consumption, an increase in initial capital makes the level of capital increases; $\partial k_t/\partial \mu = \partial k_t/\partial \lambda = 0$ when $t \to T$, this implies that neither the risk aversion parameter nor the subjective discount rate parameter affect the level of capital when *t* approaches to *T*; $\partial k_t/\partial t < 0$, this means that a change in *t* decreases the level of capital stock; and $\partial k_t/\partial T$ and $\partial k_t/\partial r$ have both ambiguous sign.



Graph 3. The level of capital as a function of *r* and *t* (Source: own elaboration).

Graph 3 illustrates the path of capital, for the average consumer, as a function of *r* and *t*; all other parameters remaining constant. It is assumed that $k_0 = 100$, λ = 0.1, α = 0.05, and $r \in (0,1]$. In this particular case, capital increases when both *r* and *t* rise. Finally, Graph 4 shows the behavior of the optimal path of capital when $\lambda \in (0,1]$ and $\mu \in (0,1]$. Notice, as expected, that capital increases when both the subjective discount rates and the risk aversion parameter.



Graph 4. The level of capital as a function of the risk aversion parameter and the subjective discount rate (Source: own elaboration).

6. Economic welfare of the average consumer

In what follows, the indirect utility or economic welfare function of the average consumer, *W*, will be computed. By substituting $c_t + \mu$ in the expected total utility in (1), it is found that

$$W = \int_0^T \frac{-\mu\lambda}{(t+\lambda)(c_t+\mu)}\mathrm{d}t = \frac{-(\mu\lambda)e^{\frac{r\lambda}{2}}\sqrt{8A\pi}\left(\Phi\left(\sqrt{r(T+\lambda)}\right)-\Phi\left(\sqrt{r\lambda}\right)\right)}{rk_0+\mu\left(1-e^{-rT}\right)}\int_0^T \frac{e^{-\frac{rs}{2}}}{(s+\lambda)^{\frac{1}{2}}}\mathrm{ds}$$

$$= \frac{-8\pi(\mu\lambda)e^{r\lambda}}{rk_0+\mu\left(1-e^{-rT}\right)}\left[\Phi\left(\sqrt{T+\lambda}\right)-\Phi\left(\sqrt{\lambda}\right)\right]\left[\Phi\left(\sqrt{r(T+\lambda)}\right)-\Phi\left(\sqrt{r\lambda}\right)\right].$$

In this case, it can be shown that

$$\frac{\partial W}{\partial k_0} > 0, \quad \frac{\partial W}{\partial \mu} < 0, \quad \frac{\partial W}{\partial T} > 0, \quad \frac{\partial W}{\partial \lambda} < 0, \quad \text{and} \quad \frac{\partial W}{\partial r} > 0 \text{ or } \frac{\partial W}{\partial r} < 0.$$

The first derivative is almost intuitive and it means that if the average consumer increased his/her initial stock of capital, then the welfare would increase. In a similar way, when the time horizon is extended, $t \to T$, the welfare function augments its value. On the other hand, there exists a negative relation between the preference parameters and the welfare function. Finally, the relation between the risk free rate and the welfare function is ambiguous.

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