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Price and quantity competition in network goods duopoly: a reversal result

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Abstract

This paper revisits the classic profit-ranking of Cournot and Bertrand equilibria and the issue of endogenous choice of strategic variables for product market competition, but for a network goods duopoly. It demonstrates that in the case of strong network externalities and imperfect-substitute goods (a) the classic profit-ranking is reversed - each firm earns higher profit under Bertrand competition than that under Cournot competition and (b) firms face a prisoners' dilemma type of situation while choosing between a price contract and a quantity contract and end up with Pareto inferior outcomes, unlike as in the case of standard non-network goods duopoly.

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1. Introduction

In their seminal paper, using the standard differentiated goods duopoly framework à la Dixit (1979), Singh and Vives (1984) demonstrate that (1) Cournot competition yields higher (lower) profits than Bertrand competition and (2) choosing quantity (price) contract is the dominant strategy for each firm, if the goods are substitutes (complements). This paper reexamines these two results, but for a network goods duopoly. It shows that, in the case of strong network externalities and imperfect-substitute goods, the standard Cournot-Bertrand profit ranking is reversed. Moreover, firms face a prisoners' dilemma type of situation while choosing between a quantity contract and a price contract, unlike as in the case of standard non-network goods duopoly à la Singh and Vives (1984).

Considering the theoretical and practical importance of Singh and Vives (1984)'s results, several authors have examined the robustness of these results. For example, while Cheng (1985), Vives (1985), Okuguchi (1987), Tanaka (2001) and Tasnadi (2006) argue that these results are quite robust, some other studies show that these results might not hold true under (a) asymmetric costs (Dastidar, 1997; Hackner, 2000), (b) managerial delegation (Miller and Pazgal, 2001), (c) union-firm bargaining (Lopez and Nayor, 2004) and (d) mixed duopoly (Ghosh and Mitra, 2010; Matsumura and Ogawa, 2012; Scrimitore, 2013). These studies offer useful insights to understand the applicability of Singh and Vives (1984)'s results in many different market structures, but for standard non-network goods only.

This paper is closely related to two studies that examine the implication of managerial delegations on equilibrium outcomes in network goods duopoly. Bhattacharjee and Pal (2013) focus on examining the role of Miller and Pazgal (2001)'s relative-performance based managerial delegation contracts in the presence of network externalities. On the other hand, Chircoa and Scrimitore (2013) consider Fershtman and Judd (1987)-type managerial delegation contracts and attempt to examine the second result of Singh and Vives (1984), but they do not analyze the nature of the equilibrium and, thus, fail to recognize the possibility of emergence of a prisoners' dilemma type of situation in the case of network goods duopoly. Moreover, the question of whether Singh and Vives (1984)'s profit ranking, i.e., the first result, remains valid in the case of network goods duopoly remains unanswered. This paper attempts to fill these gaps in the literature.

2. The model

We consider an economy with a network goods sector with two firms (firm 1 and firm 2), each one produces a differentiated good and incurs constant marginal (average) cost of production c (≥ 0), a competitive numeraire sector and many consumers. Following Hoernig (2012), we consider that demand functions of network goods are as follows.

$$x_i = \alpha + ny_i - p_i + \beta p_j, \; i, j = 1, 2, \, i \neq j; \tag{1a}$$

where x_i and p_i denote quantity and price, respectively, of good i and y_i denotes consumers' expectation about firm i's total sale. α $(> c)$, $\beta \in (0,1)$ and $n \in [0,1)$ are demand parameters. Lower value of parameter β corresponds to the case of higher degree of product differentiation. The parameter n $(=\frac{\partial x_i}{\partial y_i})$ measures the strength of network externalities - lower value n indicates weaker network externalities. Note that above demand functions are similar to that in Singh and Vives (1984), except for the term ny_i. From (1a), we get the corresponding inverse demand functions as follows.

$$p_i = \frac{\alpha\,(1+\beta) - x_i - \beta\, x_j}{1-\beta^2} + \frac{n\,(y_i + \beta\, y_j)}{1-\beta^2},\ i, j = 1,\, 2,\, i \neq j. \tag{1b}$$

Note that, as in Economides (1996), network externalities enter additively in demand functions. Also, $\frac{\partial p_i}{\partial y_i} = \frac{n}{1-\beta^2} > \frac{\partial p_i}{\partial y_j} = \frac{\beta\, n}{1-\beta^2} > 0$, $\forall n \in (0,1)$ and $\forall \beta \in [0,1)$. These demand functions can be derived from the following form of the representative consumer's utility function.

$$\begin{aligned} U(x_1, x_2, y_1, y_2) =& \frac{\alpha(x_1 + x_2)}{1-\beta} - \frac{x_1^2 + 2\beta x_1 x_2 + x_2^2}{2(1-\beta^2)} \\ &+ n\frac{(y_1 + \beta y_2)x_1 + (y_2 + \beta y_1)x_2}{1-\beta^2} + nf(y_1, y_2) + m; \end{aligned} \tag{2}$$

where $f(.)$ is a symmetric function of consumers' expectations y_1 and y_2, and m is the amount of the numeraire good.

For the utility function to be well behaved and 'rational expectations' by consumers to be consistent, we consider that $f(y_1, y_2) = -n\frac{y_1^2 + 2\beta y_1 y_2 + y_2^2}{2(1-\beta^2)}$. Clearly, for any given consumption bundle $(x_1,\, x_2)$, (a) $U(.)$ is strictly concave in y_1 and y_2, and (b) utility is maximum when $y_1 = x_1$ and $y_2 = x_2$, i.e., correct expectations lead to highest level of utility. Also note that, if $n = 0$, the above mentioned quasi-linear form of the utility function is comparable to that considered in Singh and Vives (1984) and most of the subsequent studies on Cournot-Bertrand comparison. Further, consideration of the above form of the utility function is useful to keep the analysis simple and to clearly alienate the implication of network externalities on equilibrium outcomes. Nonetheless, if we consider alternative forms of the utility function, qualitative results of this paper are likely to go through.

2.1 Cournot competition

In the case of Cournot competition, taking x_j, y_i and y_j as given, firm i decides x_i to maximize its profit $\pi_i = (p_i - c)x_i$, where p_i is given by equation (1b). Solving firm i's problem, we obtain firm i's quantity reaction function (RF_i^C) as follows.[1]

$$x_i = \frac{[\alpha - c\,(1-\beta)]\,(1+\beta) + n\,(y_i + \beta\, y_j) - \beta\, x_j}{2},\ i, j = 1,\, 2,\, i \neq j. \tag{3}$$

[1]SOCs for maximization and stability conditions are always satisfied.

Clearly, RF_i^Cs are downward sloping in x_1x_2-plane. Given x_j, higher y_i and/or y_j shift RF_i^C outward, unless $n = 0$. The extent of such outward-shift is greater, if network externalities are stronger.

Following Katz and Shapiro (1985) and Hoernig (2012), we consider that consumers' form 'rational expectations', which implies that in equilibrium $y_i = x_i$. Solving RF_1^C and RF_2^C together with $y_1 = x_1$ and $y_2 = x_2$, we obtain the equilibrium quantities and resulting prices, profits, consumers' surplus (CS) and social welfare (SW) as follows.[2]

$$x_1^C = x_2^C = x^C = \frac{[\alpha - c\,(1-\beta)]\,(1+\beta)}{2-n+(1-n)\,\beta},\ p_1^C = p_2^C = p^C = \frac{\alpha + c\,(1-n)\,\left(1-\beta^2\right)}{(1-\beta)\,[2-n+(1-n)\,\beta]},$$
$$\pi_1^C = \pi_2^C = \pi^C = \frac{[\alpha - c\,(1-\beta)]^2\,(1+\beta)}{(1-\beta)\,[2-n+(1-n)\,\beta]^2},\ CS^C = \frac{(1-n)\,(\alpha - c\,(1-\beta))^2(1+\beta)^2}{(1-\beta)\,(2-n+(1-n)\,\beta)^2}, \quad (4)$$
$$\text{and } SW^C = \frac{[\alpha - c\,(1-\beta)]^2\,(1+\beta)\,[3-n+(1-n)\,\beta]}{(1-\beta)\,[2-n+(1-n)\,\beta]^2},$$

where superscript 'C' indicates Cournot equilibrium.

It is easy to check that (a) $\frac{\partial p^C}{\partial n} > \frac{\partial x^C}{\partial n} > 0$, $\frac{\partial \pi^C}{\partial n} > 0$ and $\frac{\partial SW^C}{\partial n} > 0$, $\forall\, n \in [0, 1)$; and (b) $\frac{\partial CS^C}{\partial n} > (<) 0$, if $0 \leq n < \frac{\beta}{1+\beta}$ ($\frac{\beta}{1+\beta} < n < 1$). Clearly, due to demand-shifting effect of network externalities, stronger network externalities lead to higher output and price and, thus, higher profit of each firm in Cournot equilibrium.

2.2 Bertrand competition

Now, consider the situation in which firm i sets its price p_i, taking p_j, y_i and y_j as given, to maximize its profit $\pi_i = (p_i - c)x_i$, where x_i is given by equation (1a). Solving firm i's problem, we obtain its price reaction function (RF_i^B) as follows.

$$p_i = \frac{\alpha + c + n\,y_i + \beta\,p_j}{2},\ i, j = 1,\,2,\, i \neq j. \quad (5)$$

Note that RF_i^Bs are upward sloping in p_1p_2-plane. Interestingly, unlike RF_i^C, RF_i^B is not directly dependent on y_j.

Solving RF_1^B, RF_2^B, $y_1 = x_1$ and $y_2 = x_2$, we obtain the Bertrand equilibrium prices and corresponding quantities, profits, consumers' surplus and social welfare as follows.

$$p_1^B = p_2^B = p^B = \frac{\alpha + (1-n)\,c}{2-n-\beta},\ x_1^B = x_2^B = x^B = \frac{\alpha - c\,(1-\beta)}{2-n-\beta},$$
$$\pi_1^B = \pi_2^B = \pi^B = \frac{[\alpha - c\,(1-\beta)]^2}{(2-n-\beta)^2},\ CS^B = \frac{(1-n)\,[\alpha - c\,(1-\beta)]^2}{(1-\beta)\,(2-n-\beta)^2}, \quad (6)$$
$$\text{and } SW^B = \frac{[\alpha - c\,(1-\beta)]^2\,(3-n-2\,\beta)}{(1-\beta)\,(2-n-\beta)^2},$$

where superscript 'B' indicates Bertrand equilibrium.

[2] $SW = U(.) - cx_1 - cx_2$ and $CS = SW - \pi_1 - \pi_2$.

Interestingly, firms behave less aggressively and set higher prices, if network externalities are stronger: $\frac{\partial p^B}{\partial n} > 0$, $\forall\ n \in [0, 1)$. It is also easy to check that $\frac{\partial p^B}{\partial n} = \frac{\partial x^B}{\partial n} > 0$, $\frac{\partial \pi^B}{\partial n} > 0$ and $\frac{\partial SW^B}{\partial n} > 0$, $\forall\ n \in [0, 1)$; but $\frac{\partial CS^B}{\partial n} > (<) 0$, if $0 \leq n < \beta$ $(\beta < n < 1)$.

3. Cournot versus Bertrand equilibria

Comparing the equilibrium outcomes under Cournot and Bertrand competition, from (4) and (6), we get the following.

Lemma 1: $p^B < p^C$ *and* $x^B > x^C$, $\forall\ n \in [0, 1)$.
Proof: See Appendix 1.

Proposition 1: *In the presence of strong network externalities* $(n > n_0)$*, profits under Bertrand equilibrium are higher compared with that under Cournot equilibrium; where* $n_0 = 1 - \sqrt{\frac{1-\beta}{1+\beta}}$, $0 < n_0 < 1\ \forall\ \beta \in (0, 1)$. *Otherwise, if network externalities are weak* $(n < n_0)$*, the reverse is true.*
Proof: See Appendix 2.

The intuition behind this result is as follows. Lemma 1 implies that firms are more aggressive in the product market under Bertrand competition than that under Cournot competition, although $\frac{\partial x^C}{\partial n} > 0$ and $\frac{\partial p^B}{\partial n} > 0$, $\forall n \in [0, 1)$. Now, in network goods duopoly, more aggressive play affects firms' profits through two channels: (a) it leads to lower prices and, thus, has a *direct negative effect* on profits and (b) it has an *indirect positive effect* on profits via consumers' expectations, which is higher in the case of more aggressive play. If network externalities are strong $(n > n_0)$, the *indirect positive effect* of more aggressive play on profits dominates the associated *direct negative effect*.

It is straightforward to check that (a) $0 < \frac{\partial x^C}{\partial n} < \frac{\partial x^B}{\partial n}$, $\forall n \in [0, 1)$ and $\beta \in (0, 1)$; (b) $0 < \frac{\partial p^B}{\partial n} < \frac{\partial p^C}{\partial n}$, if $\beta \in (0, 1)$ and $0 \leq n < n_0$; and (c) $0 < \frac{\partial p^C}{\partial n} < \frac{\partial p^B}{\partial n}$, if $\beta \in (0, 1)$ and $n_0 < n < 1$. Thus, if both conditions (a) and (c) are satisfied, the classic Cournot-Bertrand profit ranking is reversed.

We note here that $CS^B > CS^C$ and $SW^B > SW^C$, $\forall n \in [0, 1)$ (see Appendix 3).

4. Endogenous modes of competition

We, now, consider the following two stage game. In stage-1 firms choose between a price contract and a quantity contract, simultaneously and independently, and then compete accordingly in stage-2. The reduced-game in stage-1 can be represented as in Figure 1, where the first (second) entry in each cell of the payoff-matrix is the profit of firm 1 (firm 2) corresponding to the associated strategy-pair of firms. π^Q (π^P) is the equilibrium profit of the quantity(price)-setting firm under asymmetric competition, in which one firm competes in price and the other firm competes in quantity (see Appendix 4 for details).

		Firm 2	
		Price	*Quantity*
Firm 1	*Price*	π^B, π^B	π^P, π^Q
	Quantity	π^Q, π^P	π^C, π^C

Figure 1: Firms' choice over strategic variables.

It can be checked that $\pi^Q > \pi^B$ and $\pi^C > \pi^P$, $\forall\ n \in [0,1)$ and $\beta \in (0,1)$. Therefore, as in Singh and Vives (1984), Cournot equilibrium constitutes the sub-game perfect Nash equilibrium (SPNE) of this game regardless of strengths of network externalities, as noted in Chircoa and Scrimitore (2013). However, note that $\pi^B > \pi^C$, if $\beta \in (0,1)$ and $n > n_0$ (by Proposition 1). That is, in the presence of network externalities, the possibility of emergence of a prisoners' dilemma type of situation cannot be ruled out. This is another new result.

Proposition 2: (a) *Choosing quantity contract is the dominant strategy for each of the two firms, regardless of the strength of network externalities.*
(b) *Unless network externalities are weak, firms face a prisoners' dilemma type of situation while choosing strategic variables, price vis-à-vis quantity, and end up with Pareto inferior outcomes.*
Proof: See Appendix 5.

So far, we have considered that goods are imperfect substitutes. It is straightforward to check that Lemma 1 holds true even when goods are complements ($-1 < \beta < 0$). However, unlike as in the case of substitute goods, both $0 < \frac{\partial x^C}{\partial n} < \frac{\partial x^B}{\partial n}$ and $0 < \frac{\partial p^C}{\partial n} < \frac{\partial p^B}{\partial n}$ hold true $\forall n \in [0,1)$ in the case of complementary goods duopoly. As a result, if goods are complements, $\pi^B > \pi^C$ holds true regardless of the strength of network externalities. It can also be checked that, if goods are complements, the strategy pair ($Price$, $Price$) constitutes the dominant strategy Nash equilibrium in stage-1 and, thus, the possibility of prisoners' dilemma type of situation does not arise.

5. Conclusion

We have demonstrated that the Cournot-Bertrand profit differential does not only depend on whether goods are substitutes or complements, as is often argued. Other characteristics of goods may influence it as well. In this paper, we have considered an example of network goods. It might be interesting to examine the implications of exclusive network goods (such as clubs), congestion effects and negative consumption externalities.

Appendix

1. Proof of Lemma 1

From (4) and (6), we get

$p^B - p^C = -\frac{(1-n)\{\alpha - c(1-\beta)\}\beta^2}{(1-\beta)(2-n-\beta)\{2-n+(1-n)\beta\}}$ and $x^B - x^C = \frac{\{\alpha - c(1-\beta)\}\beta^2}{(2-n-\beta)\{2-n+(1-n)\beta\}}$.

Also, we have $0 \leq n < 1$, $0 < \beta < 1$ and $0 \leq c < \alpha$. Therefore, $p^B - p^C < 0$ and $x^B - x^C > 0$, $\forall\ n \in [0, 1)$.

2. Proof of Proposition 1

From (4) and (6), we get

$\pi^B - \pi^C = \frac{\{\alpha - c(1-\beta)\}^2}{(1-\beta)(2-n-\beta)^2\{2-n+(1-n)\beta\}^2}[(1-\beta)\{(2+\beta) - n(1+\beta)\}^2 - (2-\beta-n)^2(1+\beta)]$.

Clearly ,

$$\begin{aligned} sign\,(\pi^B - \pi^C) &= sign\,[(1-\beta)\{(2+\beta) - n(1+\beta)\}^2 - (2-\beta-n)^2(1+\beta)] \\ &= sign\left[n - \left(1 - \sqrt{\frac{1-\beta}{1+\beta}}\right)\right]. \end{aligned}$$

Therefore, if $n > (<) 1 - \sqrt{\frac{1-\beta}{1+\beta}} = n_0$, $\pi^B > (<) \pi^C$. Note, since $0 < \beta < 1$, $0 < n_0 < 1$.

3. Consumers' surplus and social welfare

From (4) and (6), we get

$CS^B - CS^C = \frac{(1-n)\{\alpha - c(1-\beta)\}^2\beta^2\{2(1-n)(1+\beta)+(2-\beta^2)\}}{(1-\beta)(2-n-\beta)^2\{2-n+(1-n)\beta\}^2} > 0$ and

$SW^B - SW^C = \frac{\beta^2\{\alpha - c(1-\beta)\}^2\{(2-\beta^2)(1-n)+2(1-\beta)\}}{(1-\beta)(2-n-\beta)^2\{2-n+(1-n)\beta\}^2} > 0$, since $0 \leq n < 1$, $0 < \beta < 1$ and $0 \leq c < \alpha$.

4. Asymmetric competition: Equilibrium

Let us consider that problems of firm i and firm j are $\underset{x_i}{Max}\,\pi_i(x_i, p_j)$ and $\underset{p_j}{Max}\,\pi_j(x_i, p_j)$, respectively, $i, j = 1, 2$, $i \neq j$. [3] From the FOCs, we obtain reaction functions of firm i and firm j, respectively, as follows.

$$x_i = \frac{\alpha - c + n\,y_i + \beta\,p_j}{2} \tag{7a}$$

$$p_j = \frac{\alpha\,(1+\beta) + c\,\left(1-\beta^2\right) + n\,(\beta\,y_i + y_j) - \beta\,x_i}{2\,(1-\beta^2)}. \tag{7b}$$

[3] $\pi_i(x_i, p_j) = (p_i - c)x_i$ and $\pi_j(x_i, p_j) = (p_j - c)x_j$, where $p_i = \alpha + \beta\,p_j - x_i + n\,y_i$ and $x_j = \alpha\,(1+\beta) - \left(1-\beta^2\right)\,p_j + n\,(\beta\,y_i + y_j) - \beta\,x_i$.

Solving (7a), (7b), $y_i = x_i$ and $y_j = x_j$, we get

$$
\begin{aligned}
x^Q &= \frac{\{\alpha - c\,(1-\beta)\}\{2 - n\,(1-\beta) - \beta\}\,(1+\beta)}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2},\ x^P = \frac{\{\alpha - c\,(1-\beta)\}\,(2-n+\beta)\left(1-\beta^2\right)}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2},\\
p^Q &= \frac{\alpha\left\{(2-\beta)\,(1+\beta) - n\left(1-\beta^2\right)\right\} + c\,(1-n)\,\left\{2 - n\left(1-\beta^2\right) + \beta\left(1-\beta-\beta^2\right)\right\}}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2}\\
p^P &= \frac{\alpha\,(2-n+\beta) + c\,(1-n)\,\left\{2-n+\beta-(2-n)\,\beta^2\right\}}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2}, \qquad (8)\\
\pi^Q &= \left[\frac{\{\alpha - c\,(1-\beta)\}\{2 - n\,(1-\beta) - \beta\}\,(1+\beta)}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2}\right]^2,\ \text{and}\\
\pi^P &= \left[\frac{\{\alpha - c\,(1-\beta)\}\,(2-n+\beta)}{(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2}\right]^2\left(1-\beta^2\right),
\end{aligned}
$$

where θ^P and θ^Q denote the equilibrium $\theta = \{x, p, \pi\}$ of price-setting firm and quantity-setting firm, respectively, since firms are otherwise identical.

5. Proof of Proposition 2

From (4), (6) and (8), we get

$$
\pi^Q - \pi^B = \frac{(1-n)\,\{\alpha - c\,(1-\beta)\}^2\,\beta^3\,\left[2\,(2-n)^2 - 2\,\{3-(3-n)\,n\}\,\beta^2 + (1-n)\,\beta^3\right]}{(2-n-\beta)^2\left[(2-n)^2 - \{3-(3-n)\,n\}\,\beta^2\right]^2},
$$

and

$$
\pi^C - \pi^P = \frac{(1-n)\,\{\alpha - c\,(1-\beta)\}^2\,\beta^3\,(1+\beta)\left[2(2-n)^2 - 2\{3-(3-n)\,n\}\beta^2 - (1-n)\,\beta^3\right]}{(1-\beta)\,\{2-n+(1-n)\,\beta\}^2\left[(2-n)^2 - \{3-(3-n)\,n\}\beta^2\right]^2}.
$$

Clearly, $\pi^Q - \pi^B > 0$ and $\pi^C - \pi^P > 0$, since $0 \le n < 1$, $0 < \beta < 1$ and $0 \le c < \alpha$.

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