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Input price and industry concentration in a Cournot oligopoly

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Abstract

The impact of input price changes on industry concentration in a Cournot oligopoly depends on the type of firm heterogeneity and on the curvature of the demand function. Firms might be heterogeneous in their ability to use the input undergoing the price change, or in their ability to use complementary inputs. For the same demand function,it is possible that industry concentration can increase with one type of heterogeneity and decrease with the other. Conditions are derived for the industry concentration, as measured by the Herfindahl index, to increase (decrease) for each type of heterogeneity. In all cases, the change in the Herfindahl index is proportional to the variance of the initial unit cost distribution among firms.

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1. Introduction

Although there is a long literature that has examined the determinants of industry concentration, the impact of changes in input price on industry concentration has not been studied before. This paper fills the gap in the literature, and derives the effect of a change in input price on the Herfindahl index, a commonly used measure of industry concentration, in a Cournot industry. Understanding the relationship between input prices and industry concentration is important because decreasing input price is a characteristic feature of many industries, especially high technology industries.

Many authors, including Nelson and Winter (1978) and Mansfield (1983), have examined the causes of changes in industry concentration in high technology industries. Sutton (2001) has developed industry models that characterize industry concentration and R&D decisions of firms as joint equilibrium outcomes. While technological change through R&D investments is key in some industries, in many others technological change often takes the form of reduction in the price of capital equipment, of a raw material or of an intermediate component used in the production process. In the computer industry for example, decline in prices of semiconductor chips, the principal intermediate component used in computers, has been the central factor that has enabled better computers.[1] In the solar industry, the price of polysilicon, the main raw material used in the manufacture of solar panels, has decreased by a factor of over fifty during 1970-2010 (see Pillai and McLaughlin (2013)). These examples, and many other similar ones, motivate the question studied in this paper, of the impact of declining input price on industry concentration.

This paper finds that the type of firm heterogeneity and the curvature of the demand function together determine the impact of input price reduction on industry concentration. One can distinguish two types of firm heterogeneity in the context of reduction in an input price. Firms can differ in their ability to use inputs other than the one undergoing price reduction, or firms can differ in their ability to use the input undergoing price reduction. These two different types of firm heterogeneity may cause input price reduction to have different impacts on industry concentration, depending upon the properties of the demand function.

The focus of this paper is on changes in industry concentration that arise from reallocation of production across firms in response to the change in price of a common input used by all firms. An extensive strand of literature examines the effect on concentration arising from mergers (see Holmes and Gowrisankaran (2004)) and from entry and exit of firms. While merger, entry and exit can have effects supplemental to the ones derived here, they have been ignored in the model to bring out sharply the impact of reallocation effects on industry concentration. The next section lays down the model and derives the results.

[1] Aizcorbe (2006) notes that quality adjusted prices of semiconductor chips fell at an annual rate of 36% during 1993-1999.

2. Model

Manufacture of products in many high technology industries often require a fixed quantity of an intermediate component or raw material, for example a microprocessor chip in a computer or a battery in an electric car. For ease of exposition, this input will be just called as the primary input. All the other inputs needed in the production process can be bundled together and thought of as a single composite input, and would be referred to as the complementary input. The terms primary and complementary are arbitrary and is used only for ease of exposition. The purpose of the model developed below is to help understand what happens to industry concentration as the price of the primary input falls.[2] A characteristic feature of many of these industries, is that firms in the industry differ in the productivity with which they use the primary input or complementary inputs.[3] These assumptions can be captured through the Leontief production function,

$$q_i = \min(\frac{L_i}{a_i}, \frac{M_i}{b_i}).$$

where L_i is the quantity of the complementary input, M_i is the quantity of the primary input, and a_i and b_i are the number of units of the complementary and primary input needed to produce one unit of output.[4] Note that $\frac{1}{a_i}$ and $\frac{1}{b_i}$ are measures of the productivity of the firm in using the corresponding inputs. The aggregate output in the industry is $Q = \sum_{i=1}^{N} q_i$, where N is the number of firms in the industry, taken as fixed. Firms engage in Cournot

[2]The decrease in input price is taken as exogenous to the firms, to focus on the qualitative impact of input price changes on downstream industry concentration.

[3]The computer industry would be an example of an industry where firms differ in their ability to use complementary inputs. Computer firms like Dell or Lenovo differ not in their abilities to make use of the inputs whose prices are falling (like semiconductor chips or software), but in their ability to run assembly lines or to deliver products to customers. The semiconductor chip manufacturing industry would be an example of the second type of heterogeneity. Adoption of newer vintages of capital equipment reduces the capital costs of semiconductor chip manufacturers. Upstream firms manufacture and sell the same type of capital equipment to all downstream firms, but the downstream firms differ in their ability to incorporate the new lower cost vintages of capital equipment into the manufacturing process. Leading semiconductor firms like Intel and Samsung are better than their rivals in achieving high production yield with the new capital equipment.

[4]The Leontief assumption was used because the intermediate components used in many high-tech products are not readily substitutable. The Leontief assumption also makes it possible to incorporate cleanly the distinction between heterogeneity in the ability to use the primary input (whose price is falling) and a complementary input, and analyze the impact of this distinction on industry concentration.

competition.[5] The inverse demand function in the industry is $P = f(Q)$ and is assumed to satisfy $f'(Q) < 0$ and the condition,

$$2f'(Q) + Qf''(Q) < 0, \tag{1}$$

which ensures the existence and uniqueness of the Cournot equilibrium.[6]

Every firm purchases the primary input from an upstream industry. Each firm takes the input price, denoted by r, as given, and solves the problem,

$$\max_{L_i,M_i} \pi_i = f(Q)q_i - rM_i - L_i,$$
$$s.t \quad q_i = \min(\frac{L_i}{a_i}, \frac{M_i}{b_i}).$$

where the cost of complementary input has been set to 1. It is optimal for the firm to set $q_i = \frac{M_i}{b_i} = \frac{L_i}{a_i}$. Hence the firm's problem can be rewritten as,

$$\max_{q_i} \pi_i = f(Q)q_i - (a_i + rb_i)q_i.$$

In the following sections, $Q(r)$ denotes the equilibrium industry output, and $s_i(r)$ denotes the equilibrium market share of firm-i, at input price r. To avoid clutter, the argument r is sometimes ignored. Solving the maximization problem above, it is easily seen that the optimal quantity produced by firm i satisfies,

$$f'(Q)q_i + f(Q) = a_i + rb_i \tag{2}$$

where the left hand side is the marginal revenue of firm i and the right hand side is its marginal cost. Adding equation (2) across i gives,

$$f'(Q)Q + Nf(Q) = N(\bar{a} + r\bar{b}). \tag{3}$$

Substituting for $f(Q)$ from equation (3) into equation (2) gives,

$$q_i(r) = \frac{a_i - \bar{a} + r(b_i - \bar{b})}{f'(Q)} + \frac{Q}{N}$$

Substituting for $f'(Q)$ from equation (3) and dividing through by Q gives the equilibrium market share of firm-i,

$$s_i(r) = \frac{q_i(r)}{Q(r)} = \frac{1}{N} + \frac{\bar{a} - a_i + r(\bar{b} - b_i)}{N[f(Q) - \bar{a} - r\bar{b}]}. \tag{4}$$

[5]The Cournot model was used because it brings out the basic mechanisms in the model quite clearly. Delipalla and Keen (1992) also use a Cournot model to study a similar question relating to the impact of taxation.

[6]See Novshek (1985) for more general results on existence and uniqueness of the equilibrium.

For characterizing the impact of decline in input price r, it is useful to define the following functions,

$$\begin{aligned}\theta(r) &= \frac{Q(r)f''(Q(r))}{f'(Q(r))}\\ \mu(r) &= \frac{rQ'(r)}{Q(r)}.\end{aligned}$$

The first is the elasticity of the slope of the inverse demand curve at the equilibrium level of industry output, and the second is the elasticity of equilibrium industry output with respect to the input price. Note that equation (1) implies that $\theta(r) > -2$. The next lemma captures the impact of decline in input price on market shares of firms.

Lemma 1. *If firms differ only in the productivity with which they use the complementary input (a_i), then a decrease in r results in an increase in the market share of the less productive firms (i.e $a_i > \bar{a}$) and a decrease in the market shares of the more productive firms (i.e $a_i < \bar{a}$), if $\theta(r) > -1$. The reverse result obtains if $-2 < \theta(r) < -1$.*

Proof. Differentiating (3) with respect to r and re-arranging gives,

$$N\left(\frac{\bar{b}}{f'(Q)Q'(r)} - 1\right) = \frac{Q(r)f''(Q(r))}{f'(Q(r))} + 1 = \theta(r) + 1 \tag{5}$$

Since $b_i = \bar{b}$, equation (4) reduces to,

$$s_i(r) = \frac{1}{N} + \frac{(\bar{a} - a_i)}{N(f(Q) - \bar{a} - r\bar{b})}$$

Differentiating with respect to r, and re-arranging gives,

$$s_i'(r) = \frac{\bar{a} - a_i}{N(f(Q) - \bar{a} - r\bar{b})^2} f'(Q)Q'(r)\left(\frac{\bar{b}}{f'(Q)Q'(r)} - 1\right) \tag{6}$$

Substituting equation (5) in equation (6) gives,

$$s_i'(r) = \frac{f'(Q)Q'(r)}{[N(f(Q) - \bar{a} - r\bar{b})]^2}(\bar{a} - a_i)(\theta(r) + 1) \tag{7}$$

By assumption $f'(Q) < 0$ and using the assumption in equation (1), it can be shown that $Q'(r) < 0$. Hence,

$$\begin{aligned}\theta(r) > -1 &\Rightarrow\ s_i'(r) > 0 \ \text{ for } \ a_i < \bar{a}, \ \text{ and } \ s_i'(r) < 0 \ \text{ for } \ a_i > \bar{a}.\\ \theta(r) < -1 &\Rightarrow\ s_i'(r) < 0 \ \text{ for } \ a_i < \bar{a}, \ \text{ and } \ s_i'(r) > 0 \ \text{ for } \ a_i > \bar{a}.\end{aligned}$$ □

The intuition for this result is as follows. The parameter $\theta(r)$ determines the relative steepness of the marginal revenue curves of firms with different market shares in equilibrium.[7] A higher value of $\theta(r)$ corresponds to steeper marginal revenue curves for firms with higher market shares, relative to firms with lower market shares.[8] Hence, as $\theta(r)$ increases, a larger firm needs to increase its quantity by lower amounts to achieve the same increase in marginal revenue as a smaller firm. Since all firms experience the same change in marginal cost, their marginal revenues must also increase by the same amount, and hence higher values of $\theta(r)$ lead to relatively lower quantity increments for larger firms as compared to smaller firms. The value of -1 for $\theta(r)$ denotes the critical point above which the quantity increases needed to get equal increases in marginal revenue are less than proportionate to current market shares for larger firms ($a_i < \bar{a}$), leading to a decrease in their market shares.[9]

Lemma 2. *If firms differ only in the productivity with which they use the primary input (b_i), then a decrease in r results in an increase in the market shares of the less productive firms ($b_i > \bar{b}$) and a decrease in the market shares of the more productive firms ($b_i < \bar{b}$), if $\theta(r) > \frac{1}{\mu(r)} - 1$. The reverse result obtains if $-2 < \theta(r) < \frac{1}{\mu(r)} - 1$.*

Proof. Since $a_i = \bar{a}$, equation (4) reduces to,

$$s_i = \frac{1}{N} + r\frac{(\bar{b} - b_i)}{N(f(Q) - \bar{a} - r\bar{b})}$$

Taking derivative with respect to r, and re-arranging gives,

$$s_i'(r) = \frac{\bar{b} - b_i}{N(f(Q) - \bar{a} - r\bar{b})^2}\left(f(Q) - \bar{a} - rf'(Q)Q'(r)\right)$$

Substituting for $f(Q) - \bar{a}$ from equation (3) gives,

$$s_i'(r) = \frac{\bar{b} - b_i}{N(f(Q) - \bar{a} - r\bar{b})^2}\left(r(\bar{b} - f'(Q)Q'(r)) - f'(Q)\frac{Q}{N}\right)$$

[7] The elasticity $\theta(r)$ is a measure of the curvature of the demand function, and other authors have highlighted the role of this elasticity in a number of fields not related to industry concentration, for example Goldberg and Knetter (1997) in the context of exchange rate pass-throughs in international trade and Delipalla and Keen (1992) in the context of tax incidence.

[8] To see this, note that $\frac{\partial}{\partial q_i} MR_i = f'(Q)(\theta(r)s_i + 2)$.

[9] To see this, note that $\frac{d}{dr} MR_i = f'(Q)Q'(r)\left((\theta(r) + 1)s_i + 1 + Q\frac{s_i'(r)}{Q'(r)}\right)$. If all the firms have to have the same change in marginal revenue in response to a small change in r, it must be that $\dfrac{d}{dr}MR_i$ is the same for all firms. Hence if $\theta(r) > -1$, and noting that $Q'(r) < 0$, it must be that $s_i'(r) > s_j'(r)$ if $s_i > s_j$. That is, if there is a decrease in r, either the larger firm shrinks and the smaller firm expands ($s_i'(r) > 0$ and $s_j'(r) < 0$) or the larger firm expands less when compared with a smaller firm ($s_i'(r) < 0$, $s_j'(r) < 0$ and $|s_i'(r)| < |s_j'(r)|$.)

Substituting for $(\bar{b} - f'(Q)Q'(r))$ from equation (5) gives,

$$s_i'(r) = \frac{\bar{b} - b_i}{[N(f(q) - \bar{a} - r\bar{b})]^2} r f'(Q)Q'(r) \left(\theta(r) + 1 - \frac{1}{\mu(r)}\right) \tag{8}$$

Hence,

$$\theta(r) > \frac{1}{\mu(r)} - 1 \Rightarrow \; s_i'(r) > 0 \;\text{ for }\; b_i < \bar{b}, \;\text{ and }\; s_i'(r) < 0 \;\text{ for }\; b_i > \bar{b}.$$
$$\theta(r) < \frac{1}{\mu(r)} - 1 \Rightarrow \; s_i'(r) < 0 \;\text{ for }\; b_i < \bar{b}, \;\text{ and }\; s_i'(r) > 0 \;\text{ for }\; b_i > \bar{b}. \qquad \square$$

As in Lemma 1, this result is also driven by the role of $\theta(r)$ in determining the effect of a firm's market share s_i on the steepness of its marginal revenue curve. In this case however, firms differ in b_i and hence experience different changes in marginal cost as r decreases. For the same decrease in r, the marginal cost of a large firm (one with a lower b_i) decreases by a lower amount than the marginal cost of a small firm. Hence, in equilibrium, marginal revenue of a large firm has to decrease by a lower amount than that of a small firm, further exacerbating the effect in Lemma 1. Hence a less stringent condition, $\theta(r) > -1 + \frac{1}{\mu(r)}$ (where $\mu(r) < 0$), is sufficient for the result in Lemma 1 to hold.

Proposition 1. *(i) If firms differ only in the productivity with which they use the complementary input (a_i), then a decrease in r leads to a decrease in the Herfindahl index if $\theta(r) > -1$, and to an increase in Herfindahl index if $\theta(r) < -1$. The rate of change of Herfindahl index increases with the variance of unit input requirements of the complementary input across firms.*

(ii) If firms differ only in the productivity with which they use the primary input (b_i), then a decrease in r leads to a decrease in the Herfindahl index if $\theta(r) > -1 + \frac{1}{\mu(r)}$, and to an increase in the Herfindahl index if $\theta(r) < -1 + \frac{1}{\mu(r)}$. The rate of change of Herfindahl index increases with the variance of the unit input requirements of the primary input across firms.

Proof. The Herfindahl index at input price r is given by $H(r) = \sum_{i=1}^{N} s_i(r)^2$. Hence

$$H'(r) = 2\sum_{i=1}^{N} s_i s_i'(r)$$

Case (i)

Substituting for $s_i(r)$ and $s_i'(r)$ from equations () and (6) respectively, and noting that

$\sum_{i=1}^{N}(\bar{a}-a_i)=0$ gives,

$$H'(r)=\frac{N-1}{N}\left[\frac{f'(Q)Q'(r)}{[N(f(Q)-\bar{a}-r\bar{b})]^3}\right][\theta(r)+1]Var(a_i)$$

Hence, $H'(r)>0$ if $\theta(r)>-1$, and $H'(r)<0$ if $\theta(r)<-1$.

<u>Case (ii)</u>

Substituting for $s_i(r)$ and $s_i'(r)$ from equations () and (8) respectively, and noting that $\sum_{i=1}^{N}(\bar{b}-b_i)=0$ gives,

$$H'(r)=\frac{N-1}{N}\left[r^2\frac{f'(Q)Q'(r)}{[N(f(Q)-\bar{a}-r\bar{b})]^3}\right][\theta(r)+1-\frac{1}{\mu(r)}]Var(b_i)$$

Hence, $H'(r)>0$ if $\theta(r)>\frac{1}{\mu(r)}-1$, and $H'(r)<0$ if $\theta(r)<\frac{1}{\mu(r)}-1$. □

The equations on the impact of changes in r on the Herfindahl index follows directly from Lemmas 1 and 2. In cases where smaller (less efficient) firms gain market share, the Herfindahl index decreases and in cases where larger (more efficient) firms gain market share, the Herfindahl index increases. The additional insight in Proposition 1 is that a change in r produces a bigger change in concentration in industries where the current dispersion of costs (or productivities) is higher.

3. Discussion

Figure 1 illustrates the parameter values and the corresponding impact of input cost reduction on the Herfindahl index.

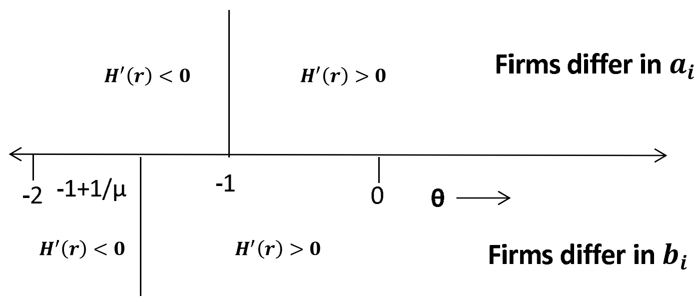

Figure 1: Firm Heterogeneity and Industry Concentration.

If $\theta > -1$ or $-2 < \theta < \frac{1}{\mu} - 1$, then industry concentration is impacted the same way, irrespective of the type of heterogeneity. Concentration decreases with input price reduction in the former case, and increases in the latter case. If $\frac{1}{\mu} - 1 < \theta < -1$, then impact on concentration depends on the type of heterogeneity. Concentration increases if firms differ in their ability to use complementary inputs, and decreases if firms differ in their ability to use the primary input.[10] Two examples illustrate the results.

First, consider the case of the linear demand curve, with inverse demand being given by $P = f(Q) = A - BQ$. In this case, the two critical elasticities are given by, $\theta(r) = 0, \quad \mu(r) = -\dfrac{\bar{b}}{A - a - r\bar{b}}$. It is easily seen that, $\theta(r) > -1, \quad \theta(r) > \dfrac{1}{\mu(r)} - 1$. If firms differ in a_i, then the former inequality implies that reduction in input price will lead to a decrease in industry concentration. If firms differ in b_i, then the latter inequality implies that reduction in input price will lead to a decrease in industry concentration in this case also. Next, consider the constant elasticity demand curve, with inverse demand being given by $P = f(Q) = DQ^{\frac{1}{\eta}}$, with $\eta < 0$. In this case, the two critical elasticities are given by, $\theta(r) = -1 + \dfrac{1}{\eta}, \quad \mu(r) = \dfrac{\eta r\bar{b}}{a + r\bar{b}}$. It is easily seen that, $\theta(r) < -1, \quad \theta(r) > \dfrac{1}{\mu(r)} - 1$. Hence if firms differ in a_i, the former inequality implies that reduction in input price will lead to an increase in industry concentration. If firms differ in b_i, then the latter inequality implies that reduction in input price will lead to a decrease in industry concentration.

4. Conclusion

The role of curvature of demand curve in determining the extent of price-cost pass-throughs and over-shifting of taxes has been known in the literature. This paper shows that curvature of the demand curve also affect the industry concentration. In addition, in an industry with falling input prices, industry concentration also depends on the nature of firm heterogeneity, whether the downstream firms differ in their ability to use the input whose price is falling or in their ability to use a complementary input. For the same demand function, it is possible that industry concentration can increase with one type of heterogeneity and decrease with the other. The results in this paper have been derived in the context of a Cournot model, and future work could explore the applicability of these results for other industry structures.

References

[10]While direct testing of the theory would be difficult, it might be possible to have indirect approaches to see if industries fall into different categories based on inferred values of the two variables. Such an approach would be similar to the one followed in Sutton (2001), who tries to explain differences in industrial concentration across a cross-section of industries based on inferred values of key variables (different from the variables in this paper) that his model shows to be important in determining concentration.

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