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Aggressive Bidding of Weak Bidders in All-Pay Auction

Yizhaq Minchuk
Department of Industrial Engineering and Management, Shamoon College of Engineering, Israel

Abstract

We study an asymmetric all-pay auction with a general utility function. We show that high-type bidders in all-pay auction with lower density, are bidding more aggressively than bidders with higher density. This result is contradictory to the result in Parreiras and Rubinchik (2010) on aggressive bidding of strong bidders.

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Contact: Yizhaq Minchuk - yizhami@sce.ac.il.
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1. Introduction

Parreiras and Rubinchik (2010) studied an asymmetric private value all-pay auction with $n \geq 3$ risk averse bidders and one prize. They characterized the equilibrium in continuous and discontinuous bidding.

We show that in an asymmetric all-pay auction with a general utility function a weak bidder can be more aggressive than a strong bidder in contradiction to Parreiras and Rubinchik (2010).

2. The Model

We consider an all-pay auction where bidders compete over one indivisible prize. A bidder i has a private valuation for the prize v_i which is drawn independently from the continuously differentiable distribution function $F_i(v)$ over the support $[0,1]$ with a strictly positive density $F_i' = f_i > 0$. Each bidder i submits a bid $b_i(v_i)$ independently of other bidders. Assume that there exists an asymmetric, monotonic and differentiable equilibrium bid function $b_i(v_i)$[1]. Let us define bidder i function as $x_i = b_i(v_i)$, and inverse bid function as $y_i(x_i)$. Let the $u_i(x_i, v_i) = u_i(v_i - x_i)$ be a utility function that is twice continuously differentiable and satisfies $\frac{\partial u_i(\cdot)}{\partial v_i} > 0$, $\frac{\partial u_i(\cdot)}{\partial x_i} < 0$, $u_i(0) = 0$ for all i.

Assumption 1: Let us assume that all bidders have the same utility function.

3. The Main Result

Under Assumption 1 we show that high type bidders (bidders with a high valuation for the item) with lower densities bid more aggressively than others in contradiction to Parreiras and Rubinchik (2010).

The maximization problem is given by

$$\max_x u_i(x) = \left(\prod_{\substack{j=1 \\ j \neq i}}^{n} F_j(y_j(x)) u_i(v_i - x) + (1 - \prod_{\substack{j=1 \\ j \neq i}}^{n} F_j(y_j(x))) u_i(-x) \right) \quad i = 1, .., n. \qquad (1)$$

When

$$y_i(1) = \bar{b}.$$

The highest bid which is equal for all bidders, is proved by Amann and Leininger (1996) for two bidders case and extended for general case by Parreiras and Rubinchik (2010), Fibich and Oren (2014).

In the following proposition we use a technique that was developed in Fibich *et al.* (2002) and Minchuk (2013).

[1] Amann and Leininger (1996) proved the existence of a pure strategy, monotonic equilibrium in two risk neutral bidders case. Athey (2001) generalized the result to n risk averse bidders. A General case of existence is established by Govindan and Wilson (2010).

Proposition 1 *If a utility function satisfies Assumption 1, $f_i(1) > f_j(1)$ when $i \neq j$, $i, j = 1, .., n$ and v is high ($v \to 1$), then $b_i(v) < b_j(v)$.*

Proof: Differentiating (1) with respect to x and substituting $y_i(x_i) = v_i$ we get

$$\begin{aligned}\frac{\partial V_i(x)}{\partial x} &= u_i(y_i(x_i) - x) \sum_{\substack{j=1 \\ j \neq k}}^{n} \left(\prod_{\substack{k=1 \\ k \neq i,j}}^{n} F_k(y_k(x)) \right) f_j(y_j(x)) y_j'(x) - \prod_{\substack{j=1 \\ j \neq i}}^{n} F_j(y_j(x)) u_i'(y_i(x_i) - x) + \\ & -u_i(-x) \sum_{\substack{j=1 \\ j \neq k}}^{n} \left(\prod_{\substack{k=1 \\ k \neq i,j}}^{n} F_k(y_k(x)) \right) f_j(y_j(x)) y_j'(x) - (1 - \prod_{\substack{j=1 \\ j \neq i}}^{n} F_j(y_j(x))) u_i'(-x) = 0\end{aligned}$$

rearranging and submitting $x = \bar{b}$ we get

$$\frac{\partial V_i(\bar{b})}{\partial x} = u_i(1 - \bar{b}) \sum_{\substack{j=1 \\ j \neq i}}^{n} f_j(1) y_j'(\bar{b}) - u_i'(1 - \bar{b}) - u_i(-\bar{b}) \sum_{\substack{j=1 \\ j \neq i}}^{n} f_j(1) y_j'(\bar{b}) = 0$$

and thus

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} f_j(1) y_j'(\bar{b}) = \frac{u_i'(1 - \bar{b})}{u_i(1 - \bar{b}) - u_i(-\bar{b})}. \tag{2}$$

Then, substracting (2) for i from (2) for j under Assumption 1 we get

$$f_j(1) y_j'(\bar{b}) = f_i(1) y_i'(\bar{b}) \quad i, j = 1, .., n.$$

Thus, $y_j'(\bar{b}) > y_i'(\bar{b})$ namely, $b_i'(1) > b_j'(1)$ yielding the result.□

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