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An experimental test of the mixed strategy equilibrium in price-quantity oligopolies

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Abstract

We experimentally study price-quantity competitions in duopoly and triopoly markets and compare the results with the respective mixed strategy equilibria. The equilibrium does not predict experimental outcomes adequately, but we find indications for a reactive strategy in the experiments.

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1. Introduction

Since Shubik (1955), a vast stream of literature studies competition with both prices and quantities as decision variables. Although Gertner (1986) established the mixed strategy equilibrium for price-quantity (PQ) games with non-increasing production cost, it remained unrecognised in the literature. As a consequence, no study so far has tested the mixed strategy equilibrium of the price-quantity (PQ) game using empirical data.[1]

In this article, we study a PQ game with constant marginal cost. Whilst the game is easy to understand, it comprises a non-trivial mixed strategy equilibrium (Gertner, 1986) and is therefore ideal for an experimental study. Our contribution is three-fold: (i) we discuss the mixed strategy equilibrium of the n-firm PQ game under the assumption of linear demand and constant marginal cost, first presented in Gertner (1986); (ii) we compare the equilibrium prediction to experimental data for duopolies and triopolies; (iii) we find that the equilibrium has low predictive power for the experimental data and briefly discuss a reactive strategy as an alternative explanatory approach.

2. The model

In this section we present the general model used for the experiments along with theoretical results from the literature. We start by explaining the duopoly game, before stating some results for a general game with n (> 2) firms. Let us therefore initially consider a game of two firms $(i = 1, 2)$ that decide simultaneously on their price p_i and their production level q_i. Products are assumed to be homogeneous between the firms and the market demand is a given function $D(p)$. The game follows the winner-takes-all-rule, i.e. the firm i with the lower price sells its full output q_i up to the market demand $D(p_i)$. The firm j $(j \neq i)$ that decided on the higher price can now satisfy the residual demand, which is given through the efficient rationing rule

$$D(p_j|p_i) = D(p_j) - s_i\,,$$

[1] The existing works of Brandts and Guillen (2007) and Davis (2013) study the PQ game without any reference to the theoretical equilibrium.

where s_i is the amount sold by the lower-price competitor i.[2] For the case of equal prices ($p_1 = p_2$), the market demand is shared equally between the firms, as far as the production levels q_i allow. These rules can be summed up by the following equation for the sales s_i of firm i (Gertner, 1986),

$$s_i(p_1, q_1, p_2, q_2) = \begin{cases} \min[q_i, D(p_i)] & , \text{ if } p_i < p_j \,, \\ \min[q_i, D(p_i) - s_j] & , \text{ if } p_i > p_j \,, \\ \min\left[q_i, D(p_i) - \min\left\{q_j, \frac{D(p_j)}{2}\right\}\right] & , \text{ if } p_i = p_j \,. \end{cases} \quad (1)$$

To find an expression for the payoff π_i of firm i we introduce the production cost $C(q)$, which is assumed to be equal for both firms. Using s_i as given in (1), the payoff π_i is given by

$$\pi_i = p_i s_i - C(q_i) \,.$$

Gertner (1986) explains that a pure strategy equilibrium does not exist in this game. Hence, we focus on a mixed strategy equilibrium, i.e. each of the firms' strategies can be described by the probability density function $g_i(p_i, q_i)$ that formally states the probability of firm i to play the strategy (p_i, q_i). If we denote by G_i the probability distribution function related to g_i, then, according to Shubik (1959) $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ form a mixed strategy equilibrium, if the integrals

$$\bar{V}_i = \int_0^\infty \int_0^\infty \pi_i(p_1, q_1, p_2, q_2) dG_j(p_j, q_j) \,,$$

are constant for all strategies (p_i, q_i) played with positive probability according to $G_i(p_i, q_i)$. Shubik (1959) refers to $\bar{V}_i$ as the value of the game for firm i, i.e. the maximum guaranteed profit it can achieve if the strategy of the opposition player is known. Note that in the case of the symmetric game considered here, the mixed strategy equilibrium is also symmetric, which means $G_1 \equiv G_2$. For our experiments we make the following simplifying assumptions of linear demand and cost curves:

$$D(p_i) = a - bp_i \,, \qquad C(q_i) = cq_i \,,$$

[2] Davidson and Deneckere (1986) discuss different rationing rules. In general, the choice of the rationing rule can have a major impact on the equilibrium of an oligopoly game. For the model presented here, however, Gertner (1986) shows that the results are not affected by choosing efficient rationing instead of proportional rationing.

where a, b and c are non-negative constants. We are therefore considering a game with constant marginal cost, for which Gertner (1986) proved that all Nash equilibria satisfy $\bar{V}_i = 0$. The mixed strategy equilibrium derived in Gertner (1986) has the property that all strategies with positive probabilities are situated on the line $p = D(q)$, i.e. each firm always produces exactly the market demand $D(p_i)$ corresponding to the chosen price p_i. The probability distribution for the prices is given through the distribution function

$$F(p) = \begin{cases} 0\,, & \text{for } p < c\,, \\ 1 - c/p\,, & \text{for } c \leq p < a\,, \\ 1\,, & \text{for } p \geq a\,. \end{cases} \tag{2}$$

In particular, this implies that each firm has two options: (i) it can leave the market by choosing $p_i = a$ with a (non-zero) probability of c/a or (ii) it can stay in the market and choose a price from the interval $[c, a)$ using the distribution function $F(p)$ as given in (2). Looking at the probability density function corresponding to $F(p)$, we see that firms are more likely to play lower prices than higher prices. The lower price firm earns a positive profit, while the other firm faces losses equal to its production costs $C(q)$, but expected profits are equal to zero.

One can easily generalise the rules of the game for an arbitrary number ($n \geq 2$) of firms. The existence of a mixed strategy equilibrium can be generalised from the duopoly to the oligopoly game (Gertner, 1986). The distribution function related to the mixed strategy equilibrium in the oligopoly settings takes the form

$$F_n(p) = \begin{cases} 0\,, & \text{for } p < c\,, \\ 1 - (c/p)^{\frac{1}{n-1}}\,, & \text{for } c \leq p < a\,, \\ 1\,, & \text{for } p \geq a\,. \end{cases} \tag{3}$$

In particular, this implies that with increasing n the probability of market entry decreases, but the average price played in case of market entry increases. Similarly to the duopoly game, the expected profit for each of the firms is zero.

3. Experimental Procedure

Our experiment was designed to analyse the classical PQ game in a duopoly (*PQ2*) and a triopoly (*PQ3*) treatment. At the beginning of the experiment, we randomly assigned subjects to groups of 2 or 3 that remained fixed for the rest of the experiment with each of the subjects in a group controlling one of the symmetric firms A, B (or C). Each treatment consists of a two-stage game. In the five rounds of the first stage, we let each firm act in a monopolistic market to allow the participants time to get used to the game. Afterwards, in the 20 rounds of the second stage, firms competed in a common (duopoly or triopoly) market.

We programmed the experimental software using z-Tree (Fischbacher, 2007). We used the simplified linear demand function $D(p) = 100 - p$ $(a = 100, b = 1)$ and the constant marginal production cost $c = 10$. Firms had to choose prices and production levels in the range $[0, 100]$.[3] For both price and quantity choices we allowed for 0.001 increments. This small increment was chosen, because we have shown that the mixed strategy equilibrium in our discrete PQ game converges to the one in the continuous game if the increment is sufficiently small (Cracau and Franz, 2011).

At the beginning of each stage, we gave subjects a what-if-calculator to help them get comfortable with the residual demand and profit calculation. After the firms' simultaneous decisions, profits were calculated according to the model presented in Section 2. Then, all players were shown a summary with prices, production levels and profits. At the end of the experiment, firms' total payoff consists of the sum of the payoff of all 25 rounds. Because firms earned a starting budget from the monopoly stage, bankruptcy during the course of the game was not considered.[4] In *PQ3*, we provided a lump-sum payment of 3 Euro at the end of the experiment to compensate for low payoffs.

We collected ten independent observations for *PQ2* and nine independent observations for *PQ3* at the MaXLab experimental laboratory at the University of Magdeburg. All participants were students from economic fields, recruited via ORSEE (Greiner, 2004). On average, participants in *PQ2* treatment earned 10.69 Euro (≈ 15.25 USD) and participants in *PQ3* earned 9.18 Euro (≈ 13.10 USD) in a 45-minute session.

[3]For reasons of simplicity, production levels were limited to demand at the chosen price. Thus, choosing a price equal to 100 automatically corresponds to a market exit.

[4]For firms with a negative total balance at the end of the second stage, we set earnings equal to zero (2 firms in the *PQ3* treatment).

4. Results

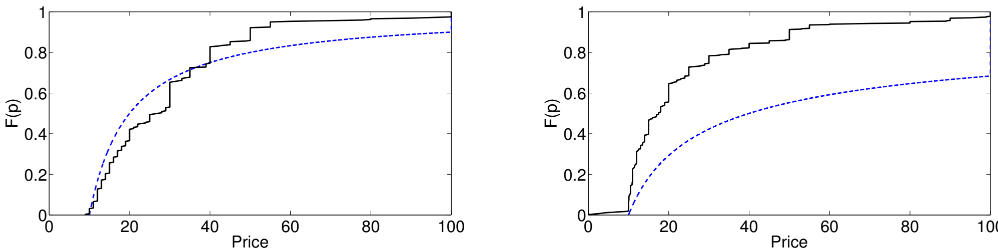
In the first (monopolistic) stage of the game, subjects earned in total 88% (96% in the last round) of the possible monopoly profits. As this is in line with the literature (Potters et al., 2004), we conclude that all participants understood the experimental procedure and produced reliable observations.

Table 1: Comparison of equilibrium prediction and experimental results.

		AWP	Production	Profits
$n = 2$	mixed eq.	19	134.95	0
	PQ2	28.20	108.32	635.68
$n = 3$	mixed eq.	23.68	140.26	0
	PQ3	15.19	150.63	-381.44

Table 1 summarises the experimental results for both treatments and the corresponding benchmarks. We follow Brandts and Guillen (2007) in presenting the average weighted market prices (AWP). Thereby, the prices at which units are sold are weighted by their respective market shares.
Figure 1(a) illustrates that the distribution of prices in *PQ2* differs visibly from the mixed strategy equilibrium prediction (2-sample KS test, $p = 0.001$, two-sided). We observe a greater fraction of prices in the range $[10, 55]$ (i.e. between marginal cost and the monopoly level) than predicted. In total, we observe less than 6% of the prices above the monopoly / cartel price ($p = 55$), compared to the 18.2% predicted by the mixed strategy equilibrium, which suggests that firms perceive prices above the monopoly level as implausible. Whilst the equilibrium predicts 10% market exits, we do not observe those frequently. In particular, prices equal to 100 were not observed at all, but we observe 9 decisions with quantities equal to zero which we also denote as a market exit.
In *PQ2*, we find no significant effects in the development of AWP, production or profits over time.[5] For the AWP, we observe an initial drop, because the players were slightly biased towards the monopoly price from the first stage

[5] This is in contrast to Brandts and Guillen (2007). In their experiment, prices converge to towards the monopoly level in PQ duopolies and triopolies due to either collusion or bankruptcies.



(a) *PQ2* (b) *PQ3*

Figure 1: Price distribution. Observations (solid line), prediction (dashed line).

of the game. This bias, however, disappears quickly as the participants get used to the new situation and the AWP stays on the lower level.
The average market production is below the expected value of the mixed strategy equilibrium but nevertheless above total market size. We observe overproduction because firms had to decide on their production level before knowing their actual demand. In the experiment, we see that only 46% of the production decisions satisfied the equilibrium prediction $q = D(p)$. On average the value of $q/D(p)$ in *PQ2* is 0.83, i.e. players are looking to satisfy on average 83% of the market demand at their chosen price. This results in a positive residual demand for the higher price firm in 27% of all rounds.[6] Because we find no significant difference in profits between decisions that serve the full market demand and those that did not, we conclude that firms had no disadvantage from deviating from the equilibrium condition $q = D(p)$. The total payoff per participant is significantly positive (WSR test, $p = 0.04$, two-sided), contradicting the equilibrium prediction of zero expected profits.

Result 1. *In the PQ duopoly, the observed behaviour differs markedly from the equilibrium predictions, as can be seen in the different price distribution, the lower than expected production levels and the positive average profits.*

Figure 1(b) illustrates that the distribution of prices in *PQ3* does not fit the mixed strategy equilibrium prediction given in (3) ($n = 3$). As in *PQ2*, we observe the vast majority of prices in the range $[10, 55]$. In total, we

[6]In the rest of the rounds, the price of the high price firm was too high to guarantee any residual demand.

only observe about 6% prices above the monopoly price. Overall, the AWP in *PQ3* stays on a significantly lower level than in the duopoly treatment (MWU test, $p < 0.001$, two-sided).

Result 2. *In contrast to the equilibrium prediction, observed prices in the PQ triopoly were lower than in the PQ duopoly.*

This observation can be explained by the low number of observed market exits (2.4% vs. 31.62% predicted) that incorporate price choices of $p = 100$. In turn, if firms enter the triopoly market in *PQ3*, they choose lower prices than in *PQ2*. This finding of lower observed prices in markets with more firms is persistent in the experimental literature.[7]

For production in *PQ3*, we find a negative trend after the first periods. Total production is above market size and above the mixed strategy equilibrium prediction. In this treatment only 28% of the decisions satisfied the equilibrium condition $q = D(p)$. In *PQ3*, the average value of $q/D(p)$ is 0.65 and therefore lower than in *PQ2* (83%). Although firms produced less than they could, total production is higher than in the mixed strategy equilibrium, because the number of market exits is much smaller than predicted.

Except for the first round, average profits in *PQ3* are negative throughout the whole game. This difference to the equilibrium prediction of zero profits is significant (WSR test $p = 0.04$, two-sided). We find a slight positive trend with profits seeming to converge to zero. Overall, we find that the total payoff is negative for 20 out of 27 participants.

Result 3. *In the PQ triopoly, the observed behaviour differs markedly from the equilibrium predictions, as can be seen in the different price distribution, the low production levels and the negative average profits.*

5. Discussion and conclusion

Our experimental results indicate that the mixed strategy equilibrium does not adequately describe the price choices made by the players in our experiment. This observation is known from experimental economics and a recent

[7] See for example Dolbear et al. (1968) and Huck et al. (2004) for quantity competitions, Dufwenberg and Gneezy (2000) and Abbink and Brandts (2008) for price competitions as well as Brandts and Guillen (2007) for a PQ competition.

Table 2: Fixed-effects regression with price as dependent variable.

($F = 17.77$, $p < 0.01$)

independent variable	coefficient	standard error	t	$P > \|t\|$
Constant	19.36297	1.79147	10.81	< 0.001
Preceding price	0.36927	0.06240	5.92	< 0.001
LOSS	−3.78175	1.83629	−2.06	0.040

(a) *PQ2*

($F = 31.78$, $p < 0.01$)

independent variable	coefficient	standard error	t	$P > \|t\|$
Constant	16.62901	1.37816	12.07	< 0.001
Preceding price	0.36942	0.04634	7.97	< 0.001
LOSS	−4.39027	1.60825	−2.73	0.007

(b) *PQ3*

discussion on this topic can be found in Palacios-Huerta and Volij (2008). The authors show that subjects inexperienced in real life tasks with mixed strategy equilibria fail to play even simple mixed strategies. However, they also find evidence for mixed strategy play if lab participants are experienced with the decision situation.

To gather deeper understanding of the choices observed in our experiment, we suggest that firms may react to the result of the previous rounds rather than choosing independently at random from a mixed strategy.[8] We find that on average, winners in *PQ2* increase their prices by 5.14 while losers decrease their prices by 7.22.[9]

In Table 2 we study the dependence on previous rounds outcome using regression analysis. We model firms' price choices in dependence of the previous round price choices and a dummy, $LOSS$, that is 1 if the firm lost the previous round and 0 otherwise. We can see that in both market situations preceding

[8]Edgeworth cycles provide a further possible explanation of firms' behaviour (Brown-Kruse et al., 1994). We identify an Edgeworth cycle in one of our observations, whilst the pricing behaviour in the remaining observations cannot be explained satisfactorily by this theory.

[9]This result is in line with previous findings, see for example Neugebauer and Selten (2006) or Ockenfels and Selten (2005) for first-price sealed-bid auctions and Bruttel (2009) for a Bertrand duopoly.

round losers *ceteris paribus* chose significantly lower prices than preceding round winners. From this we conclude that winning / losing the previous round has a major impact on the price choice of a participant. This result contributes to the learning direction theory (Selten and Stoecker, 1986).

Result 4. *In the experimental data, players react directly to the outcome of a previous round. On average, winners increase their prices while losers decrease prices, with the absolute price change by losers being stronger.*

Overall, our study is a further step to gather a comprehensive understanding of firm behaviour in PQ oligopoly competition. Our presentation of the mixed strategy equilibrium together with our experimental results provides a good benchmark for further analysis of PQ oligopolies in theory as well as experiments. For example, games with endogenous timing or endogenous choice of the decision variable can be studied more comprehensively, allowing for simultaneous price-quantity choices.

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