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Does growth promote Equality? A Note on Piketty's capital on the twenty-first century

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Abstract

A simple model of the distribution of wealth is presented. In such a model the distribution of wealth follows a Pareto type II distribution whose parameters depend on the growth rates discussed in the book. Compared to previous studies of the asymptotic distribution of wealth in economies subject to multiplicative random shocks, our model can make the argument that growth promotes equality accessible to a larger audience.

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1. Introduction

In a recent book that has generated considerable interest [Piketty (2014)], the author makes interesting statements regarding the relation between the rate of growth of several economic variables and the distribution of wealth. We present a simple model that allows us to study the issue analytically. The approach presented here complements the studies referenced in the book about the asymptotic distribution of wealth in economies subject to random shocks. The stochastic approach is complex and in order to obtain the desired results the shocks must affect the variables in a multiplicative way. Moreover, in many cases the conclusions must be drawn from numerical simulations. The analysis presented here is considerably simpler and it is accessible to a larger audience.

Most of Piketty's statements on the topic are presented in a section of the book entitled Growth as Factor of Equalization. The first assertion states that growth either in the form of population growth or productivity growth decreases inequality: "Other things being equal strong … growth tends to play an equalizing role because it decreases the importance of inherited wealth: every generation must in some sense construct itself."

2. Growth and the distribution of wealth

We begin by analyzing the role of population growth brought by immigration from other countries. The key variables in the first version of our model are: r the rate of return on capital, the wage rate w and the rate of immigration m. The wage rate is assumed to be the same for everyone. Population growth comes only from the arrival of new immigrants. The flow of immigrants is equal to mP(t) where P(t) is the population at time t. Therefore P(t) is equal to $P_o e^{mt}$. It is assumed that immigrants come with no wealth. All citizens save a constant fraction, s, of their income. Given these assumptions the capital stock k owned by a member of any cohort evolves according to the following equation $\dot{k} = s(rk + w)$. Solving this differential equation and recalling that the starting value of wealth at the time of arrival is zero, it follows that the per-capita value of k for a cohort that arrived h years ago is given by $k(h) = \frac{w}{r}[e^{srh} - 1]$. At time t, the proportion of the population that that has per-capita wealth smaller than k(h) is $\frac{P(t)-P(t-h)}{P(t)} = 1 - e^{-mh}$. From these two facts one can derive the distribution of wealth.

Let Z be any arbitrary number and let F[Z] be the fraction of the population with wealth less or equal to Z. A family with exactly Z units of wealth must have arrived h periods ago where h is the solution to $Z = \frac{w}{r}[e^{srh} - 1]$, thus $h = \frac{Log[\frac{r}{w}Z+1]}{sr}$. Substituting the result into the expression for the share of the population that arrived less than h years ago we get F[Z] = 1- $\left[\frac{r}{w}Z+1\right]^{-\frac{m}{sr}}$. This is the cdf of a type II Pareto distribution[1].

Economists are more familiar with the standard Pareto distribution whose cdf is given by $1-\left[\frac{Z}{k}\right]^{-\alpha}$. A simple measure of inequality sometimes mentioned in the literature is the Gini coefficient that in the case of the standard Pareto turns out to be $(2\alpha-1)^{-1}$, for $\alpha>$ .5 [Arnold (2008)]. As the tail parameter increases the distribution becomes more equalitarian. For the distribution found here my calculations made the Gini coefficient equal to $\frac{\alpha}{2\alpha-1}$ for $\alpha>1$. Again a higher tail coefficient brings equality but the Gini coefficient cannot go below .5.

The implications of the results for the forces affecting the distribution of wealth are simple. More equality can be achieved by increasing m or by decreasing sr. The role of sr is not surprising since inequality is driven by the accumulation of wealth by families associated with old money. The fact that increasing m improves the distribution of wealth is consistent with Piketty's view: "Assuming that most immigrants arrive without much wealth, the amount of wealth passed down from previous generations is inherently fairly limited in comparison to new wealth accumulated through savings".

There is however a puzzle regarding the role of m. With no immigration there is no stable income distribution since there is a strong force towards equalization. The proportional rate of growth of k is given by $\frac{\dot{k}}{k}=s(r+\frac{w}{k})$. This expression implies that the proportionate rate of growth of the capital owned by the more affluent families is smaller than the same rate calculated for those at the bottom of the distribution. It follows that for any two arbitrary cohorts the ratio of their wealth converges towards one regardless of the initial disparity of their endowments. Immigration here is required to keep the income distribution alive and it plays a similar role to the random shocks in Piketty's model. This makes the fact that increasing m makes the distribution more equalitarian even more interesting.

It is also worthwhile to point out that immigration (or some form of population growth outside of a dynastic family such as the birth of unloved children) was found to play an important role in other type of models. When building models of growth for small open economies it was found that in the standard model of a single representative family, consumption will either grow to infinity or decrease to zero depending on the relative value of the intertemporal rate of discount versus the rate of return to capital. Weil (1989) show that this unsatisfactory result can be avoided introducing immigration into these models.

[1] The cdf of a Pareto Type II distribution is given by 1- $\left[1+\frac{Z-\mu}{\sigma}\right]^{-\alpha}$. In our case μ is zero, σ is $\frac{w}{r}$ and α is equal to $\frac{m}{r}$. The parameter α is known as the tail index and the parameter σ is called the scale index.

We now turn to population growth coming from the excess of births over deaths among the population already living in the country. We assume that once every immigrant settles he begins to reproduce at a rate n, so now the entire population grows at the rate n+m, $P(t) = P_o e^{(n+m)t}$. The newly born members of each cohort enjoy the same wealth as the older ones, because each cohort is a dynastic family and within the family each member has the same wealth. The equation for the evolution of per- capita wealth for a member of any particular cohort must be changed to $\dot{k} = (sr - n)k + sw$. It can be shown that at any point in time the proportion of the population composed by immigrants and their descendants that arrived less than h years ago is $1 - e^{-mh}$ the same as before[2]. Following the same steps discussed before one gets the following expression for the cdf of the distribution of wealth: F[Z] = 1- $\left[\frac{r-n}{w}Z + 1\right]^{-\frac{m}{sr-n}}$. As long as sr – n remains positive, introducing population growth promotes equality while preserving the statistical type of the income distribution. These results are consistent with Piketty who argues that population growth decreases the importance of inherited wealth on the process of wealth accumulation.

Finally we introduce exogenous productivity growth. It is assumed to be of the "labor augmenting" type, so at time t wages are given by $w\theta(t)$ and the productivity index θ is assumed to grow at a rate γ, $\frac{\dot{\theta}}{\theta} = \gamma$. The per-capita wealth of member of a given cohort measured in efficiency units is denoted by $\tilde{k}(t) = \frac{k(t)}{\theta(t)}$ where k is as before wealth per-capita. It follows that $\dot{\tilde{k}} = \big(sr - (n + \gamma)\big)\tilde{k} + sw)$. The distribution of $\tilde{k}$ is again a Pareto II distribution with tail parameter $\frac{m}{sr-(n+\gamma)}$. As long as $sr - (n + \gamma)$ remains positive increases in the rate of productivity growth decrease the inequality of the distribution of wealth.

3. Final Remarks

We close with some general comments about the model. One concerns the assumption of a constant savings rate. The distribution of wealth is quite sentitive to the assumptions made concerning consumption. For example it is well known that assuming intertemporal utility maximization and assuming the same rate of discount for all consumers leaves the steady distribution of wealth indeterminate: any distribution can be supported. The assumption made here has a bias towards equality (remember the remark on the ratio of wealth for two given cohorts). This bias is probably responsible for the fact that our distribution does not incorporate a property mentioned by Piketty that a significant proportion of the population does not leave any bequests. Through his many papers on the subject he has proposed several stochastic models of the distribution of wealth and some of them incorporate the desired result [Piketty and Saez

[2] The change in the total population during the period starting at time t-h and ending at time t is given by $P(t - h)e^{(m+n)h} - P(t - h)$. Of those $P(t - h)e^{nh} - P(t - h)$ were births from people already living at time t-h. These results and the fact $P(t) = P(t - h)e^{(m+n)h}$ yield the expression in the text.

(2012)]. The advantage of our approach is its simplicity that allows making a formal presentation of the main ideas in a model accessible to a larger audience and that it can serve as an introduction to the more complex models.

Other issues involve the plausible values of the parameters of the model. All the analysis carried out here has relied on the assumption that sr exceeds a narrow definition of the growth rate $n + \gamma$. That assumption is central to our approach which is based on the idea that the poorer cohorts are always the most recent ones[3]. It is probably roughly consistent with the data.

A more delicate issue is whether the model implies some sort of unreasonable macro behavior particularly in relation to the capital labor and capital income ratios. We want to show that it is possible to find combinations of the parameters such that the assumptions of the model hold and the economy grows along a balanced path. Let K(t) be the aggregate capital stock. For simplicity we assume no technological change so $\dot{K} = srK + swP_0E^{(m+n)t}$. Two cases can arise. In the first one the rate of population growth m +n exceeds sr. In such a case the capital stock will eventually grow at the rate m+n and the capital labor ratio converges asymptotically to $\frac{sw}{m+n-sr}$. Moreover in this scenario the ratio of capital to output approaches $\frac{s}{m+n}$. This is the more interesting case because if one introduces the additional inequality n < sr < n+m, the tail coefficient of the distribution of wealth discussed before is a positive number and long run inequality and balanced growth can coexist. However, in an economy satisfying those constraints the tail coefficient of the distribution of wealth must always be larger than one. According to our previous discussion regarding the Gini coefficient the restriction does not seem to be very important. The second case considered here is less interesting and takes place when $sr > m + n$. That case is incompatible with the steady state solution to Solow's growth model because it implies that capital labor ratio will approach infinity while the economy will eventually grow at the rate sr and the capital income ratio converges to $\frac{1}{r}$. It is the sort of scenario that some economists fear will prevail if computers and robots take over the economy and the share of labor in income becomes negligible.

However Piketty seems to be more interested in considering medium range forecasts without making predictions about the eventual steady state. He seems particularly interested in situations where sr is quite large and the capital income ratio is increasing. However it appears that he does not necessarily associate this scenario with the second case described in the previous paragraph because in Piketty's own words "too much capital kills the return on capital" so the process may not continue indefinitely and the rate of return should eventually fall. However according to Piketty this process is quite slow and his data does not support its relevance: "experience suggests that the predictable rise in the capital income ratio will not necessarily lead to a significant drop in the return on capital. There are many uses for capital in the long run, and this

[3] A similar idea is exploited in Rodriguez (1985).

fact can be captured by noting that the long run elasticity of substitution of capital for labor is probably greater than one”.

Finally it is important to point out that they may be other interesting connections between growth and income distribution not included in the model. Growth can also promote equality when it comes from innovations that increase the fortunes of those who develop them. Unless the innovators come from families associated with old money (they usually do not) their success improves the distribution of wealth. Current endogenous growth models are ill equipped to incorporate this because the zero profit condition does not allow inventors to become rich.

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