Economics Bulletin

Volume 34, Issue 2

Mandatory retirement and economic growth: An inverted U-shaped relationship

Lars Kunze
TU Dortmund

Abstract

The aim of this note is to investigate the relationship between economic growth and mandatory retirement in an overlapping generations model where private investment in human capital is the engine of endogenous growth. We derive a novel result in the retirement-growth literature by showing that the relationship between the mandatory retirement age and economic growth is inverted U-shaped.

I am grateful to Wolfram F. Richter for helpful comments and suggestions. Any remaining errors are mine.
Citation: Lars Kunze, (2014) "Mandatory retirement and economic growth: An inverted U-shaped relationship", *Economics Bulletin*, Vol. 34 No. 2 pp. 885-891.
Contact: Lars Kunze - lars.kunze@tu-dortmund.de.
Submitted: March 17, 2014. **Published:** May 06, 2014.

1 Introduction

The sole aim of this note is to investigate the relationship between economic growth and mandatory retirement in an overlapping generations model where private investment in human capital is the engine of endogenous growth.

Rapid population aging and slow economic growth pose a serious threat to the financial stability of public pension systems in many countries. To address these threats, increases in the mandatory retirement age (among other things) are underway or planned in 28 out of the 34 OECD countries (OECD, 2012). Therefore, the economics of retirement have recently received much attention in the economic literature. However, most of the existing work focusses either on the normative implications of retirement (e.g. Hu (1979), Lacomba and Lagos (2006)), or is concerned about its determinants in politico-economic equilibrium (e.g. Conde-Ruiz and Galasso (2004), Gonzalez-Eiras and Niepelt (2012)). By contrast, this paper addresses the question how an increase in the mandatory retirement age impacts on economic growth depending on preferences and technology. The few existing theoretical results on the growth-retirement relationship remain inconclusive: Whereas Zhang and Zhang (2009) find a positive growth effect in an OLG model where parents' time investment into the education of their children ultimately determines long-run growth[1], Gonzalez-Eiras and Niepelt (2012) demonstrate a negative effect when growth is driven by public investment and increasing the mandatory retirement age affects growth through changes in savings and labour supply.

In contrast to these findings, this note derives a novel result in the literature on the mandatory retirement-growth nexus. Specifically, we show that there exists an inverted U-shaped relationship between growth and the mandatory retirement age: Increasing the mandatory retirement age enhances the return to education and therefore slows down physical capital accumulation as individuals substitute spending on education for voluntary savings. If the direct effect of higher educational spending outweighs the indirect effect of slower capital accumulation, economic growth increases.

Hence, the essence of our findings is a rate-of-return argument which is absent in previous studies: An increase in the mandatory retirement age raises the relative attractiveness of human capital which in turn promotes growth but decreases savings and physical capital accumulation and therefore wages and the return to education in equilibrium. Consequently, the positive effect of mandatory retirement (as in Zhang and Zhang (2009)) or the negative effect (as in Gonzalez-Eiras and Niepelt (2012)) are the result of the particular assumptions made on the way human capital is accumulated. In this note, however, we assume that young individuals may invest into their own education in order to enhance the stock of knowledge or skills and thereby permitting a higher flow of labour services during old age, as in Azariadis and Drazen (1990).

The remainder is organized as follows. The next section introduces the model and derives the growth effects of an increase in the mandatory retirement age. Section 3 shortly concludes.

2 The Model

The basic framework is a two-period overlapping-generations model in the tradition of Diamond (1965) where the size of each generation is normalized to one. In the first period of life, each indi-

[1] In fact, they assume a linear human capital technology and are thus not able to account for general equilibrium effects resulting from the accumulation of physical capital.

vidual gives birth to one child and inelastically supplies h_t efficiency units of labor.[2] She receives the market wage w_t and spends her disposable income on consumption c_t, private education e_t and savings s_t:

$$w_t h_t = c_t + e_t + s_t \tag{1}$$

During old-age, second period consumption d_{t+1} equals the return to voluntary savings $R_{t+1}s_t$ plus the proceeds of labor income in old age $w_{t+1}h_{t+1}\rho\chi$:

$$d_{t+1} = R_{t+1}s_t + w_{t+1}h_{t+1}\rho\chi \tag{2}$$

where R_{t+1} is the gross interest factor[3] at $t+1$, $\rho \in [0,1]$ is the retirement age (or, more precisely, the fraction of the period that on old household is required to work) and $\chi \in (0,1]$ the labor productivity of old relative to young workers.

We further assume that all individuals have access to a common training technology. Consequently, a period-t individual, which is effectively born with a labor efficiency of h_t, can improve her productivity in the second period of life by investing an amount e_t of her disposable income when she is young.[4] Her future productivity depends upon the inherited stock of skills and her investment in education according to:[5]

$$h_{t+1} = De_t^{\delta}h_t^{1-\delta} = D\bar{e}_t^{\delta}h_t \tag{3}$$

where D is a scale parameter, $\delta \in (0,1)$ is the elasticity of the education technology with respect to private educational spending and $\bar{e}_t \equiv e_t/h_t$ private educational spending per unit of human capital. Note that h_t represents both the skills that old members of generation t acquired through schooling, as also the skills that the young in period t inherit from their preceding generation. The modeling of the human capital accumulation process follows Azariadis and Drazen (1990).

Individual preferences are assumed to be logarithmic and depend on first and second period consumption:[6]

$$U_t = \ln(c_t) + \beta\ln(d_{t+1}) \tag{4}$$

[2]Allowing labor supply to be endogenous would not affect our main result qualitatively but only complicate the theoretical analysis. For reasons of simplicity, we therefore abstract from labor supply choices.

[3]For reasons of simplicity, we assume that capital depreciates completely in one period.

[4]An alternative and equivalent formulation with time instead of monetary investments into education does not affect our results.

[5]A specification of this type is standard in the literature, see e.g. Lambrecht et al. (2005). Note further that, for reasons of simplicity, we abstract from public educational spending. However, it is straight forward to show that the main result is robust against this simplification. See, e.g., Kaganovich and Zilcha (1999) for a model in which education is jointly financed by public and private spending.

[6]Note that labor supply of both young and old individuals is assumed to be inelastic (as in Zhang and Zhang (2009, section 4)). Yet, in a previous version of this paper (available at .../Mandatory_retirement_and_economic_growth.pdf) we showed that our main result is robust against this simplification. Specifically, following the recent literature on mandatory retirement (Gonzalez-Eiras and Niepelt, 2012) and assuming that young and non-retired old households work the same number of hours, it turns out that the relationship between long-run growth and the level of mandatory retirement is inverted U-shaped as in the simpler model of the present version. An alternative assumption would be to assume that young individuals supply labor inelastically whereas old individuals also care about leisure. In this case, however, as long as the mandatory retirement age is not binding, an increase in the mandatory retirement age has no effect on economic growth as a higher mandatory retirement age is completely offset by a corresponding increase in leisure time (see the discussion in Zhang and Zhang (2009, p.339)).

where $\beta \in (0, 1)$ is a discount factor. Each individual maximizes utility (4) subject to the constraints (1), (2) and (3) by choosing c_t, e_t, s_t and d_{t+1}. The first order conditions determining optimal savings and private educational spending are:

$$\frac{\partial U_t}{\partial s_t} = -\frac{1}{c_t} + \frac{\beta R_{t+1}}{d_{t+1}} = 0 \tag{5}$$

$$\frac{\partial U_t}{\partial e_t} = -\frac{1}{c_t} + w_{t+1} D\delta e_t^{\delta-1} h_t^{1-\delta} \frac{\beta\rho\chi}{d_{t+1}} = 0 \tag{6}$$

Combining (5) and (6) gives

$$w_{t+1}\rho\chi D\delta e_t^{\delta-1} h_t^{1-\delta} = R_{t+1} \tag{7}$$

Equation (7) captures the essence of our rate-of-return argument: Increasing the mandatory retirement age raises the return to education but does not affect, all other things being equal, the rate of return to savings.

In every period t, firms produce a single output good according to a Cobb-Douglas production function combining physical capital K_t and human capital H_t:

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} \tag{8}$$

where $\alpha \in (0, 1)$ denotes the capital share. Profit maximization gives the usual marginal productivity conditions:

$$w_t = (1-\alpha)AK_t^{\alpha} H_t^{-\alpha} = (1-\alpha)Ak_t^{\alpha}, \quad R_t = \alpha AK_t^{\alpha-1} H_t^{1-\alpha} = \alpha Ak_t^{\alpha-1} \tag{9}$$

where $k_t = K_t/H_t$ is the physical to human capital ratio.

In equilibrium, the market clearing conditions for the labor and the capital market are:

$$H_t = (1+\rho\chi)h_t \tag{10}$$

$$K_t = s_{t-1} \tag{11}$$

Combining (5), (6) and (9), we obtain[7]

$$\bar{e}_t^{1-\delta} = \frac{1}{\alpha}(1-\alpha)\rho\chi D\delta k_{t+1}. \tag{12}$$

The above expression shows that, for k_{t+1} given, a higher mandatory retirement age increase private educational spending per unit of human capital. On the other hand, an increase in ρ has a negative impact on k_{t+1} via savings. This raises the question whether an increase in the mandatory retirement age speeds up or slows down economic growth. Using (3), equation (12) implies that

$$k_{t+1}h_{t+1} = \frac{\alpha}{(1-\alpha)} \frac{1}{\rho\chi\delta} \bar{e}_t h_t \tag{13}$$

[7] The derivation of the equilibrium levels of consumption, savings and private educational spending follows Lambrecht et al. (2005) and Kunze (2014).

We can now determine individual savings s_t (from (10), (11) and (13))

$$s_t = (1+\rho\chi)k_{t+1}h_{t+1} = \frac{\alpha(1+\rho\chi)}{(1-\alpha)\rho\chi\delta}\bar{e}_t h_t \tag{14}$$

and consumption c_t (using (5), (9), (10), (11) and (14)):

$$c_t = \frac{d_{t+1}}{\beta R_{t+1}} = \left(\frac{1+\rho\chi\beta}{\beta} + \frac{(1-\alpha)\rho\chi}{\beta\alpha}\right) k_{t+1}h_{t+1} = \frac{1}{\beta(1-\alpha)}\frac{1}{\rho\chi\delta}\left(\alpha+\rho\chi\right)\bar{e}_t h_t \tag{15}$$

Plugging (14) and (15) into (1) and solving for $\bar{e}_t$ gives

$$\bar{e}_t = \frac{(1-\alpha)}{\bar{B}(\rho)} A k_t^{\alpha} \tag{16}$$

with

$$\bar{B}(\rho) = 1 + \frac{\alpha(1+\rho\chi)}{(1-\alpha)\rho\chi\delta} + \frac{\alpha+\rho\chi}{\beta(1-\alpha)\rho\chi\delta} \tag{17}$$

Clearly, for given stocks of physical and human capital, k_t and h_t, the educational spending per unit of human capital e_t increases with ρ (note that $\partial\bar{B}(\rho)/\partial\rho < 0$), whereas savings decrease (as $\partial((1+\rho\chi)/(\rho\bar{B}(\rho)))/\partial\rho < 0$) and the effect on first period consumption is generally ambiguous.

The dynamics of the physical to human capital ratio k_t result from combining (12) and (16):

$$\left[\frac{1}{\alpha}(1-\alpha)\rho\chi D\delta k_{t+1}\right]^{\frac{1}{1-\delta}} = \bar{e}_t = \frac{(1-\alpha)}{\bar{B}(\rho)} A k_t^{\alpha} \tag{18}$$

or, equivalently,

$$k_{t+1} = \left[\frac{(1-\alpha)}{\bar{B}(\rho)} A\right]^{1-\delta} \frac{\alpha}{(1-\alpha)\rho\chi D\delta} k_t^{\alpha(1-\delta)} \tag{19}$$

which converge monotonically towards a steady state $(k, \bar{e})$. To assess the growth effect of an increase in the mandatory retirement age, we derive the long-run educational spending per unit of human capital. It is obtained by rearranging (18) in steady state:[8]

$$\bar{e}^{1-\alpha(1-\delta)} = \frac{(1-\alpha)}{\bar{B}(\rho)} A \left[\frac{\alpha}{(1-\alpha)\rho\chi D\delta}\right]^{\alpha} \tag{20}$$

Further inspection of equation (20) reveals:

Proposition 1 *There exists a growth-maximizing mandatory retirement age*

$$\hat{\rho} = \frac{(1-\alpha)\alpha(1+\beta)}{(1+(\delta+\alpha(1-\delta))\beta)\alpha\chi} \tag{21}$$

such that growth increases (decreases) with the retirement age if $\rho < \hat{\rho}$ ($\rho > \hat{\rho}$). When individuals are sufficiently impatient, i.e. $\beta < \frac{\alpha}{1-\alpha} \equiv \bar{\beta}$, this maximum is interior ($\hat{\rho} < 1$).[9]

[8] Note that the growth factor of the economy equals $g = h_{t+1}/h_t = D\bar{e}^{\delta}$.

[9] For example, suppose $\alpha = 0.3$, $\delta = 0.3$, $\beta = 0.5$ and $\chi = 0.8$. Then, $\hat{\rho} = 0.297$.

Proof: *Equation (20) can be rewritten as*

$$\bar{e}^{1-\alpha(1-\delta)} = \bar{M}\left(\frac{1}{\rho^{\alpha}\bar{B}(\rho)}\right) \tag{22}$$

where $\bar{M}$ is a constant. The logarithmic derivative of $\partial\bar{e}^{1-\alpha(1-\delta)}/\partial\rho$ then has the same sign as the function

$$\Psi(\rho) = (1-\alpha)(\beta\alpha+\alpha) - \rho\chi\alpha(1+\beta(\alpha(1-\delta)+\delta))$$

with $\Psi(0) > 0$ and $\Psi'(\rho) < 0$. Solving $\Psi(\rho) = 0$ for ρ gives equation (21). A sufficient condition for $\hat{\rho} < 1$ can be derived as follows: Note that

$$\hat{\rho} < 1 \quad \Leftrightarrow \quad \chi < \frac{(1-\alpha)\alpha(1+\beta)}{\alpha(1+\beta(\delta+\alpha(1-\delta)))} \equiv \bar{\chi} \tag{23}$$

where $\bar{\chi}$ is decreasing in δ. Setting $\delta = 0$ and solving $\bar{\chi} < 1$ gives $\bar{\beta}$.

Intuitively, an increase in the mandatory retirement age lowers savings, and thus physical capital accumulation and wages, due to a raise in old-age labor income. At the same time, however, it fosters educational spending as the return to human capital increases, implying that the overall effect on economic growth turns out to be ambiguous.

It is important to note that a non-linear relationship between mandatory retirement and economic growth can alternatively be obtained by assuming that private educational spending is motivated by altruism (as in Zhang and Zhang (2009)) and that labour productivity in old age is proportional to the average productivity in the economy. Put differently, the crucial element for our result is the trade-off between investment in education and savings rather than the underlying motivation for private educational investment.

3 Conclusion

This note shows that the relationship between mandatory retirement and economic growth is inverted U-shaped. In view of the current reform efforts in many OECD countries, these findings are highly relevant from a policy perspective. Specifically, they imply that starting from a relatively low mandatory retirement age, the net effect of a further increase in the retirement age is to raise the growth rate whereas the net effect turns out to be negative in countries in which the retirement age is already sufficiently high.

In the present model, the overall growth effect is determined by the balance of two opposing effects: A direct positive effect through increases in the rate of return to education and a negative general equilibrium effect lowering the wages of workers and thus decreasing the return to education. Our model could be extended to study how increasing the mandatory retirement age affects individuals' welfare or how both the size of the transfer to old households and the mandatory retirement age are jointly determined within a politico-economic equilibrium.

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