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Life expectancy, schooling and lifetime labor supply: the Ben-Porath mechanism revisited

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Abstract

Hansen and Lønstrup [Journal of Population Economics, 2012] construct a three-period, life-cycle model to study the famed Ben-Porath mechanism and attempt to reconcile it with the empirical findings in Hazan [Econometrica, 2009]: increased life expectancy has a positive effect on schooling but a negative effect on expected lifetime labor supply. This paper shows that within the framework of Hansen and Lønstrup (2012), as life expectancy increases, expected lifetime labor supply may not decline even when labor supplies at the two end stages of life do. I fully characterize the conditions under which the model in Hansen and Lønstrup (2012) can produce the empirical findings in Hazan (2009).

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1. Introduction

The relationship between life expectancy and individual education attainment has been studied, both theoretically and empirically, starting from the seminal work of Ben-Porath (1967) which argues that reductions in mortality induce more investment in human capital. The intuitive reasoning behind this causal relationship is that as life expectancy increases, the horizon over which investment in schooling is paid off will also be increased, thereby inducing individuals to invest more in education.

Recently, Hazan (2009) raises an important question in the context of the Ben-Porath mechanism. He shows that a necessary condition for the Ben-Porath mechanism to hold is that increased life expectancy must also *increase* expected lifetime labor supply. He then estimates the empirical counterpart of expected lifetime labor supply by computing the expected total working hours over a lifetime (henceforth, ETWH) of consecutive cohorts of American men born between 1840 and 1970, and of all American individuals born between 1890 and 1970. More specifically, in his estimation, the ETWH is determined by three factors: the age specific mortality rates, and the labor supply decisions along both the extensive and intensive margins at each age. Hazan's data analysis suggests that the reduction in labor supply along both the extensive and intensive margins outweighs the gains in life expectancy, leading to a *decline* in expected lifetime labor supply, the exact opposite of the needed necessary condition. In other words, the data seem to suggest a negative relation between expected lifetime labor supply and life expectancy while the theory underlying the Ben-Porath mechanism requires the same relation to go the other way.

Ever since this issue was raised, a growing research effort has been devoted to study the relationship between life expectancy, schooling and lifetime labor supply. A recent contribution is Cervellati and Sunde (2013), who use a prototype Ben-Porath model with age-specific survival rates to show that an increase in expected lifetime labor supply is neither a necessary nor a sufficient condition for greater life expectancy to increase optimal schooling. Another important piece of work is Hansen and Lønstrup (2012) – hereafter HL. They construct a three period life-cycle model and assume a) no capital markets for the young, and b) no annuity markets for the middle-aged. They show, theoretically, that increased life expectancy has both a positive effect on schooling and a negative effect on retirement leading them to conclude that expected lifetime labor supply must go up. In their judgment, they reconcile the empirical finding in Hazan (2009) with the theoretical Ben-Porath mechanism in their paper.

In this paper, I point out that the empirical results in Hazan (2009) cannot be directly connected to the theoretical result on lifetime labor supply and life expectancy in HL. Hazan (2009) estimated *expected* lifetime labor supply, which is affected both by age-specific mortality rates and labor supply at each age. However, HL only show that young-age labor supply and old-age (conditional on reaching old age) each decrease, but they don't consider the effect of increased survival itself on expected lifetime labor supply. The main contribution of this note is to see if their model can generate a negative relationship between life expectancy and the correct measure of expected lifetime labor supply. Using the framework of HL, I show that as life expectancy increases, expected lifetime labor supply may not decline even when labor supplies at the two end stages of life do.

The plan for the rest of the note is as follows. Section 2 outlines the Hansen and Lønstrup

(2012) model of life expectancy, schooling and retirement. Section 3 contains a detailed analysis of the effect of life expectancy on expected lifetime labor supply. Section 4 concludes. Proofs of central results are to be found in the appendices.

2. The model

The model (and the notation) is based entirely on HL. Consider a small open economy where the wage rate, w, and the gross real interest rate, R, are exogenously given. An individual can live at most for three periods. In each alive period, he is endowed with one unit of time. In the first period, he has one unit of human capital. The unit time endowment is divided between schooling time, e, and labor supply $1-e$. It's assumed a young individual is unable to borrow to finance consumption during the first (schooling) period. With this assumption, the choices on schooling and the consumption path are interdependent, and spending more time on schooling implies a lower consumption when young. This assumption is important because if individuals can smooth consumptions between the first and second period, then schooling is decided only with the objective of maximizing present value lifetime income. As a consequence, schooling is strictly increasing in expected lifetime labor supply, and we would never be able to reconcile the empirics in Hazan (2009). The income in the first period is used solely for consumption, $c_1 = w\left(1-e\right)$.[1] Individuals survive with certainty onto the second period where they supply $h = h\left(e\right)$ units of efficient labor inelastically. The wage income is divided between consumption, c_2, and saving, $s : c_2 = wh\left(e\right) - s$. We assume $h'\left(e\right) > 0, h''\left(e\right) < 0$ and $h\left(0\right) = 1$.

Survival becomes uncertain at the end of the second period. Let $\phi \in (0,1)$ denote the probability of surviving onto the third period. Contingent on survival, individuals divide the unit time endowment between leisure, l, and working time, $1-l$. To facilitate interpretation, I label l as retirement. Labor market income and prior saving are used to fund third-period consumption, c_3. Here, it is assumed, just as in HL, annuity markets are absent, so the return to the saving is unaffected by the survival probability ϕ. Also, there are no bequests. One can assume unclaimed saving is taxed away and used on useless government consumption. Third period consumption is given by $c_3 = wh\left(e\right)\left(1-l\right) + Rs$.

The expected lifetime utility is represented by

$$U = \psi \ln c_1 + \beta \ln c_2 + \phi\beta^2 \left(\ln c_3 + \theta \ln l\right), \tag{1}$$

where $\psi \geq 0$ is an inverse measure of the taste for acquiring knowledge from education, $\beta > 0$ is a time discount factor, and $\theta > 0$ is the taste for leisure in the third period.[2] Each individual maximizes (1) subject to the budget constraints above. I obtain the following solutions for schooling time, savings and retirement:

$$e = \frac{1+\beta\phi}{1+\beta\phi+\frac{\psi}{\beta\mu}}, \tag{2}$$

[1] HL point out that by assuming individuals cannot borrow when young does not exclude the possibility of positive savings to smooth consumption across periods. However, in the schooling period, this is a theoretical curiosity, since higher earnings later in life and a desire to smooth consumtion will pull in the direction of borrowing rather than saving.

[2] HL specialize to the logarithmic utility function to obtain their main result – Proposition 4.

$$s=\frac{1+\theta-\frac{1}{\beta\phi R}}{1+\theta+\frac{1}{\beta\phi}}wh\left(e\right)\equiv g\left(\phi\right)wh\left(e\right), \tag{3}$$

and

$$l=\frac{\theta\left[1+Rg\left(\phi\right)\right]}{1+\theta}=\frac{\theta\left(1+R\right)}{1+\theta+\frac{1}{\beta\phi}}, \tag{4}$$

where $\mu\equiv\frac{h'(e)}{h(e)}e>0$ is the constant elasticity of human capital with respect to schooling time, and $g\left(\phi\right)\equiv\frac{1+\theta-\frac{1}{\beta\phi R}}{1+\theta+\frac{1}{\beta\phi}}<1$, $g'\left(\phi\right)>0$. [3]From equations (2) and (4), we can derive

$$\frac{de}{d\phi}=\frac{\psi}{\mu\left(1+\beta\phi+\frac{\psi}{\beta\mu}\right)^{2}}>0, \tag{5}$$

and

$$\frac{dl}{d\phi}=\frac{\theta Rg'\left(\phi\right)}{1+\theta}>0. \tag{6}$$

(5) and (6) imply that an exogenous rise in the survival probability has a positive effect on schooling time and a negative effect on the retirement age – Proposition 4 in HL.

The intuition is clear. A higher probability of entering old age induces the middle-aged to save more. The consumption-smoothing motive induces agents to want to distribute these savings across youth and middle age thereby lowering consumption in each. Since capital markets are absent in youth, consumption smoothing may be achieved by spending more time on education. Those alive in the third period begin with a higher wealth because they saved more (in terms of the saving ratio) in the past, and the return to saving is fixed. In the convenient, logarithmic case where the income and substitution effects of a higher wage rate (caused by more schooling time e) cancel each other, the only income effect from the higher initial wealth induces these old people to retire earlier.

3. Expected lifetime labor supply

HL argue that their Proposition 4 implies a negative relation between schooling time and lifetime labor supply, which, in their view, squares well with the empirical findings in Hazan (2009). However, in Hazan (2009), the empirical counterpart of lifetime labor supply is *expected* total working hours over a lifetime – ETWH – and in this three-period model, that is defined as:

$$ETWH=\left(1-e\right)+1+\phi\left(1-l\right), \tag{7}$$

[3]Notice that in an economy without annuity market, borrowing is not allowed at the end of the second period, i.e., $s\geq 0$. So here, we restrict the analysis to $\phi\in\left[\frac{1}{\beta R(1+\theta)},1\right)$. Also, we assume interior solutions for both e and l. Clearly, $e\in(0,1)$ is satisfied. $l\in(0,1)$ requires $R<\frac{1}{\theta}\left(1+\frac{1}{\beta\phi}\right)$. If we want this to hold for all $\phi\in\left[\frac{1}{\beta R(1+\theta)},1\right)$, then it requires $R<\frac{1}{\theta}\left(1+\frac{1}{\beta}\right)$.

and not as

$$TWH = (1-e) + 1 + (1-l)\,.$$

TWH is the lifetime labor supply of only those who survive to the third period. So if we follow Hazan (2009), and look at the *expected* lifetime labor supply at the beginning of each life cycle, then the relevant measure of expected lifetime labor supply is given by (7).

An increase in schooling, together with early retirement doesn't necessarily imply a decrease in expected lifetime labor supply. This is because although people retire earlier conditional on being alive in the third period, they have a higher survival probability at the end of their middle age, so the change of the *expected* old-age labor supply, $\phi(1-l)$, is indeterminate. If the gains in life expectancy outweigh the reductions in labor supply, then the expected lifetime labor supply will actually increase.

Equations (2)(4) and (7) lead to the following proposition:

Proposition 1 *Consider an economy identical to that in Hansen and Lønstrup (2012). In particular, if the utility function is logarithmic and if the elasticity of human capital with respect to schooling time is constant, the effect of an exogenous rise in the survival probability on the expected lifetime labor supply is, in general, indeterminate. In particular, an exogenous rise in the survival probability raises (reduces) lifetime labor supply iff*

$$R \lessgtr \check{R} \equiv \frac{1}{1+\theta+\frac{2}{\beta\phi}}\left[1-\frac{\psi\left(1+\theta+\frac{1}{\beta\phi}\right)^2}{\mu\theta\left(1+\beta\phi+\frac{\psi}{\beta\mu}\right)^2}+\frac{1}{\theta}\left(1+\frac{1}{\beta\phi}\right)^2\right].$$

The proof of Proposition 1 is in Appendix A. Proposition 1 suggests an increase in schooling time and an earlier retirement doesn't guarantee a negative relationship between life expectancy and lifetime labor supply. The gains in life expectancy may outweigh the reduction in labor supply leading to an increase in lifetime labor supply.

HL provide a robustness test for their results. Here I use exactly the same functional forms and parameter values in HL to check the effect of life expectancy on lifetime labor supply. The utility function is:

$$U = \psi\frac{c_1^{1-\sigma}}{1-\sigma}+\beta\frac{c_2^{1-\sigma}}{1-\sigma}+\phi\beta^2\left(\frac{c_3^{1-\sigma}}{1-\sigma}+\theta\frac{l^{1-\gamma}}{1-\gamma}\right), \tag{8}$$

and

$$h(e) = 1 + Ae^{\mu}. \tag{9}$$

The parameters of the models are set as follows: $\psi = 0.69, \theta = 0.8, \gamma = 1.5, \sigma = 0.8, \beta = \frac{1}{R} = 0.8, A = 3,$ $w = 1$ and $\mu = \frac{1}{3}$. HL set $R = 1.25$ in their example. I add another case with a higher interest rate, $R = 2$ (other parameter values remain unchanged), to illustrate the two vastly different responses of ETWH with changing life expectancy.[4]

[4] By considering the length of each period to be 25 years, the annual interest rate is around 0.9% for the case $R = 1.25$, and 2.8% for the case $R = 2$. In both cases, we only consider the range of ϕ over which savings are positive. Also, we always have interior solutions for e and l in this example.

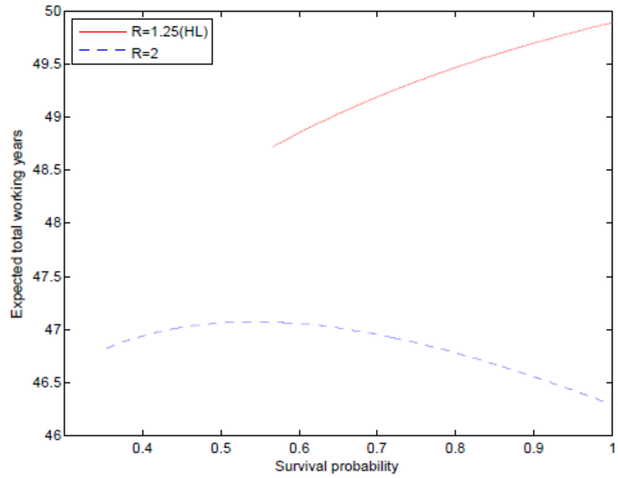

Figure 1: Life expectancy and ETWH

Figure 1 shows that in different cases, the effect of life expectancy on expected lifetime labor supply is different. In the case with a relatively low interest rate, $R = 1.25$, an increase in life expectancy leads to an increase in expected lifetime labor supply, which is not supported by the empirics of Hazan (2009). In the case with a relatively high interest rate, $R = 2$, the effect is non-monotonic. As Proposition 1 hinted – see definition of $\check{R}$ – the relationship is itself dependent on the level of ϕ. It suggests that in the case with $R = 2$, the effect of life expectancy on expected lifetime labor supply may depend on the stage of development of an economy. In particular, in an early stage of development, the probability of surviving to an old age, say, 75, conditional on reaching a middle age, say, 50, is relatively low ($\phi < 0.54$ in Figure 1). In this economy, an increase in life expectancy leads to an increase in expected lifetime labor supply. Whereas in a later stage of development, the economy begins with a high survival probability ($\phi > 0.54$ in Figure 1), then an increase in life expectancy will lead to a decrease in expected lifetime labor supply. So in the case of $R = 2$, only with a high survival probability, the empirical findings in Hazan (2009) will be supported. Notice in this example only the interest rate is allowed to vary, but there could be other factors, such as parameters capturing individuals' preferences, that differ and generate the differences in effects.

4. Conclusion

This paper studies the effect of life expectancy on expected lifetime labor supply based on the life-cycle model in Hansen and Lønstrup (2012). I show that in an economy without annuity markets, as life expectancy increases, although schooling time increases and individuals retire earlier conditional on being alive in the third period, since the survival probability also increases, the effect on the expected lifetime labor supply is ambiguous. Only when the interest rate sufficiently high does increasing life expectancy decrease expected lifetime labor supply.

Appendix

A Proof of Proposition 1

Using (2)(4) and (7), we can calculate the effect of an increase in survival probability on expected lifetime labor supply:

$$\begin{aligned}
\frac{dETWH}{d\phi} &= -\frac{de}{d\phi}+1-l-\phi\frac{dl}{d\phi} \\
&= -\frac{\psi}{\mu\left(1+\beta\phi+\frac{\psi}{\beta\mu}\right)^2}+1-\frac{\theta\left(1+R\right)}{1+\theta+\frac{1}{\beta\phi}}-\frac{\theta\left(1+R\right)}{\beta\phi\left(1+\theta+\frac{1}{\beta\phi}\right)^2} \\
&= -\frac{\psi}{\mu\left(1+\beta\phi+\frac{\psi}{\beta\mu}\right)^2}+\frac{\left(1+\frac{1}{\beta\phi}\right)^2+\theta\left(1-R\left(1+\theta+\frac{2}{\beta\phi}\right)\right)}{\left(1+\theta+\frac{1}{\beta\phi}\right)^2}.
\end{aligned}$$

So

$$\frac{dETWH}{d\phi}\gtrless 0 \ \text{ iff } \ R\lessgtr \check{R}\left(\phi\right)\equiv\frac{1}{1+\theta+\frac{2}{\beta\phi}}\left[1-\frac{\psi\left(1+\theta+\frac{1}{\beta\phi}\right)^2}{\mu\theta\left(1+\beta\phi+\frac{\psi}{\beta\mu}\right)^2}+\frac{1}{\theta}\left(1+\frac{1}{\beta\phi}\right)^2\right].$$

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