Economics Bulletin

Volume 35, Issue 1

Pooling promises with moral hazard

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Abstract

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Citation: Luca Panaccione and Catarina Goulão, (2015) "Pooling promises with moral hazard", *Economics Bulletin*, Volume 35, Issue 1, pages 460-465
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Submitted: February 05, 2014. **Published:** March 11, 2015.

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1 Introduction

In this paper, we study a model in which risk-averse consumers face uncertain endowments. Consumers can influence the likelihood of the states of nature by undertaking a costly action. Since the action is unverifiable, there is moral hazard. Contrary to the traditional literature on insurance with moral hazard (see e.g. Arnott and Stiglitz 1988), we do not consider that consumers buy insurance contracts from perfectly competitive insurance companies. Instead, we assume that consumers commit to contribute a fraction of their endowments to a common pool, and, therefore, gain the right to receive a fraction of the total return of the pool, proportional to their promises. As in Dubey and Geanakoplos (2002), consumers take the return of the pool as given and they are free to choose how much to promise to the pool.

We show that, even if consumers are equal ex-ante, they may choose to promise differently, and, as a consequence, choose different actions. This illustrates how such a pool of voluntary promises can induce redistribution from those with the highest expected endowment towards those with the lowest. This is an important feature of our model in contrast with other contributions that consider mutual arrangements in which participants have to pay a uniform contribution to the pool, see e.g. Guinnane and Streb (2011).

The framework first proposed by Dubey and Geanakoplos (2002) had, as its main purpose, to overcome the problem of existence of equilibrium in the competitive model with adverse selection of Rothschild and Stiglitz (1976). Other authors have extended and applied their framework in setups with adverse selection (see, among others, Martin 2007, and Fostel and Geanakoplos 2008). To our knowledge our contribution is the first to consider a pool of promises in the presence of moral hazard.

Our paper is set out as follows: in section 2, we introduce the model; in section 3, we present our results, illustrate them through examples, and discuss their main implications.

2 The model

We consider a pure exchange economy with a single consumption good. The economy is populated by a large number of ex-ante identical consumers, and it lasts for two periods $t = 0, 1$. At $t = 0$ there is no consumption, and at $t = 1$ each consumer has verifiable endowments that depend on a state of nature. There are two possible states $s = G, B$, and we let $w = (w_G, w_B) \in \mathbb{R}^2_+$ denote the vector of endowment, with $w_G > w_B \geqslant 0$.

Consumers may influence the likelihood of states of nature by undertaking an action $a \in \mathcal{A} = \{L, H\}$, which is not verifiable. Let π_a denote the probability of the state G when action a is chosen, with $1 > \pi_H > \pi_L > 0$. The (dis)utility of the action is c_a, and we assume $c_H > c_L = 0$. Preferences are represented by an expected utility function $U(x, a)$: $\mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}$, which depends on state contingent consumption bundle $x = (x_G, x_B) \in \mathbb{R}^2_+$ and a as follows:

$$U(x, a) := \pi_a u(x_G) + (1 - \pi_a) u(x_B) - c_a \, , \tag{1}$$

with u twice differentiable, strictly increasing and strictly concave.

Inspired by Dubey and Geanakoplos (2002), we propose the following insurance mechanism: at $t = 0$, each consumer voluntarily promises to make a delivery to a common pool, proportional to his endowment at $t = 1$. In exchange, at $t = 1$, the consumer receives a share of the total resources of the pool in proportion to his promise, and not to his actual delivery. More precisely, suppose that a fraction q of consumers choose $a = H$ and promise

θ_H, while a fraction $1-q$ choose $a = L$ and promise θ_L. In this case, total deliveries to the pool equal $q\theta_H\bar{w}_H + (1-q)\theta_L\bar{w}_L$, where $\bar{w}_a = \pi_a w_G + (1-\pi_a)w_B$ is the average (aggregate) endowment when action a is undertaken.[1] Obviously, probabilities, and hence the fraction of consumers in each state, depend on the action chosen by consumers. Let κ denote the return per promise, given by:

$$\kappa = \frac{q\theta_H\bar{w}_H + (1-q)\theta_L\bar{w}_L}{q\theta_H + (1-q)\theta_L}. \tag{2}$$

Note that, since all consumers participate in the pool, the idiosyncratic uncertainty is wiped out, hence κ is not state contingent. Additionally, (2) implies that $w_B < \bar{w}_L \leqslant \kappa \leqslant \bar{w}_H < w_G$, and therefore that net deliveries to the pool $\theta(w_s - \kappa)$ are positive for consumers in the good state of nature, and negative for consumers in the bad state, irrespective of the action chosen. Indeed, state contingent consumption bundles are given by:

$$x_s = w_s - \theta(w_s - \kappa), \tag{3}$$

with $s = G, B$. Hence, consumers in state G consume less than their endowment, while those in state B consume more than their endowment. Therefore, the pool actually works as an insurance mechanism.

Consumers take the return per promise κ as given and choose their promises and actions so as to maximize expected utility. Formally, the consumers' problem can be written as:

$$\max_{\theta\in\Theta, a\in\mathcal{A}} \quad v(\theta, a) = \pi_a u(w_G - \theta(w_G - \kappa)) + (1-\pi_a)u(w_B - \theta(w_B - \kappa)) - c_a, \tag{4}$$

with $\Theta = [0, \bar{\theta}]$, and $\bar{\theta} = w_G/(w_G - \kappa)$ being the maximum value θ can take to ensure $x_G > 0$. Note that $0 \leqslant \theta$ implies $x_G \leqslant w_G$. Moreover, since $\bar{\theta} > 1$, $x_B > x_G$, is admitted. Also, $\theta \leqslant \bar{\theta}$ implies $x_G \geqslant 0$, and since $w_B < \kappa$, $\theta \geqslant 0$ also implies $x_B \geqslant 0$. In what follows, $\psi(\kappa) \subset \Theta \times \mathcal{A}$ denotes the set of solutions to problem (4). It is easy to verify that $\psi(\kappa)$ is not empty.

3 Results and discussion

We propose the following definition of equilibrium:

Definition 1. *An equilibrium with a pool of promises is a collection of promises $\tilde{\theta}$, actions $\tilde{a}$, return per promise $\tilde{\kappa}$ and distribution of consumers $(\tilde{q}, 1-\tilde{q})$ such that:*

1. *$(\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa})$,*
2. *$\tilde{\kappa}$ satisfies (2),*
3. *$\tilde{q}$ satisfies:*

(a) $\tilde{q} = 0$ *if* $\tilde{a} = L$ *for every* $(\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa})$ (Action L Equilibrium)

(b) $\tilde{q} = 1$ *if* $\tilde{a} = H$ *for every* $(\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa})$ (Action H Equilibrium)

(c) $\tilde{q} \in (0,1)$ *if* $(\tilde{\theta}_L, L), (\tilde{\theta}_H, H) \in \psi(\tilde{\kappa})$ *for some* $\tilde{\theta}_H, \tilde{\theta}_L \in \Theta$. (Mixed Action Equilibrium)

The above definition states that the equilibrium values of q must be properly related to the optimal choices of consumers. In particular, $q = 0$ ($q = 1$) can only arise in equilibrium

[1] We assume that each consumer faces uncertainty independently of other consumers. This assumption, in addition to the fact that there is a large number of consumers, rules out aggregated uncertainty.

if $a = L$ ($a = H$) is the optimal choice for every consumer. Similarly, for $q \in (0,1)$ to arise in equilibrium, both $a = H$ and $a = L$ must be optimal choices of consumers. Given indifference, consumers will split themselves between the two admissible actions in a way that is consistent with the equilibrium value of κ. Propositions 1-3 identify the possible types of equilibrium and their properties are discussed in Section 3.2.

Proposition 1. [Impossibility of a high cost action equilibrium]
There cannot be an equilibrium in which all consumers undertake the action H, i.e., if $(\tilde{\theta}, H, \tilde{q}, \tilde{\kappa})$ is an equilibrium, then $\tilde{q} \neq 1$.

Proof. Let $\phi(\kappa, a) \subset \Theta$ denote the solution set of $\max_{\theta\in\Theta} \; v(\theta, a)$, and $\chi(\kappa, \theta) \subset \mathcal{A}$ the solution set of $\max_{a\in\mathcal{A}} \; v(\theta, a)$. Both $\phi(\kappa, a)$ and $\chi(\kappa, \theta)$ are non empty and $\phi(\kappa, a)$ is a singleton, because of the strict concavity of u. Note that $(\tilde{\theta}, \tilde{a}) \in \psi(\kappa)$ implies $\tilde{\theta} = \phi(\kappa, \tilde{a})$ and $\tilde{a} \in \chi(\kappa, \tilde{\theta})$. Now, suppose, by way of obtaining a contradiction, that $\tilde{q} = 1$. Then, (2) implies $\kappa = \bar{w}_H$. If $(\tilde{\theta}, H)$ is an equilibrium choice, then $(\tilde{\theta}, H) \in \psi(\bar{w}_H)$. This implies, $H \in \chi(\bar{w}_H, \tilde{\theta})$ and, thus, $v(\tilde{\theta}, H) \geqslant v(\tilde{\theta}, L)$. Moreover, $(\tilde{\theta}, H) \in \psi(\bar{w}_H)$ also implies $\tilde{\theta} = \phi(\bar{w}_H, H)$ and, hence, $\tilde{\theta} = 1$. In this case, however, $v(\tilde{\theta}, L) > v(\tilde{\theta}, H)$, which is the desired contradiction. □

Proposition 1 states that if $q = 1$ and $a = H$, then κ does not satisfy (2). Indeed, if consumers anticipate the high return per promise $\kappa = \bar{w}_H$, which is implied by $q = 1$, their optimal choice is to overinsure themselves and to choose $a = L$. The non-existence of the high action equilibrium can be ascribed to the inability of the pool to limit promises. If, on the contrary, promises could be limited, $\bar{\theta}$ could be set sufficiently low so as to ensure that $a = H$ would be incentive compatible, given the resulting equilibrium level of κ.

Proposition 2. [Low cost action equilibrium]
Let $\hat{\theta}_H = \phi(\bar{w}_L, H)$ and $\hat{\theta}_L = \phi(\bar{w}_L, L)$ be the consumers' optimal promises when $\kappa = \bar{w}_L$ conditional on choosing, respectively, $a = H$ and $a = L$. If $v(\hat{\theta}_L, L) \geq v(\hat{\theta}_H, H)$, then a low action equilibrium exists.

Proof. $\phi(\kappa, a)$ is introduced in Proposition 1. When $q = 0$, (2) implies $\kappa = \bar{w}_L$. If $\kappa = \bar{w}_L$, then consumers' optimal promise is $\hat{\theta}_L$ when $a = L$ and $\hat{\theta}_H$ when $a = H$. If $(\hat{\theta}_L, L)$ is preferred to $(\hat{\theta}_H, H)$, then every consumer will choose $a = L$ and hence $q = 0$. □

Proposition 2 states that if all consumers choose $a = L$, then κ satisfies (2) and, thus, it identifies the conditions under which a low action equilibrium exists. When $\kappa = \bar{w}_L$, consumers prefer $a = L$ if the cost of $a = H$ is not compensated by an increase in the expected return per promise, either because c_H is relatively high or because π_H is relatively low. In such cases, a low action equilibrium exists.

We are also interested in a mixed action equilibrium. Yet, since consumers are ex-ante equal, this can only happen if they are all indifferent to undertaking action H or action L. Proposition 3 states the condition under which this happens.

Proposition 3. [Mixed action equilibrium]
If $v(\hat{\theta}_L, L) < v(\hat{\theta}_H, H)$, then a mixed action equilibrium exists.

Proof. When $v(\hat{\theta}_L, L) < v(\hat{\theta}_H, H)$, by adapting lemma 3.2 in Hellwig (1983) it is possible to show that there exist $\hat{\kappa} \in (\bar{w}_L, \bar{w}_H)$, $\theta_H < 1$ and $\theta_L > 1$ such that $\psi(\hat{k}) = \{(\theta_H, H), (\theta_L, L)\}$. By definition of equilibrium it must be that $\hat{\kappa}$ satisfies (2) and $q \in (0,1)$. From (2) we get:

$$q = \left[1 + \frac{\theta_H\left(\hat{\kappa} - \bar{w}_H\right)}{\theta_L\left(\bar{w}_L - \hat{\kappa}\right)}\right]^{-1}.$$

Since $\bar{w}_L < \hat{\kappa} < \bar{w}_H$, we immediately verify that $q \in (0,1)$. □

Proposition 3 says that there exists $\hat{\kappa}$ such that consumers are indifferent between either action H or L when choosing two different promises. In this case, they split into the two actions in the proportion $q \in (0,1)$ required to ensure that $\hat{\kappa}$ satisfies (2). Note that there is an implicit ex-ante redistribution from those undertaking a high effort action with higher expected endowment towards those undertaking a low effort action with lower expected payoff. Also note that this equilibrium is (second-best) inefficient: if, given the same return per promise, consumers choosing $a = L$ switched to $a = H$ and promised θ_H, their utility would not change and yet resources would be left to the pool.[2]

Figure 1, inspired by Dubey and Geanakoplos (2002), illustrates a mixed action equilibrium. The initial contingent endowments are $(w_G, 0)$. Indifference curves are steeper when $a = H$ than when $a = L$. Therefore, they cross below the certainty line and make a kink. Combining the state contingent consumption levels, as given by (3), with a view to eliminating θ, we set

$$x_B = \frac{(w_G - w_B)\kappa}{w_G - \kappa} - \left(\frac{\kappa - w_B}{w_G - \kappa}\right) x_G. \tag{5}$$

This equation shows that, by giving up $(w_G - \kappa)$ units of consumption in the state G, a consumer gets $(\kappa - w_B)$ units of consumption in the state B. In Figure 1, we plot three downward sloping lines corresponding to (5) when $\kappa = \bar{\kappa}, \hat{\kappa}, \underline{\kappa}$, where $\bar{\kappa} = \bar{w}_H$, $\underline{\kappa} = \bar{w}_L$, and $\hat{\kappa}$ is the value emerging in a mixed action equilibrium. Alternatively, we can relate x_B and x_G by eliminating κ:

$$x_B = x_G - (w_G - w_B)(1 - \theta). \tag{6}$$

This equation shows how much is left over for consumption in the bad state of nature for a promise θ. In particular, when $\theta = 1$, then $x_G = x_B$, and when $\theta < 1$ ($\theta > 1$), then $x_G > x_B$ ($x_G < x_B$). In Figure 1 we plot two of these curves: one associated with the action H promise; and the other associated with the action L promise. These are the upward sloping curves, respectively below and above the 45° line. The mixed action equilibrium consumption bundles are those at the intersection of the two lines identified by (5) and (6).

3.1 Examples of mixed action equilibria

We present two examples that illustrate how the mixed action equilibrium appears.

Example 1: Let $u(x) = \log(x)$, $w = (1.5, 0)$, $c_H = 0.21$, and $(\pi_H, \pi_L) = (2/3, 1/3)$. In the mixed action equilibrium, $\hat{\kappa} = 0.52$ and $\hat{q} = 0.1$. In this case, $\psi(\hat{k}) = \{(0.51, H), (1.02, L)\}$. The level of utility achieved is $v(\theta_H, H) = v(\theta_L, L) = -0.65$, where $\theta_H = 0.51$ and $\theta_L = 1.02$.

[2]We thank the referee for pointing this feature out to us.

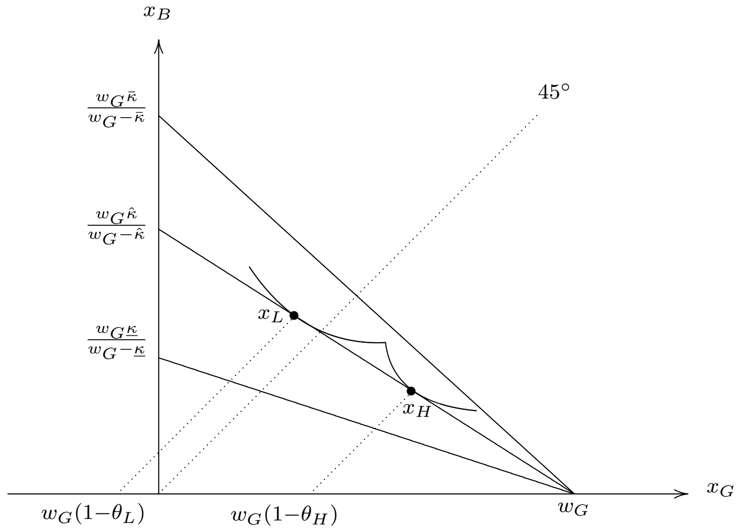

Figure 1: Mixed action equilibrium

Example 2: Let $u(x) = x^\gamma/\gamma$ with $\gamma = 0.5$, $w = (1, 0)$, $c_H = 0.163$, and $(\pi_H, \pi_L) = (2/3, 1/3)$. In the mixed action equilibrium, $\hat{\kappa} = 0.4$ and $\hat{q} = 0.56$. In this case, $\psi(\hat{k}) = \{(0.23, H), (1.21, L)\}$. The level of utility achieved is $v(\theta_H, H) = v(\theta_L, L) = 1.27$, where $\theta_H = 0.23$ and $\theta_L = 1.21$.

4 Conclusion

We analyze a model of pool of promises in a setting with moral hazard. We show that the economy may end up in a mixed action equilibrium with some consumers undertaking one type of action and some others another type.

In our view this framework is of particular interest in developing countries. As Pauly et al (2006) suggest, it seems reasonable to think of insurance cooperatives as an adequate form of insurance organization for these countries.[3] Indeed, in this type of countries tax systems are often more deficient, which compromises a compulsory public insurance scheme. On the other hand, the population of these countries is poorer and more often excluded from the market. Therefore, we argue that a voluntary mutual insurance scheme, such as the pool of promises, could be implemented at the national level.

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[3]See, for example, Cabrales et al (2003) who analyze a specific mutual fire insurance scheme used in Andorra, De Weerdt and Dercon (2006) who find evidence of risk-sharing across networks within a village of Tanzania, and Murgai et al (2002) who study water transfers along two water courses in Pakistan.

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