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Collusion and demand volatility

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Abstract

In this article, we use a simple mean-variance set up, where two symmetric firms are competing in quantities, and investigate the impact of demand variability on the stability of collusion.

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1. Model

Using a real option model, Hassan (2006) presented an analysis of optimal cartel defection timing. Members of the cartel are competing in prices and demand shocks are described by a Brownian motion. He concludes that "volatility serves as a hitherto unidentified collusion facilitating factor." Also in a real option set up, Wong (2008) shows that an increase in the underlying market demand uncertainty leads to a "U-shaped pattern of the optimal defection trigger against the market demand volatility.". In the model below we use a simple mean-variance set up, where two symmetric firms are competing in quantities, and investigate the impact of demand variability on the stability of collusion.

Consider a market where two firms, labelled Firm 1 and Firm 2, are operating and selling a homogeneous good. The inverse demand function is assumed linear and given by:

$$P = L - Q$$

where P is the market price of the good, L is a random variable of mean 1 and variance σ^2, and Q is the total production of the two firms. We assume that the two firms' aversion to risk is embodied in a mean-variance utility function so that, for $i = 1, 2$, the welfare of Firm i is given by:

$$\begin{aligned} W_i &= E\left[\left(L - q_1 - q_2 - c\right) q_i\right] - \rho \mathrm{Var}\left[\left(L - q_1 - q_2\right) q_i\right] \\ &= \left(1 - q_1 - q_2 - c\right) q_i - q_i^2 \sigma^2 \rho, \end{aligned}$$

where $c < 1$ is the marginal production cost. $\rho$is the risk aversion parameter.

2. The Cournot and collusion equilibrium outcomes

If the two firms choose their production independently, individual concave welfare maximization yields the following conditions:

$$\begin{aligned} -c - 2q_1 - q_2 - 2\sigma^2 \rho q_1 + 1 &= 0 \\ -c - 2q_2 - q_1 - 2\sigma^2 \rho q_2 + 1 &= 0 \end{aligned}$$

which reduces to

$$q_i^e = \frac{1 - c}{2\sigma^2 \rho + 3}, i = 1, 2.$$

Notice that both demand variability and risk aversion have a negative impact on the equilibrium production in the Cournot market. The equilibrium welfare of each firm under Cournot competition is then

$$W^e = (1 - c)^2 \frac{\sigma^2 \rho + 1}{\left(2\sigma^2 \rho + 3\right)^2}. \tag{1}$$

Now assume that the two firms choose production levels maximizing their joint welfare, given by

$$\sum_{i=1}^{2} \left(1 - q_1 - q_2 - c\right) q_i - q_i^2 \rho \sigma^2.$$

In that case, the optimal production is

$$q^c = \frac{1-c}{2\sigma^2\rho + 4}$$

and the corresponding collusion equilibrium welfare is then

$$W^c = \frac{1}{4}\frac{(1-c)^2}{\sigma^2\rho + 2}, \tag{2}$$

which is obviously always higher than the firms' welfare under Cournot competition.

3. Collusion sustainability

As is well known, in a static setting collusion is not an equilibrium, since there is an incentive for each firm to unilaterally deviate from the agreed-upon production and capture a larger share of the profit. To ensure collusion sustainability, one needs a repeated game setting, where firms threaten to revert to the Cournot equilibrium quantity as soon as a deviation from the cooperative production is detected (Friedman, 1971).

Suppose that Firm 1 can deviate from the collusion production quantity for a short period of time, after which the other firm reverts to the Cournot equilibrium quantity. During the period where Firm 1 is deviating from the collusion equilibrium while Firm 2 is not, the welfare of Firm 1 is given by

$$\left(1 - q_1 - \frac{1-c}{2\sigma^2\rho + 4} - c\right) q_1 - q_1^2\rho\sigma^2.$$

The maximization of this concave function yields the following deviation quantity:

$$q^d = \frac{(1-c)}{4}\frac{2\sigma^2\rho + 3}{(\sigma^2\rho + 1)(\sigma^2\rho + 2)}$$

and corresponding deviation welfare:

$$W^d = \frac{(1-c)^2}{16}\frac{(2\sigma^2\rho + 3)^2}{(\sigma^2\rho + 1)(\sigma^2\rho + 2)^2}. \tag{3}$$

Collusion can be sustained if

$$W^d(1-\delta) + \delta W^e < W^c \tag{4}$$

where $\delta \in (0,1)$ is a discount factor that depends on the duration of the detection period and on the time value of money (see Appendix). From (1)-(4), collusion can be sustained if

$$\begin{aligned} \delta &> \frac{W^d - W^c}{W^d - W^e} \\ &= \frac{(2\sigma^2\rho + 3)^2}{8\sigma^4\rho^2 + 24\sigma^2\rho + 17} \end{aligned}$$

that is, if the detection period is sufficiently short and/or the interest rate is sufficiently small. In fact, in that case, the threshold value for the discount factor δ only depends on demand variability and risk aversion. The impact of demand variability and of risk aversion on the threshold value of δ is negative:

$$\begin{aligned} & \frac{d}{d\left(\rho\sigma^2\right)}\left(\frac{\left(2\sigma^2\rho+3\right)^2}{8\sigma^4\rho^2+24\sigma^2\rho+17}\right) \\ = & -4\frac{2V+3}{\left(8V^2+24V+17\right)^2}<0, \end{aligned}$$

indicating that the threshold value above which collusion can be sustained is decreasing with demand variability. We conclude that demand variability is indeed a collusion facilitating factor in this model where firms compete in quantities.

4. An alternative model of collusive behavior

In this section, we use an alternative interpretation of collusion, where the two colluding firms behave as a single entity and maximize

$$(1-Q-c)\,Q-Q^2\rho\sigma^2.$$

The profit function above is similar to that of a quantity setting cartel acting as a monopolist. It assumes symmetric firms in terms of costs and an equal sharing of the cartel profit or equivalently an equal sharing of the quantity produced at the same market price. This is different from the previous collusion case (section 3) whereby each firm is choosing its production to maximize the total profit of the industry.

Notice that, when welfare includes a risk aversion component, minimizing the variability of the total revenues in the industry is not the same as minimizing the sum of the revenue variabilities of the two firms

$$\operatorname{Var}\left[L\left(q_1+q_2\right)\right]\neq\operatorname{Var}\left[Lq_1\right]+\operatorname{Var}\left[Lq_2\right],$$

where it is obvious that the l.h.s. is always larger than the r.h.s. When the two colluding firms behave as a single entity, extreme variations in the industry are more penalized and variations in demand have a greater impact on the firms' decisions. In that case, it is no longer obvious that the individual firm's welfare is greater under collusive behavior than under Cournot competition.

In that alternative model, the optimal industry production of each firm is

$$q^I=\frac{1-c}{4\left(\sigma^2\rho+1\right)},$$

which is lower than q^c. The corresponding firm welfare is then

$$W^I=\frac{(1-c)^2}{16}\frac{3\sigma^2\rho+2}{\left(\sigma^2\rho+1\right)^2}.$$

Collusion is interesting for individual firms if

$$W^I - W^e = \frac{(2\sigma^2\rho+1)(2-2\sigma^4\rho^2-\sigma^2\rho)}{16}\frac{(1-c)^2}{(\sigma^2\rho+1)^2(2\sigma^2\rho+3)^2} > 0,$$

or equivalently if $\sigma^2\rho(2\sigma^2\rho+1) < 2$, that is, risk aversion and/or demand variability are relatively small.

During the period where deviation from the collusive solution is not detected, the production and welfare of the deviating Firm are given respectively by

$$\begin{aligned} q^D &= \frac{(1-c)}{8}\frac{4\sigma^2\rho+3}{(\sigma^2\rho+1)^2} \\ W^D &= \frac{1}{64}(1-c)^2\frac{(4\sigma^2\rho+3)^2}{(\sigma^2\rho+1)^3}. \end{aligned}$$

Collusion can be sustained if

$$\delta\left(W^D - W^e\right) > W^D - W^I,$$

or equivalently, if

$$\delta > \left(2\sigma^2\rho+1\right)\frac{(2\sigma^2\rho+3)^2}{16\sigma^4\rho^2+34\sigma^2\rho+17},$$

which is only possible if the r.h.s. is smaller than 1, that is if $\sigma^2\rho(2\sigma^2\rho+1) < 2$.

Again, it is easy to check that the impact of demand variability on the threshold for δ is positive:

$$\begin{aligned} & \frac{d}{d(\sigma^2\rho)}\left(2\sigma^2\rho+1\right)\frac{(2\sigma^2\rho+3)^2}{16\sigma^4\rho^2+34\sigma^2\rho+17} \\ = & \ 4\left(2\sigma^2\rho+3\right)\frac{44\sigma^2\rho+44\sigma^4\rho^2+16\sigma^6\rho^3+17}{(34\sigma^2\rho+16\sigma^4\rho^2+17)^2} > 0. \end{aligned}$$

This means that in that alternative model of collusion, an increase in demand variability facilitates collusion, as long as it does not exceed a certain limit; if demand variability becomes too large, then firms are no longer interested in cooperating. A similar non-linear effect has been reported by Wong (2008) for a price setting cartel although the drivers of such shapes are different. If the quantity is the strategic variable, then the increasing part of the U shaped curve is triggered by the negative value of $W^I - W^e$,which shows that the non cooperative profit becomes higher than the collusive outcome. In the real options set up used in Wong (2008), the optimal defecting trigger, which is the size of the demand at which defecting from the cartel is execised, is subject to two effects: a positive effect whereby the increase in volatility increases the option value to wait, hence supporting collusion. The negative effect is due to the reduction of the value of the option to wait due to the discount factor. Wong (2008) shows that under certain conditions the negative effect dominates the positive effect at high volatility values thereby facilitating defection.

6. Conclusion

Demand volatility has been shown to facilitate collusion within cartels where competition is a la Bertrand. In the case of Cournot competition, where two firms are deciding about production quantities, we show that a similar result can be retrieved with a simple model of mean-variance utility. For high volatility values, collusion is not sustainable.

Appendix

In a discrete-time, infinite horizon setting, assume that a deviation by a defecting firm is detected after T periods. The total discounted welfare of a firm when there is no defection is given by $\frac{W^c}{(1-\beta)}$, where $\beta \in (0,1)$ is the one-period discount factor of the firms. On the other hand, a defecting firm enjoys the higher welfare W^d during T periods, after which the other firm will apply the Cournot quantity forever. The total discounted welfare when a firm defects is therefore equal to

$$\frac{\left(1-\beta^T\right)}{(1-\beta)} W^d + \beta^T \frac{W^e}{(1-\beta)};$$

Setting $\delta \equiv \beta^T$, defecting is not interesting and collusion can be sustained if

$$W^d\left(1-\delta\right) + \delta W^e < W^c.$$

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