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Optimal ticket pricing in professional sports: a social identity approach

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Abstract

A frequently observed phenomenon in professional sports is apparent underpricing of tickets. The concept of social identity may explain this pricing behavior without giving up the assumption of profit-maximizing behavior. Repeated match attendance increases spectators' identification with the team and their willingness to pay for attendance. In this paper, we set up a model to analyze a profit-maximizing team's optimal pricing decision including such spectator identification. Conditions are derived under which incentives to underprice arise.

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1. Introduction

Studies in the field of sports economics regularly find evidence for ticket underpricing in spectator sports. For instance, Krautmann and Berri (2007) present a recent list of articles reporting inelastic ticket pricing in a variety of professional sports leagues, including studies on the Major League Baseball (Fort & Quirk, 1996), the National Football League (Depken, 2001), and the Spanish First Division Soccer League (Garcia & Rodriguez, 2002). Further evidence of ticket pricing in the inelastic range of demand is reported for US basketball (NBA), English soccer, Scottish soccer and English cricket (Fort, 2006).

The economic literature proposes different potential explanations as to why producers may choose to underprice their products. Kahneman, Knetsch, and Thaler (1986) emphasize the role of fairness. Facing a surge in demand, a firm that decides to raise the price in the absence of increased costs may suffer from a reputation for being exploitative. However, the authors note that transaction terms presently considered unfair gradually gain acceptance in the market, an effect that cannot rationalize persistently low prices. Becker (1991) suggests that the existence of excess demand serves as a signal for popularity, therefore increasing customers' willingness to pay (WTP). Yet, while the indication of popularity may play a role in other product markets, the need for additional signaling of popularity appears doubtful for many nationally televised and media-hyped matches in major sports leagues.

Courty (2003) categorizes event ticket buyers into two groups: "busy professionals" realize only close to the event date whether they can attend, whereas committed "diehard fans" wish to secure tickets well in advance, albeit only at a comparably lower price. Courty concludes that profit-maximizing event promoters abstain from raising prices in the primary market because they cannot effectively clear the market given the dichotomy of customers' commitment ability and the lack of price competitiveness in the broker-dominated secondary market. Eichhorn and Sahm (2010) rationalize the existence of underpriced event tickets by assuming the price to be an instrument a two-product monopolist uses to reach a favorable type distribution of spectators characterized as being more cheerful. Provided an enhanced atmosphere among spectators exerts a positive externality on demand in a second related product market, as in the market for sponsorship contracts, the monopolist may maximize aggregate profit from both markets by setting the lower ticket price. While the authors do not examine the role of intangible goods for the emergence and development of demand in the ticket market itself, we take up this aspect and analyze its implications for the ticket pricing decision.

In this paper, we provide a different rationale for apparent underpricing in the primary ticket market by considering the role of social identity in building spectator demand. In line with empirical evidence, we propose that the experience of sports matches affects spectators' consumption choices by eliciting an increased WTP for attendance. The approach thus shares common elements with models of habit formation (e.g. Pollak, 1970). Spectators are assumed to experience a shift in their preferences over time toward attending matches, which rests on social identification motives. Following this rationale, we develop a model to explain ticket underpricing in tandem with the assumption of profit-maximizing behavior on the part of the sports team. Conditions are derived under which incentives to underprice arise.

The remainder of this paper is organized as follows. In the next section, we motivate a social identity approach to ticket underpricing and review empirical evidence. In the section that follows, we introduce a model to formalize the suggested rationale and derive a profit-maximizing team's optimal pricing decision. The last section concludes the paper.

2. A Social Identity Approach to Underpricing

In this paper, we suggest that the (repeated) experience of sports matches increases the individual WTP for attendance over time. This rise comes from the process of identifying with the team and its supporters. In this light, teams that underprice their tickets can increase a spectator's frequency of match attendance and thereby maximize long-run ticket revenues. This *group identification effect* in spectator sports then suggests an immanent investment character. It may incentivize team owners to forgo myopic short-term gains if an increase in future revenues can be induced by an amount which more than offsets the initial sacrifice.

The theoretical basis for this line of reasoning has its origin in the social psychological literature. According to *Social Identity Theory* (SIT), individuals define themselves in part by their social identity. Social identity relates to multiple social selves that derive from the particular social categories or groups that an individual perceives herself to be a member of and identifies with (Tajfel & Turner, 1979). Ashforth and Mael (1989, p.21) define social identification as "the perception of oneness with or belongingness to some human aggregate." Identification with a social group allows the individual to vicariously partake in the group's accomplishments (Katz & Kahn, 1978). In the given context, we argue that spectators tend to categorize themselves as and identify with the members of a focal group composed of the team and its supporters (i.e., "I am a supporter of this team"). Match attendance thereby serves as a means to establish group contact and affects individuals in social terms.

There is ample evidence in the pertinent literature suggesting that higher identification with a focal group is associated with more group contact in terms of frequency and duration (e.g. Bhattacharya et al., 1995; Gwinner & Swanson, 2003; Mael & Ashforth, 1992; Wann & Branscombe, 1993). By repeatedly attending a team's matches, spectators affiliate with a peer group of like-minded supporters and participate in an intensive, immediate, and highly involving activity exposing them to a paramount sense of group identification.

Gwinner and Swanson (2003) show that the number of contacts individuals have with their favorite NCAA football team is antecedent to perceived team identification. According to Dutton, Dukerich, and Harquail (1994), more contact with an organization increases the attractiveness of a member's social identity and leads to a higher degree of identification with the organization. Sutton et al. (1997) argue that group identification is strengthened by factors such as visibility of affiliation, group-specific rituals, shared goals, or common symbols, all constituting essential ingredients of sporting events. Previous studies in the fields of sport marketing and sport psychology have found that group identification, in turn, results in more frequent group contact and group supportive behavior.[1] Wann (2006, p. 365) reviews the related literature and concludes that "[…] not only is level of team identification a significant independent predictor of game attendance, it may well be the most powerful factor." For instance, Wann and Branscombe (1993) find that individuals high in identification with their focal basketball team are willing to invest greater amounts of money in tickets. Fisher and Wakefield (1998) find that higher team identification among professional hockey fans results in individuals attending more matches regardless of whether, as fans, they are affiliated to a successful hockey team or to an unsuccessful one. Similarly, Mahony et al. (2002) identify spectators' level of attachment and identification with their favorite J. League soccer team as the strongest predictor of frequency of match attendance.

We argue that group contact increases a spectator's identification with a team and its supporters. Higher levels of identification, in turn, bias the consumption preferences toward match attendance. In the following, we introduce the proposed group identification effect into the ticket pricing problem, describing that the WTP for attendance, and to seek contact with the focal group, is likely to be increasing in the number of matches experienced.

3. The Model

3.1 The Demand for Tickets

Suppose that a potential spectator determines to consume a bundle of goods τ and γ at time *t*. Denote τ_t as the number of matches that the individual attends in the given period and γ_t as the number of units of an alternate leisure good. The number of matches that can possibly be attended in a given period is limited to $\hat{\tau}_t$, hence, $\tau_t \leq \hat{\tau}_t$. We assume that a spectator's utility can be described by a Cobb–Douglas function.

Assumption 1. A spectator maximizes her utility described by the function

$$U_t = \tau_t^{\alpha_t} \gamma_t^{(1-\alpha_t)}, \tag{1}$$

which is subject to the budget constraint $M \geq p_t \tau_t + q\gamma_t$, where M is a constant budget, p_t denotes the ticket price in period *t*, and q is the price of the alternate good. In addition, we assume that α_t in (1) has the following properties:

Assumption 2. $\alpha_t = \alpha_t(n_{t-1})$ with $\dfrac{d\alpha_t}{dn_{t-1}} > 0$, $\dfrac{d^2\alpha_t}{dn_{t-1}^{\ 2}} < 0$, and $\alpha_t \in (0,1]$, (2)

where $n_{t-1} = \tau_{t-1} + n_{t-2}$ denotes the aggregate number of matches the individual has attended in previous periods. We define $a \equiv \alpha_t(0)$ for the situation without prior match attendance, and assume that α_t is increasing in n_{t-1} and converging to a given saturation level $\overline{\alpha}$. Hence, we let previously attended matches influence the present consumption choice. Accordingly, $d\alpha_t / dn_{t-1} > 0$ describes the proposed group identification effect in our model.

Lemma 1. *Under assumptions (1) and (2), and for* $\alpha_t < p_t \hat{\tau} / M$, *the individual ticket demand is strictly increasing in the number of previously attended matches.*

Proof. Taking the Lagrangian $L = U_t + \lambda(p_t \tau_t + q\gamma_t - M)$, and using first-order conditions for τ_t, γ_t, and λ yields the *unconstrained* ticket demand $\dot{\tau}_t = \alpha_t M / p_t$. From $\tau_t \leq \hat{\tau}_t$, it follows that the individual ticket demand in period *t* is given by

$$\tau_t = \min\{\alpha_t(n_{t-1}) M / p_t, \hat{\tau}_t\}. \tag{3}$$

With $d\alpha_t / dn_{t-1} > 0$, equation (3) implies $\partial \tau_t / \partial n_{t-1} > 0$ for $\alpha_t < p_t \hat{\tau}_t / M$. □

3.2 The Pricing Problem

A team owner's objective to maximize ticket profits is governed by the choice of the optimal ticket price vector $P_T^* = \left(p_1^*,..., p_T^*\right)$.[2] Any costs linked to the pricing decision are considered negligible for the marginal analysis. The team's overall profits from ticket sale are given by $\Pi = \sum_{t=1}^{T} \delta^{t-1}\Pi_t$, where δ is a positive discount factor. We first scrutinize the case where the venue capacity does not become a binding constraint. Hence, periodic profits are

$$\Pi_t = p_t \sum_{i=1}^{N} \tau_t^i \,, \tag{4}$$

where τ_t^i is added up over N spectators and denotes the individual ticket demand in period *t*.[3] For simplicity, suppose that ticket demand originates from two types of consumers: *fans* and *casual spectators*. The assumption of diminishing returns in (2) is accentuated as follows: Fans represent $(1-z)N$ of all spectators and have a high WTP, that is, $\alpha^F = 1$. On the other hand, casual spectators represent zN of all spectators and have a low initial WTP, that is, $\alpha^C < 1$, and $d\alpha_t / dn_{t-1} = b$, where b denotes the constant *group identification factor*.[4]

Without loss of generality, assume that the budget is equal to unity and the maximum supply of matches to be a constant. Setting $N = 1$, and using equation (3), the T-period pricing problem takes the following form:

$$P_T^* = \arg\max\left[\sum_{t=1}^{T} \delta^{t-1} p_t \left(z\tau_t^C + (1-z)\tau_t^F\right)\right] \quad \text{s.t. } \tau_t^i(p_t) \le \hat{\tau}\,, \tag{5}$$

where $\tau_t^i = \alpha_t^i / p_t$, $\alpha_t^C = a + bn_{t-1} < 1$, and $\alpha^F = 1$.
The first period marks the starting point characterized by a situation where no matches were attended beforehand. In later periods, however, casual spectators may experience a shift in consumption preferences dependent on the number of previously attended matches and the size of the group identification factor b.

3.3 The Optimal Ticket Price Disregarding Spectator Identification

To solve the above problem, it is useful to first define critical price levels that follow from the capacity constraint. Hence, let p_t^i denote the highest price in *t* at which the ticket demand of spectators is still equal to the upper bound, that is, $p_t^i = \max\{p_t | \tau_t^i = \hat{\tau}\}$. Using (3), it follows that $p_t^C = \alpha_t^C / \hat{\tau}$ and $p^F = \alpha^F / \hat{\tau}$ in every period. Hence, $p_t^C < p^F$ for $\alpha_t^C < \alpha^F$.

The analysis of the implications of spectator identification for the optimal ticket pricing policy is conducted by first identifying the price level that would be set by a profit-maximizing team in the absence of identification motives among spectators, that is, in a situation where $b = 0$. This ticket price then serves as the benchmark when we proceed to determine the profit-maximizing ticket price in a situation where b takes a positive value. Following this course, the next result is expressed in Lemma 2:

Lemma 2. *In the absence of spectator identification, profits are maximized by setting the ticket price equal to* p^F.

Proof. See appendix A1.

Hence, the myopic price that would be charged in disregard of identification motives equals $p_t^{My} = p^F$. The corresponding price vector is given by $P_T^{My} = P_T^F = \left(p_1^F,..., p_T^F\right)$.

3.4 The Optimal Ticket Price with Spectator Identification

Having determined the myopic ticket price, we need to examine if and how the introduction of a positive identification factor b may affect the optimal pricing decision. For that, the impact of the price choice on profits in $t = 1,..,T$ is derived. Lemma 3 summarizes the effects:

Lemma 3. *In the presence of spectator identification, the relationships (a) to (c) hold:*

(a) $\dfrac{\partial \Pi_t}{\partial p_t} > 0$ for $p_t \leq p^F$, and $\dfrac{\partial \Pi_t}{\partial p_t} \leq 0$ for $p_t > p^F$,

(b) $\dfrac{\partial \Pi_t}{\partial n_{t-1}} \geq 0$,

(c) $\dfrac{\partial n_{t-1}}{\partial p_{t-k}} \leq 0$ for $k = 1,...,t-1$.

Proof. See appendix A2.

Lemma 3 indicates that raising the price in t increases profits in that same period if the price does not exceed p^F. The results in (b) and (c), taken together, imply that a price increase in the current period may reduce future ticket revenues through a corresponding decrease in the number of matches experienced by casual spectators. We write the following lemma:

Lemma 4. *An increase in the current ticket price may reduce ticket profits in future periods; formally,* $\partial \Pi_t / \partial p_{t-k} \leq 0$ *for* $k = 1,...,t-1$.

Spectator identification may lead to a situation where relatively low prices can be beneficial for the team. If raising the current price results in higher present ticket profits but induces a reduction in future profits, thus outweighing the present gains, then the team has an incentive to refrain from such a price increase. Let p_t^* be the price a profit-maximizing team sets when the identification factor b takes a positive value. The next result is obtained:

Proposition 1. *With spectator identification, it is never optimal to set ticket prices other than* p_t^C *or* p^F*; hence,* $p_t^* \in \left\{p_t^C, p^F\right\}$.

Proof. See appendix A3.

The optimal ticket price may lie below the myopic price if spectator identification is present. It is worth analyzing the conditions under which a profit-maximizing team has an incentive to set the lower of the two prices. The parameter configurations that pinpoint the optimal pricing decision are stated in Proposition 2.

Proposition 2. *The optimal periodic ticket price* $p_t^*(b,\delta)$ *equals* p_t^C *and, thus, falls below the myopic price* p^F *if the group identification factor* b *is sufficiently high; more formally,*

$$p_{T-m}^* = \begin{cases} p_{T-m}^C & \text{if } \Omega_m(b) > \dfrac{1-z}{z}, \quad \text{for } m = 1,\ldots,T-1, \\ p^F & \text{if } \Omega_m(b) \le \dfrac{1-z}{z}, \quad \text{for } m = 1,\ldots,T-1, \\ p^F & \text{for } m = 0, \end{cases}$$

with $\Omega_m \equiv \sum_{n=1}^{m}\left[\sum_{k=n}^{m}\frac{(k-1)!}{(n-1)!(k-n)!}\delta^k b^n \hat{\tau}^n\right]$.

Proof. See appendix B.

Proposition 2 asserts that there exists a threshold $\Omega(b)$ in every decision period that governs the pricing decision, and that is strictly increasing in the group identification factor. When the threshold exceeds the fan-to-spectator ratio it is optimal to set $p_t^* = p_t^C$; otherwise $p_t^* = p^F$ is optimal. $\Omega(b)$ is increasing in δ. Thus, for a given identification factor, stronger discounting reduces the incentive to underprice as the resulting gains in future profits lose in value.

3.5 Discussion

While Proposition 2 explains temporary underpricing, it does not explain ticket underpricing in all periods given the finite time horizon under consideration. If we consider an infinite time horizon, however, and assume that $\alpha(n_{t-1})$ converges to its limit as n approaches infinity, p^C can become the permanent optimum. Lemma 3 offers a sufficient condition for this result to hold.[5] Accordingly, $P_\infty^* = P_\infty^C$ holds if, in every period *t*, the one-time revenue gain in *t* from setting p^F instead of p_t^C falls below the value of the infinite series of revenue losses resulting in all future periods.

The presented formal analysis scrutinizes the case in which the given venue capacity does not become a binding constraint. The proposed identity-based rationale may, however, also help to explain optimal ticket prices below the maximum sell-out level, for example, in a market that is characterized by potential spectators with very different individual saturation levels $\overline{\alpha}$. In particular, prices even below the maximum sell-out level may attract spectators whose identification is still low, e.g. $\alpha(i)$, but with the potential to increase to much higher levels when compared to other already saturated spectators, such that $\alpha(i) < \overline{\alpha}(j) < \overline{\alpha}(i)$. Ticket prices that are just too high to attract and stimulate such high potentials in the market may fail to maximize long-term revenues from ticket sale.

4. Conclusion

The object of this paper was to examine the impact of spectators' identification with a sports team and its supporters on the ticket pricing decision in the primary ticket market. Following the concept of social identity, the presented model has established a link between spectators' visiting frequency and the valuation of match attendance to explain ticket underpricing.

The findings indicate that the optimal periodic ticket price level in the primary market may well lie below the short-term revenue maximizing price if match attendance increases spectators' group identification and induces a rise in their WTP for tickets. While the analysis investigates the unifying effect of social identification, the experience of clamoring or hostile spectators may as well cause resentment, an effect that can interfere with the process of social identification and at times lead to social self-exclusion of spectators. In such cases, it may be difficult to stimulate the identification process that is proposed in our analysis.

In view of its seemingly universal relevance in spectator sports, the concept of social identity on the part of spectators may bear relevance to understanding inelastic ticket pricing in a variety of sports leagues.

Notes

1. Wann (2006) provides a comprehensive overview of studies investigating the impact of team identification on match attendance and spectators' consumption preferences.
2. Unlike periodic prices, ticket price vectors are indicated by a capital letter. For instance, the vector P_t contains all prices up to period *t*, that is, $P_t = (p_1,..., p_{t-1}, p_t)$.
3. Notice that lower indices indicate the point in time, whereas upper indices indicate the individual or group-specific context of a variable.
4. We therefore restrict the domain to $0 < b \leq (1-a)/n_{T-1}$. While the results do not depend on this assumption, it is useful to simplify the formal analysis. One interpretation of this specification is to consider $\alpha_t(n_{t-1})$ over a sufficiently small interval of its domain where $d^2\alpha_t / dn_{t-1}^2 \cong 0$, thus, implying a finite time horizon.
5. Note that Lemmas 3, 4, and Proposition 1 remain valid under the infinity assumption.

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Appendix A1 (Proof of Lemma 2)

The critical prices are used to restate (5): As $\alpha_t^C = a + bn_{t-1}$, and $\alpha^F = 1$, it follows for $b = 0$ that $p_t^C = a/\hat{\tau}$ and $p^F = 1/\hat{\tau}$.

With $\Pi = \sum_{t=1}^{T} \delta^{t-1}\Pi_t$ and demand $\tau_t^i = \alpha_t^i / p_t$ (Lemma 1), profits in t are:

$$\Pi_t = \begin{cases} p_t\hat{\tau} & p_t < p_t^C, \\ za + (1-z)p_t\hat{\tau} \quad \text{for} & p_t^C \le p_t \le p^F, \\ 1 + z(a-1) & p^F < p_t. \end{cases} \tag{A.1}$$

Profits are strictly increasing in p_t on $\left[0, p^F\right]$. As $\Pi_t(p_t) = 1 + z(a-1)\ \ \forall\ p_t > p^F$, periodic and overall profits are maximized by setting $p_t \ge p^F$, where the lowest price p^F serves as the benchmark for subsequent analysis. □

Appendix A2 (Proof of Lemma 3)

The critical price levels are used to restate (A.1): As $\alpha_t^C = a + bn_{t-1}$, and $\alpha^F = 1$, it follows for $b > 0$ that $p_t^C = (a + bn_{t-1})/\hat{\tau}$ and $p^F = 1/\hat{\tau}$.

With $\Pi = \sum_{t=1}^{T} \delta^{t-1}\Pi_t$ and demand $\tau_t^i = \alpha_t^i / p_t$ (Lemma 1), profits in t read:

$$\Pi_t = \begin{cases} p_t\hat{\tau} & p_t < p_t^C, \\ z(a + bn_{t-1}) + (1-z)p_t\hat{\tau} \quad \text{for} & p_t^C \le p_t \le p^F, \\ 1 + z(a - 1 + bn_{t-1}) & p^F < p_t, \end{cases} \tag{A.2}$$

where $n_{t-1} = \tau_{t-1}^C + n_{t-2}$.

(a) As $(1-z)\hat{\tau} > 0$, Π_t is strictly increasing in p_t on $\left[0, p^F\right]$, is nonincreasing for $p_t > p^F$.

(b) Since $b > 0$, Π_t is increasing in n_{t-1} for $p_t \ge p_t^C$, and independent of n_{t-1} for $p_t < p_t^C$.

(c) As $n_{t-1} = \tau_{t-1}^C + n_{t-2}$, we use the insight from (3) that $\partial\tau_t / \partial p_t \le 0$. With $\partial n_{t-1} / \partial\tau_{t-1}^C > 0$ and $\partial n_{t-1} / \partial n_{t-2} > 0$, it holds that $\partial n_{t-1} / \partial p_{t-k} \le 0$ for $k = 1,\ldots,t-1$. □

Appendix A3 (Proof of Proposition 1)

Proof of Proposition 1 is given by showing that **(i)** to **(iii)** hold true:

(i): $\forall p_t < p_t^C :\ p_t^* \neq p_t$

(ii): $\forall p_t > p^F :\ p_t^* \neq p_t$

(iii): $\forall p_t \in \left(p_t^C, p^F\right):\ p_t^* \neq p_t$

(i) $\left[0, p_t^C\right)$:

From (A.2) we know that $\partial\Pi_t / \partial p_t > 0$ on $\left[0, p_t^C\right)$ as $\hat{\tau} > 0$.

Moreover, $\partial\tau_t^C / \partial p_t = 0$ holds since $p_t^C = \max\left\{p_t \middle| \tau_t^C = \hat{\tau}\right\}$. Therefore, $\partial n_{t-1} / \partial p_{t-k} = 0$, and $\partial\Pi_t / \partial p_{t-k} = 0$, for $k = 1,...,t-1$. Hence, $\forall p_t < p_t^C :\ p_t^* \neq p_t$

(ii) $\left(p^F, \infty\right)$:

Lemma 3(a) states that $\partial\Pi_t / \partial p_t > 0$ for $p_t \leq p^F$ and $\partial\Pi / \partial p_t \leq 0$ for $p_t > p^F$. That is, the strictly positive price effect is limited to the price interval $\left[0, p^F\right]$.

Lemma 4 states that $\partial\Pi_t / \partial p_{t-k} \leq 0$ for $k = 1,...,t-1$. The latter effect is strictly negative for $p_t > p^F$ where $\Pi_t = \Pi_t\left(n_{t-1}\right)$. Hence, $\forall p_t > p^F :\ p_t^* \neq p_t$.

(iii) $\left[p_t^C, p^F\right]$:

Define $t \equiv T - m$, and $\Gamma_{T-m} \equiv \sum_{k=0}^{m} \delta^k \Pi_{T-m+k}$. As the present pricing decision in t may also affect profits from ticket sales in all future periods, the objective function in t is restated as $p_{T-m}^* = \arg\max\left[\Gamma_{T-m}\right]$, where $p_{T-m}^* \in \left[p_{T-m}^C, p^F\right]$.

As Γ_{T-m} is continuous on $\left[p_{T-m}^C, p^F\right]$, we get $\partial\Gamma_{T-m} / \partial p_{T-m} = \left(\sum_{k=0}^{m} \delta^k \partial\Pi_{T-m+k}\right) / \partial p_{T-m}$.

Because $\partial\Pi_{T-m} / \partial p_{T-m} > 0$ and $\partial^2\Pi_{T-m} / \partial p_{T-m}^2 = 0$, and in addition $\partial\Pi_{T-m+k} / \partial p_{T-m} < 0$ and $\partial^2\Pi_{T-m+k} / \partial p_{T-m}^2 > 0$ for $k = 1,..,m$, it must hold true that either:

- $\Gamma_{T-m}(p_{T-m}^C) < \Gamma_{T-m}(p_{T-m}) < \Gamma_{T-m}(p^F)$, or
- $\Gamma_{T-m}(p^F) < \Gamma_{T-m}(p_{T-m}) < \Gamma_{T-m}(p_{T-m}^C)$, or
- $\Gamma_{T-m}(p_{T-m}^C) = \Gamma_{T-m}(p^F) > \Gamma_{T-m}(p_{T-m})$,

for any p_{T-m} in the open interval (p_{T-m}^C, p^F). It follows that $p_{T-m}^* \in \left\{p_{T-m}^C, p^F\right\}$. □

Appendix B (Proof of Proposition 2)

Proof of Proposition 2 is given by proving that the implications **(I)** and **(II)** are true:

(I): $p^*_{T-m+1} = p^C_{T-m+1} \rightarrow p^*_{T-m} = p^C_{T-m}$,

(II): $\Omega_{m-1} \le \dfrac{1-z}{z} < \Omega_m \rightarrow p^*_{T-m} = p^C_{T-m}$,

with $\Omega_m \equiv \sum_{n=1}^{m}\left[\sum_{k=n}^{m} \dfrac{(k-1)!}{(n-1)!(k-n)!}\delta^k (b\hat{\tau})^n\right]$ for $m \ge 1$, and $\Omega_0 \equiv 0$.

From **(I)** and **(II)** is deduced that $\dfrac{1-z}{z} < \Omega_m \rightarrow p^*_{T-m} = p^C_{T-m}$.

(I): Having defined $t \equiv T-m$ and $\Gamma_{T-m} \equiv \sum_{k=0}^{m}\delta^k \Pi_{T-m+k}$, we use the Proposition 1 result $p^*_{T-m} \in \{p^C_{T-m}, p^F\}$ and examine the conditions under which $\Gamma_{T-m}(p^C_{T-m}) > \Gamma_{T-m}(p^F_{T-m})$.

Considering (A.2) on the price interval $[p^C_{T-m}, p^F]$, $\Pi_{T-m} = z(a+bn_{T-m-1}) + (1-z)p_{T-m}\hat{\tau}$ are the profits for $m = 1,...T-1$. For $m=0$, $p^*_T = p^F$ holds, hence, $\Pi_T = z(a+bn_{T-1}) + (1-z)$.

Inserting Π_{T-m+k} for $k = 0,...,m$ into Γ_{T-m} yields:

$$\Gamma_{T-m} = \sum_{k=0}^{m}\left[\delta^k z(a+bn_{T-m-1}) + \delta^k p_{T-m+k}(1-z)\hat{\tau}\right] + zb\sum_{k=0}^{m-1}\left[\sum_{n=1+k}^{m}\delta^n \tau_{T-m+k}\right]. \tag{B.1}$$

Since $p^*_{T-m} = p^C_{T-m}$ if $\Gamma_{T-m}(p^C_{T-m}) > \Gamma_{T-m}(p^F)$, and $p^*_{T-m} = p^F$ otherwise, we can use (B.1) to rewrite $\Gamma_{T-m}(p^C_{T-m}) > \Gamma_{T-m}(p^F)$ as:

$$b \cdot \frac{\sum_{k=0}^{m-1}\left[\sum_{n=1+k}^{m}\delta^n\left(\tau_{T-m+k}(p^C_{T-m}) - \tau_{T-m+k}(p^F_{T-m})\right)\right]}{1-(a+bn_{T-m-1})} > \frac{1-z}{z}, \tag{B.2}$$

where $p^*_{T-m} = p^C_{T-m}$ iff (B.2) holds true and $p^*_{T-m} = p^F$ otherwise.

Now let $\Delta\tau_{m,k} \equiv \sum_{n=1+k}^{m}\delta^n\left(\tau_{T-m+k}(p^C_{T-m}) - \tau_{T-m+k}(p^F_{T-m})\right)$, so that we can write:

$$b \cdot \frac{\sum_{k=0}^{m-1}\Delta\tau_{m,k}}{1-(a+bn_{T-m-1})} > \frac{1-z}{z}, \tag{B.3}$$

and define $\hat{m} \equiv m+1$. As $\Delta\tau_{m,k}$ allows factoring out $1-(a+bn_{T-m-1})$ for $k = 0,...,m-1$, it follows that if (B.3) holds, (B.4) must always hold:

$$\frac{\sum_{k=0}^{\hat{m}-1}\Delta\tau_{\hat{m},k}}{1-\left(a+bn_{T-\hat{m}-1}\right)} > \frac{\sum_{k=0}^{m-1}\Delta\tau_{m,k}}{1-\left(a+bn_{T-m-1}\right)}. \tag{B.4}$$

From (B.3) and (B.4) it follows that, if $p^*_{T-m} = p^C_{T-m}$, it must also hold that $p^*_{T-\hat{m}} = p^C_{T-\hat{m}}$, or, equivalently:

$$p^*_{T-m+1} = p^C_{T-m+1} \rightarrow p^*_{T-m} = p^C_{T-m}. \qquad \square$$

(II): Proof is given by applying backward induction to (B.2). With $p^*_T = p^F$, we solve (B.2) for $m = 1$, and derive $\delta b\hat{\tau} > (1-z)/z$, hence:

$$p^*_{T-1} = p^C_{T-1} \quad \text{if} \quad \delta b\hat{\tau} > \frac{1-z}{z}, \tag{B.5}$$

$$p^*_{T-1} = p^F \quad \text{if} \quad \delta b\hat{\tau} \leq \frac{1-z}{z}. \tag{\beta.5}$$

We now look at the case where (β.5) holds true. Hence, we assume $p_{T-1} = p^F = 1/\hat{\tau}$. Using $p_T = p_{T-1} = p^F$ to solve (B.2) for $m = 2$, we get $\delta^2(b\hat{\tau})^2 + (\delta+\delta^2)b\hat{\tau} > (1-z)/z$ and write:

$$p^*_{T-2} = p^C_{T-2} \quad \text{if} \quad \delta b\hat{\tau} \leq \frac{1-z}{z} < \delta^2(b\hat{\tau})^2 + (\delta+\delta^2)b\hat{\tau}, \tag{B.6}$$

$$p^*_{T-2} = p^F \quad \text{if} \quad \delta^2(b\hat{\tau})^2 + (\delta+\delta^2)b\hat{\tau} \leq \frac{1-z}{z}. \tag{\beta.6}$$

Continuing the process of backward induction and following the beta-branches, we find the generalized condition (B.2) for $m \geq 1$, which is given by:

$$\sum_{n=1}^{m}\left[\sum_{k=n}^{m}\frac{(k-1)!}{(n-1)!(k-n)!}\delta^k(b\hat{\tau})^n\right] > \frac{1-z}{z}.$$

Defining $\Omega_m \equiv \sum_{n=1}^{m}\left[\sum_{k=n}^{m}\frac{(k-1)!}{(n-1)!(k-n)!}\delta^k(b\hat{\tau})^n\right]$ for $m \geq 1$, and $\Omega_0 \equiv 0$, the ticket pricing conditions in generalized form read:

$$p^*_{T-m} = p^C_{T-m} \quad \text{if} \quad \Omega_{m-1} \leq \frac{1-z}{z} < \Omega_m,$$

$$p^*_{T-m} = p^F \quad \text{if} \quad \Omega_m \leq \frac{1-z}{z}. \qquad \square$$