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The stochastic volatility model with random jumps and its application to BRL/USD exchange rate.

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Abstract

This work proposes the application of a stochastic volatility model with jumps to the BRL/USD exchange rate. This model decomposes the process into transitory and permanent components that capture the jumps in the level of the unobserved volatility process. The model estimation is done using Bayesian inference, and jumps at the level of the volatility of the exchange rate are analyzed according to the main economic events in this sample. We conclude that the model is consistent with the changes in the Brazilian economy and the crises observed in the analyzed period.

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1 Introduction and Methodology

In a floating exchange rate regime, which is adopted in Brazil since July 1999, the exchange rate varies according to demand and supply of foreign currency in the market. However, Brazil, like most countries using this system, uses interventions in the currency market, having as one of its goals the reduction of its oscillations, leading to greater predictability of exchange rate and attracting more foreign investment to the country. The Central Bank interventions can soften the behavior of the exchange rates through accommodation of sudden excess demand or supply of foreign currency, controlling part of the inherent risk related to the external sector linked to excessive volatility in exchange rate transactions, as discussed in Teixeira et al. (2013).

In this context, econometric models that explain the volatility of the exchange rate series are relevant to assist the monetary authority in its role of maintaining a solid and efficient financial system through the exchange rate policy, either applying direct intervention in the currency spot market that occurs with the help of exchange rate dealers, either through with currency swaps and direct operations in future markets. Moreover, market participants like banks and companies have an interest in calculating the volatility of the exchange rate with the objective of carrying out speculative operations and hedging exchange rate risk in portfolios.

Given the importance of modeling the volatility of the exchange rate, we conducted in this work an application of a stochastic volatility model with jumps, similar to that proposed by Qu and Perron (2013), using a jump model for the level of latent volatility through a compound binomial process, and present its application to the BRL/USD exchange rate, using data that cover the period 1999-2014. The results indicate that the model can adequately explain the observed changes in the volatility of this series, explaining the economic changes and crisis observed in this period.

This model corrects some shortcomings of the usual stochastic volatility models, which represent the logarithm of the conditional volatility as a latent factor using a first order autoregressive process. However, this representation may be inadequate to capture some events observed in exchange rates in emerging markets. In particular, it is possible to notice that there are distinct volatility patterns (regimes) for some countries, which may be associated with crisis periods. Note that in the original representation of the stochastic volatility model of Taylor (1986) the probability of sudden changes in the volatility process is low, because the specification of the innovations as originating from a continuous Gaussian distribution process. Another problem is that in this representation all shocks have transitory effects, and so large shocks, possibly associated with negative news or crises has no permanent effect on the process. These two characteristics are not consistent with the permanent effects and changes in the level of volatility observed in periods of crisis in emerging countries.

For a more realistic characterization of the process of exchange rate volatility in emerging countries in our analysis we use a parameterization similar to the model proposed by Qu and Perron (2013). In this model the process of latent volatility is decomposed as the sum of two components that capture the short-term dynamics and the random level shifts in the series. In this case, the components in question are respectively a first order autoregressive process, analogous to the original model of Taylor (1986), and a compound binomial process, which represents the permanent component of volatility, in the form:

$$r_t = exp\left(\frac{h_t}{2} + \frac{\mu_t}{2}\right)\varepsilon_t \tag{1}$$

$$h_{t+1} = \phi h_t + \sigma_\upsilon \upsilon_t \tag{2}$$

$$\mu_{t+1} = \mu_t + \delta_t \sigma_\eta \eta_t \tag{3}$$

The autoregressive component is represented by h_t, while the latent factor representing the level of volatility if given by μ_t. We assume that the terms η_t, υ_t and ε_t are independent standard normal random variables and δ_t is a sequence of independent Bernoulli random variables taking the value one with unknown probability p. The main innovation in this model is the association of δ_t variable with the jumps observed in the component μ_t. If the realization of binomial variable δ in period t is zero, the variable μ_t remains with the same value of period $t-1$. However, if the δ_t variable takes value one, the process μ_t is added to a Gaussian innovation process with volatility σ_η, representing the size of the jumps observed in this permanent component.

In this process, the conditional volatility of returns is obtained as the sum of these two components, enabling a very intuitive interpretation of the volatility process. Jumps can be associated with unforeseen events that change permanently the level of the volatility process, and thus may be associated with negative news or crises, while the autoregressive process captures the mean reverting behavior of non-permanent shocks. Thus we can also understand the proposed in Qu and Perron (2013) as a regime change model, but where the number of regimes and the volatility in each regime are not fixed a priori, as assumed in the usual regime switching models, and so this specification gives greater flexibility to the volatility process.

To perform the inference procedure of inference we use Bayesian estimation using Markov Chain Monte Carlo methods, similar to that described in Qu and Perron (2013). This procedure is based on the usual MCMC methods used in Bayesian estimation of stochastic volatility models, with the main change being the use of a data augmentation method for sampling the jump process, introducing a new latent variable to capture this process. In this way the Bernoulli variable takes the value one when this latent variable is greater than a certain threshold, calibrated so that the probability of exceeding this threshold is equal to the probability of the jumps in the volatility process. We use a prior scheme similar to the used in Qu and Perron (2013), in particular using the same Beta prior for the probability of jumps in the model. We also modify the MCMC procedure to directly sample the model using directly equations (1)-(3), without the linearization used in Qu and Perron (2013), by using a Metropolis-Hastings step in this stage due to the non-linearity. The inference procedure is based on a burn-in of 8000 samples, and calculating the posterior distributions for latent factors and parameters using 12000 additional samples[1].

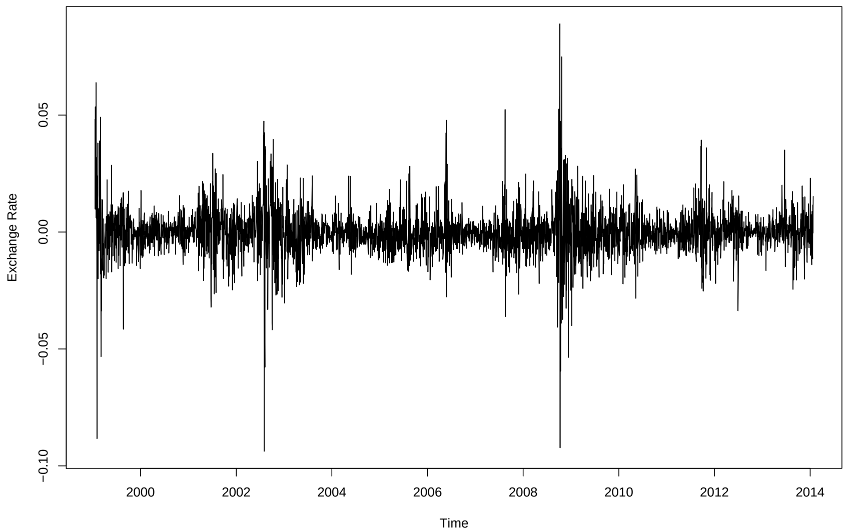
2 Results

The application of the model described above was made for daily returns of the BRL/USD exchange rate in the period between 18/01/1999 and 24/01/2014, with a sample of 3775 observations. The sample starts from the end of exchange rate target zone in Brazil to the most recent sample during the preparation of the article. Figure 1 shows the returns of the exchange rate in that period, evidencing the process of stochastic volatility and the changing patterns in this series. The estimation results of the model can be seen in figures 2, 4 and 3, which represent the posterior mean and 95 % credibility intervals for the components μ_t, h_t and smoothed probability of the jump process δ_t.

We can observe that the processes μ_t and h_t oscillate considerably in this period. The year 2002, being a time of great political transition, led to great uncertainty in the market, bringing, among other problems, the increased volatility in the exchange rate. The end of 2002 was also marked by rapid increases in the interest rates in Brazil, plus a large number of debt swap contracts to roll the public debt, which could explain the greater exchange rate fluctuation. The volatility of the level μ_t of exchange rate thereafter declined significantly, going from -3.62 in 2003 to -5.00 in March 2004. To seek an explanation for this fact, we can analyze the work of de Souza and Hoff (2006), whereby the Central Bank intervention in the foreign exchange market can be divided into six main phases in the period between the flexibilization of the Brazilian Exchange (1999) and the year 2006. In the named phase five, which runs from January 2003 through January 2005, the author mentions that Brazil has a more stable time in which the scenario was basically of a new government that increased the basic interest rate initially from 22% for 25%, and defined the primary surplus target at 4.5%. Also, the Central Bank made sales of dollar reserves in lower volume than it had done previously, which may explain the reduction in the volatility of the period. The country risk at the time was reduced and the exchange rate has appreciated considerably, reflecting

[1]Implementation details, the complete structure of priors, fit and convergence measures and comparison with other models are not presented for space reasons but can be obtained from the authors.

Figure 1: BRL/US$ Exchange Rate Returns

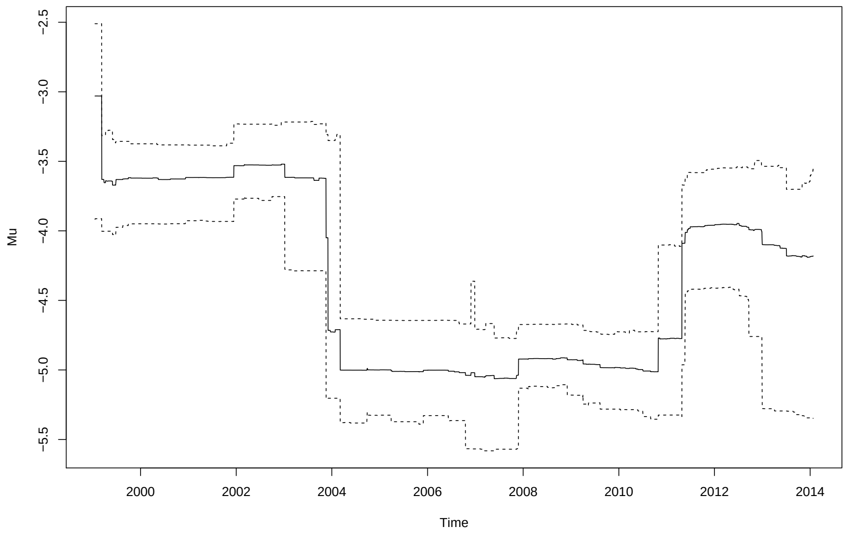


directly on the component μ_t, explaining the high probability of a negative jump in the level of volatility observed in figure 3 in this period.

According to data presented in Teixeira et al. (2013), the capital and financial account of the Brazilian balance of payments suffered considerable variation between the years 2003 and 2004, respectively registering a surplus of R$ 5.1 billion and a deficit of R$ 7.5 billion. Moreover, the Central Bank intervention in the foreign exchange market in 2004 was considerably higher than the 2003 (R$ 5.3 billion, compared to R$ 1.6 billion), while the external operations of the Central Bank and the Treasury were reduced to a deficit of R$ 9 billion. This data set shows that the government had to again perform interventions in the currency market, explicitly having as one of its goals the reduction in rate volatility. Indeed, this information supports a finding that we obtained from the model (the reduction in the volatility of the exchange rate in 2004), maintained until the year 2008, as can be seen in the processes μ_t and h_t estimated by the model.

In addition to these findings, the Figures 2 and 4 also show that the year 2008 presented considerable variations in the level of volatility of the exchange rate. The crisis that originated in the U.S. housing sector contributed greatly to the great variability in BRL/USD exchange rate. News about the intense liquidity problem experienced by Bear Stearns bank,

Figure 2: Level Component μ_t

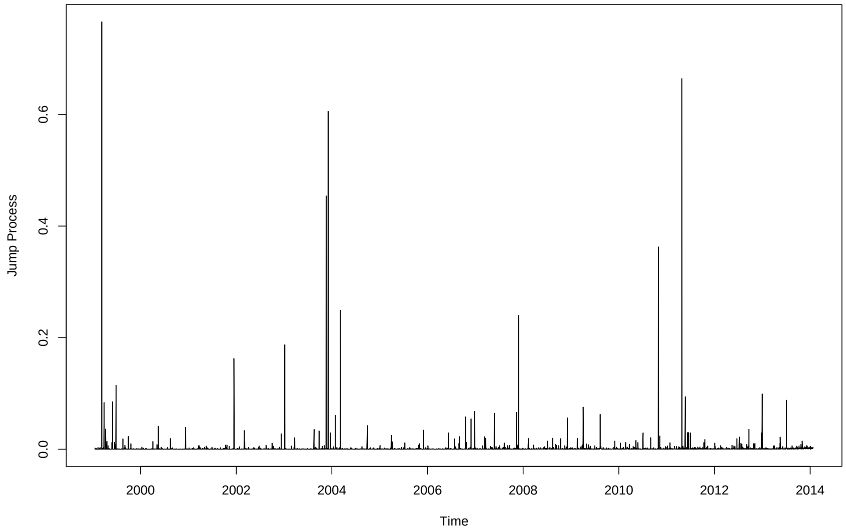


the declaration of bankruptcy of Lehman Brothers, among many other impactful events, leading to the great financial crisis after the problems of the American mortgage system contributed significantly to the increase in the volatility of the exchange rate. However, most of this period of increased volatility may be associated with transitory factor h_t, as can be seen in Figure 4.

Another important pattern is the volatility of the exchange rate in the period 2010-2011, corresponding to the other jump observed in the process μ_t. In this period the exchange rate jump from 1.54 in July 2011 to 1.90 BRL/USD in September of the same year, with high volatility. According to Rossi (2013) this period is notable for the behavior of investors, and foreign and institutional investors remained mainly sold in dollar futures positions (betting on the appreciation of the Brazilian currency), while banks generally positioned on the contrary position. This situation of high fluctuations in currency market generated the need for foreign exchange intervention by the government, which announced the tax on financial operations (IOF) of 1% on net short positions in currency derivatives, with the possibility of rate be increased to 25%.

Moreover, many events in 2011 may help explain the high volatility observed in the model. The sovereign debt crisis in Europe, for example, caused a flight of capital to assets

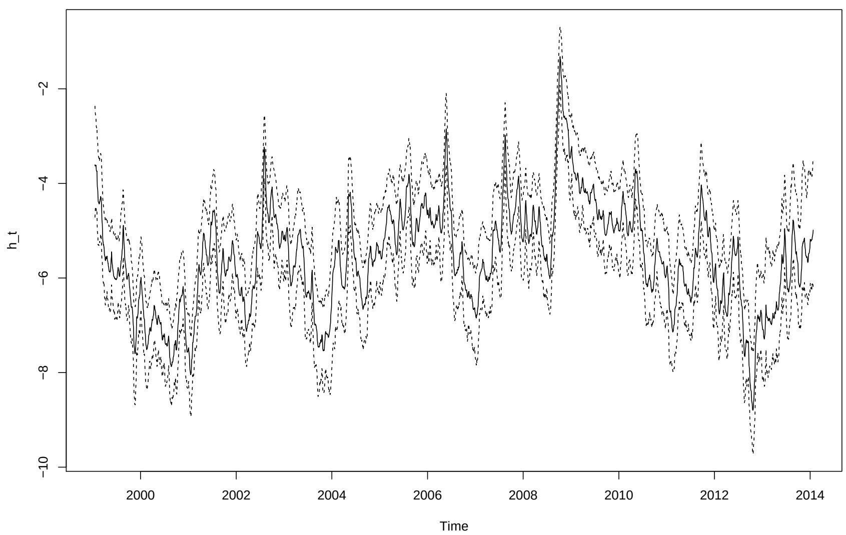
Figure 3: Posterior Jump Probability δ_t



considered safe in September, especially for the dollar, affecting many emerging countries. However, by observing Figure 4 is also clear that the magnitude of transitory component h_t rate volatility in 2012 was reduced. Between the months of July and November 2012, for example, the exchange rate remained between 2.00 and 2.05. According to information of the time, the limited volatility of exchange rates in the period is a direct result of the actions and statements of the Central Bank, with of the government economic team advocating the called "informal band" adopted for the dollar and considered ideal by the government to boost industry without creating inflationary pressures.

Figure 5 shows the good fit for the observed volatility process obtained by the sum of the components μ_t, h_t compared to the absolute returns of the exchange rate, indicating that the proposed model is an adequate representation of the volatility process of the series in the analyzed period. Figure 6 presents the posterior distribution of the relevant parameters of the stochastic volatility process, with the vertical line showing the posterior mean of each parameter. The first graph shows the posterior distribution of the parameter ϕ. We note that this component also has high persistence, with a posterior closer to, but less than one. In this case we have that the shocks are also quite persistent in the autoregressive process, although the process still keeps the mean reversion property. This feature is consistent with

Figure 4: Autoregressive Component h_t

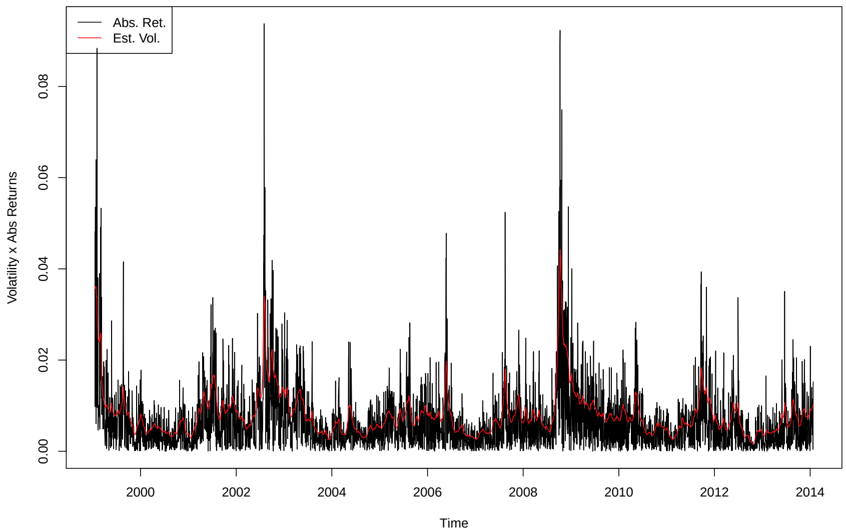


the high persistence in the volatility of shocks to the exchange rates of emerging countries. In this figure we also show the posterior distribution of the parameters of the jump volatility σ_η and the probability of jumps parameter p. We observe a large variability in the jump process volatility, consistent with the nature of the shocks observed in μ_t. Note that the probability of jumps in the process is relatively small, consistent with 3 observed jumps with posterior probability higher than .5 observed in Figure 3. Although the probability of jumps is small, the size of the jumps observed in the log-volatility process is quite high, showing the relevance of the component μ_t in modeling the volatility of this series.

3 Conclusions

This paper proposed a stochastic volatility model with random jumps applied to the BRL/USD exchange rate. We show that the exchange rate presented large jumps in the level of volatility since the year 1999 when the full floating exchange rate regime was adopted. Through the analysis of the permanent (μ_t) and transitory (h_t) components, we note that the jumps captured by the model are consistent with moments with considerable changes in the economy and emerging market crises. This model has a good fit to the data in this period, and allows

Figure 5: Fitted Volatility and Absolute Returns of BRL/US$ Exchange Rate Returns



a more appropriate structure for the volatility processes in emerging markets subject to rapid changes in volatility due to economic changes and exposure to external shocks. Further developments using this method can be performed constructing forecasts for the exchange rate and applying the model in currency hedging and risk management.

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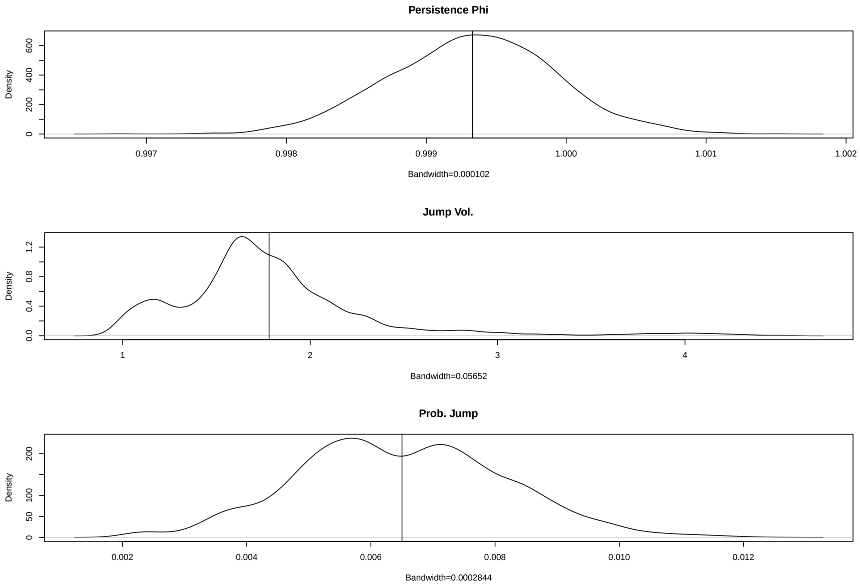
Figure 6: Estimated Posterior Distribution of Parameters



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