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A Note on Commodity Taxation and Economic Growth

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Abstract

This study reexamines the growth effect of commodity taxation in a variety-expansion model. Integrating endogenous labor supply, a linear utility function for consumption, and an additively separable utility function for consumption and leisure, we derive results that contrast with those of previous analyses in certain respects. If the elasticity of labor supply with respect to commodity taxation is sufficiently high, an increase in the commodity tax rate can decrease the short-run growth rate.

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1. Introduction

The effects of consumption taxation have been examined numerous times in the endogenous growth literature. Many studies indicate that taxes on consumption have negative or no effects on long-run growth (see King and Rebelo 1990, Rebelo 1991, Pecorino 1993, Devereux and Love 1994, Stokey and Rebelo 1995, Milesi-Ferretti and Roubini 1998, and Zeng and Zhang 2002). On the other hand, Jorgenson (1998), examining the United States, argues that implementing consumption taxes can boost the growth rate. In a notable contribution to this literature, Futagami and Doi (2004) theoretically consider the effect of specific consumption taxes (henceforth, commodity taxes) in a variety-expansion model following Grossman and Helpman (1991). They show that an increase in commodity taxes raises both short- and long-run growth rates.

The objective of the present study is to reexamine the growth effect of Futagami and Doi (2004) by considering the labor–leisure choice. Here we assume that the utility function for consumption follows a linear form, the utility function for leisure follows a constant relative risk aversion (CRRA) form, and the overall utility function is additively separable in consumption and leisure. Under these assumptions, the economy cannot converge to a balanced growth path and leisure time approaches zero as the economy grows. Although the zero leisure time result is counterfactual, we derive other relevant results. In the long run, the growth effect of commodity taxation is the same as that in Futagami and Doi (2004). However, an increase in the commodity tax rate can reduce the *short-run* growth rate if the elasticity of labor supply with respect to commodity taxation is sufficiently high.[1]

The remainder of this brief paper is organized as follows. The next section describes the model, after which we consider its equilibrium dynamics. The fourth section focuses on the effects of commodity taxation to conclude the paper.

2. Model

The factor market is perfectly competitive, while the goods market is monopolistically competitive, as explained below. Households have perfect foresight.

2.1. Households

Households maximize the following lifetime utility:

$$U_0 \equiv \int_0^\infty e^{-\rho t}\left(C_t + \frac{l_t^{1-\sigma}-1}{1-\sigma}\right)dt, \tag{1}$$

where C_t represents instantaneous utility derived from the consumption of a composite good, $\rho > 0$ is a rate of time preference, $\sigma > 0$ is the inverse intertemporal elasticity of substitution for leisure, and l_t is leisure time. C_t is given by

$$C_t = \left[\int_0^{n_t} x_t(j)^{\frac{\beta-1}{\beta}}dj\right]^{\frac{\beta}{\beta-1}}, \tag{2}$$

[1] If the utility function for consumption follows a logarithmic form instead of a linear form, Futagami and Doi's (2004) results do not change. A more detailed discussion is found in the last paragraph of Section 4.

where $x_t(j)$ denotes the consumption of good j and n_t denotes the number of available varieties of goods. $\beta > 1$ is the elasticity of substitution between any two goods. By denoting household expenditures as $E_t = \int_0^{n_t} q_t(j)x_t(j)dj$, we obtain the following demand function for good j:

$$x_t(j) = \frac{q_t(j)^{-\beta} E_t}{Q_t^{1-\beta}}, \tag{3}$$

where $q_t(j)$ is the consumer price of good j and Q_t is the price index, defined as

$$Q_t = \left(\int_0^{n_t} q_t(i)^{1-\beta} di \right)^{\frac{1}{1-\beta}}. \tag{4}$$

The maximization problem for households is as follows:

$$\begin{aligned} &max && U_0 \\ &subject\ to && \dot{A}_t = r_t A_t + w_t(L - l_t) - E_t + T_t, \end{aligned}$$

where A_t, r_t, w_t, L, and T_t represent asset holdings, the interest rate, the wage rate, the total time endowment, and lump-sum transfers from government. By substituting (3) into (2), we obtain the indirect subutility function as follows: $C_t = E_t/Q_t$. Maximization subject to the intertemporal budget constraint yields the following optimal conditions:

$$\lambda_t = \frac{1}{Q_t}, \tag{5}$$

$$l_t^{-\sigma} = \lambda_t w_t, \tag{6}$$

$$r_t - \frac{\dot{Q}_t}{Q_t} = \rho, \tag{7}$$

where λ_t stands for the costate variable attached to asset holdings.

2.2. Firms

This subsection considers producer behavior. Producers undertake two distinct activities: creating blueprints for new varieties of differentiated goods and manufacturing the differentiated goods that have been created by R&D.

We assume that one unit of labor input produces one unit of a differentiated good and each differentiated good is produced by a single firm (i.e., the good is infinitely protected by a patent). The firm manufacturing good j (firm j) maximizes its own profit, $\pi_t(j) = p_t(j)x_t(j) - w_t x_t(j)$, where $p_t(j)$ represents the producer price for firm j. Following Futagami and Doi (2004), we introduce commodity taxes into the model in the same manner. The government imposes a commodity tax on all goods at a uniform rate and redistributes the resulting tax revenue back to households as a lump-sum transfer. Assuming that the commodity tax rate is constant over time, the consumer price becomes

$$q_t(j) = p_t(j) + \tau,$$

where $\tau \geq 0$ represents the commodity tax. Firm j charges the following price: $p_t(j) = p_t = \frac{\beta w_t + \tau}{\beta - 1}$. Therefore, all goods are priced equally. This pricing rule yields brand-specific operating profits as follows:

$$\pi_t = \frac{E_t}{\beta n_t}. \tag{8}$$

We normalize the wage rate at unity, and thus, $w_t = 1$. The producer price, consumer price, and demand for goods respectively become

$$p = \frac{\beta + \tau}{\beta - 1},$$
$$q = \frac{\beta}{\beta - 1}(1 + \tau), \tag{9}$$
$$x_t = \frac{\beta - 1}{\beta}\frac{E_t}{(1 + \tau)n_t}.$$

Shareholders earn dividends π_t and capital gains or losses $\dot{v}_t$ on their stocks. The no-arbitrage condition is given by

$$r_t = \frac{\pi_t}{v_t} + \frac{\dot{v}_t}{v_t}, \tag{10}$$

where v_t denotes the value of a firm.

Next, we consider the technology involved in developing a new good.[2] The R&D firms create blueprints and expand the variety of goods available for consumption. One unit of R&D activity needs a/n_t units of labor input, where a is a parameter expressing R&D productivity. We assume that firms enter the R&D race freely. The free-entry condition is given by

$$v_t = \frac{a}{n_t} \quad if \quad \dot{n}_t > 0. \tag{11}$$

2.3. Market-clearing condition

Households supply $L - l_t$ units of labor, which is required for R&D and production. From (9), the labor market-clearing condition becomes

$$ag_t + \frac{E_t}{q} = L - l_t, \tag{12}$$

where $g_t \equiv \dot{n}_t/n_t$.

3. Equilibrium dynamics

In this section, we characterize the model's equilibrium dynamics. From (4), (5), (6), and (9), we obtain

$$l_t = Q_t^{\frac{1}{\sigma}} = q^{\frac{1}{\sigma}} n_t^{-\frac{1}{(\beta-1)\sigma}}. \tag{13}$$

Applying (7), (9), and (13) then yields

$$r_t = \rho + \frac{g_t}{1 - \beta}. \tag{14}$$

By using (8), (10), and (11), we obtain

$$r_t = \frac{E_t}{\beta a} + \frac{\dot{v}_t}{v_t}. \tag{15}$$

[2]See Grossman and Helpman (1991) for the details of the R&D process.

The free-entry condition (11) implies that $\dot{v}_t/v_t = -g_t$ holds. Thus, (14) and (15) yield

$$\frac{1}{\beta a}E_t = \rho + \frac{2-\beta}{1-\beta}g_t. \tag{16}$$

From (9), (12), (13), and (16), we obtain the following differential equation:

$$g_t = \zeta - \mu n_t^{-\frac{1}{(\beta-1)\sigma}}, \tag{17}$$

where $\zeta \equiv \frac{\frac{1+\tau}{a}L-(\beta-1)\rho}{\beta+\tau-1}$ and $\mu \equiv \frac{1}{a(\beta+\tau-1)}(1+\tau)^{\frac{\sigma+1}{\sigma}}\left(\frac{\beta}{\beta-1}\right)^{\frac{1}{\sigma}}$. Equation (17) captures the autonomous dynamic system with respect to n_t.

We illustrate the phase diagram for (17) in (n_t, g_t) space in Figure 1. $\beta > 1$ and $\tau \geq 0$ imply that $\beta+\tau-1 > 0$ and, thus, $\mu > 0$. Note that the growth rate g_t is non-negative. If $\frac{1+\tau}{a}L \leq (\beta-1)\rho$, $g_t = 0$ for all n_t.[3] To ensure the existence of positive growth, we assume $\frac{1+\tau}{a}L > (\beta-1)\rho$; that is, $\zeta > 0$.

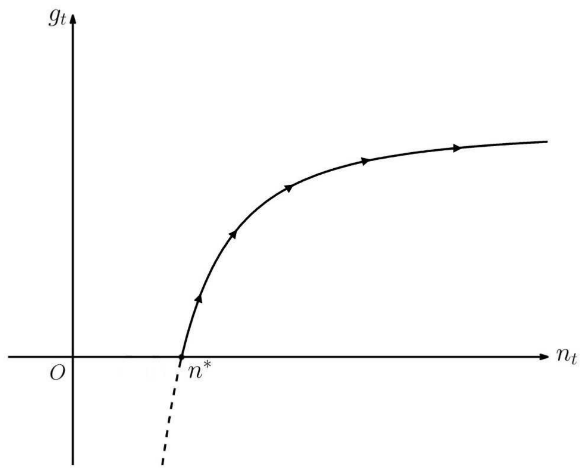

Figure 1

As shown in Appendix A, the conditions under which the growth rate, labor supply, and consumption expenditures take positive values are as follows: $\beta > 2$ and $n_0 > n^* \equiv (\mu/\zeta)^{(\beta-1)\sigma}$. Note that n^* is the number of varieties that satisfy $g_t = 0$ in (17). In addition, the condition for the discounted utility to remain finite is $\frac{1+\tau}{a}L < (\beta+\tau)(\beta-1)\rho$.[4] These results are summarized in the following proposition:

Proposition 1

When $\beta > 2$, $(\beta+\tau)(\beta-1)\rho > \frac{1+\tau}{a}L > (\beta-1)\rho$, and $n_0 > n^*$, the economy has transitional dynamics and sustained growth is possible (see Figure 1).

If the labor supply is inelastic, corresponding to the case when $\sigma \to \infty$, the economy initially jumps onto a balanced growth path. In our set-up, however, the economy has transitional dynamics because leisure depends on the variety of differentiated goods. From

[3]In this case, labor is devoted to production only. That is, the labor market-clearing condition becomes $\frac{E_t}{q} = L - l_t$.

[4]This derivation is shown in Appendix B.

(13), l_t is decreasing in n_t. The mechanism is as follows. The indirect subutility function, $C_t = E_t/Q_t$, implies that C_t is the utility of real consumption. From (13), a rise in n_t reduces the price index, Q_t, causing consumption goods to become cheaper than leisure. Households thus decrease leisure time and consume more goods. From these results, an increase in n_t reduces leisure, l_t; labor supply thus increases. In addition, an increase in labor supply allocates more labor to R&D. This increases the growth rate in the number of differentiated goods. Hence, g_t is increasing in n_t.

4. The effect of commodity taxation

In this section, we investigate the effect of commodity taxation on economic growth. Differentiating (17) with respect to τ yields

$$\frac{\partial g_t}{\partial \tau} = -\frac{\eta}{(\beta+\tau-1)^2} - \frac{\mu\Theta}{\sigma(1+\tau)(\beta+\tau-1)} n_t^{-\frac{1}{(\beta-1)\sigma}}, \tag{18}$$

where $\eta \equiv \frac{2-\beta}{a}L - (\beta-1)\rho$ and $\Theta \equiv (1+\tau) - (\sigma+1)(2-\beta)$. The assumption $\beta > 2$ implies that $\eta < 0$ and $\Theta > 0$. Thus, the sign of $\partial g_t/\partial\tau$ depends on the value of n_t. Let n' be the number of varieties at which $\partial g_t/\partial\tau = 0$. From (18), n' is defined as follows:

$$n' \equiv \left\{-\frac{(\beta+\tau-1)\mu\Theta}{\sigma(1+\tau)\eta}\right\}^{(\beta-1)\sigma}.$$

Hence, $n_t \lesseqgtr n'$ implies that $\partial g_t/\partial\tau \lesseqgtr 0$ holds. Next, we examine whether n' is larger than n^*. We calculate the following difference:

$$(n')^{\frac{1}{(\beta-1)\sigma}} - (n^*)^{\frac{1}{(\beta-1)\sigma}} = -\frac{1}{a}(1+\tau)^{\frac{1}{\sigma}}\left(\frac{\beta}{\beta-1}\right)^{\frac{1}{\sigma}}\left(\frac{\Omega}{\sigma\eta\zeta}\right),$$

where $\Omega \equiv \frac{1+\tau}{a}L - (\sigma+1)(\beta-1)\rho$. Therefore, $\Omega \gtreqless 0$ implies that $n' \gtreqless n^*$. Figure 2-1 shows how the economy responds to an increase in the commodity tax rate when $\Omega > 0$. The locus of g_t, which represents (17), shifts downward if $n^* < n_t < n'$ and shifts upward if $n_t > n'$. Figure 2-2 shows how the economy responds to an increase in the commodity tax rate when $\Omega < 0$. In this case, the locus of g_t shifts upward if $n_t \geq n^*$. We summarize these results in the following proposition:

Proposition 2

Suppose that $\beta > 2$, $(\beta+\tau)(\beta-1)\rho > \frac{1+\tau}{a}L > (\beta-1)\rho$, $n_0 > n^*$, and n_0 is sufficiently small. For an economy in which $\frac{1+\tau}{a}L > (\sigma+1)(\beta-1)\rho$, when the government increases the commodity tax rate, the short-run growth rate decreases and the long-run growth rate increases (see Figure 2-1). On the other hand, for an economy in which $\frac{1+\tau}{a}L < (\sigma+1)(\beta-1)\rho$, when the government increases the commodity tax rate, the short-and long-run growth rates both increase (see Figure 2-2).

In the model of Futagami and Doi (2004), when the government increases the commodity tax rate, households decrease consumption, leading to reduced production. Assuming constant labor supply, as the demand for production labor decreases, labor is reallocated from production to R&D; as a result, the growth rate increases. In contrast,

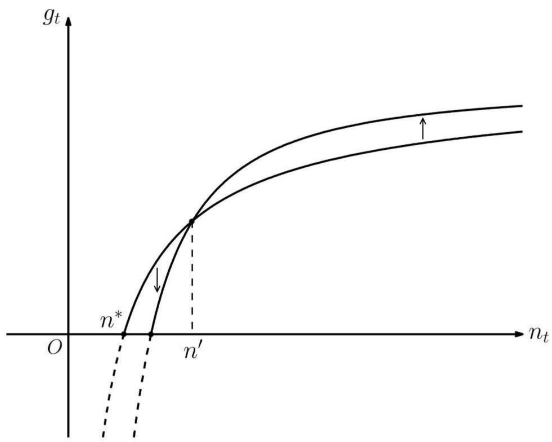

Figure 2-1

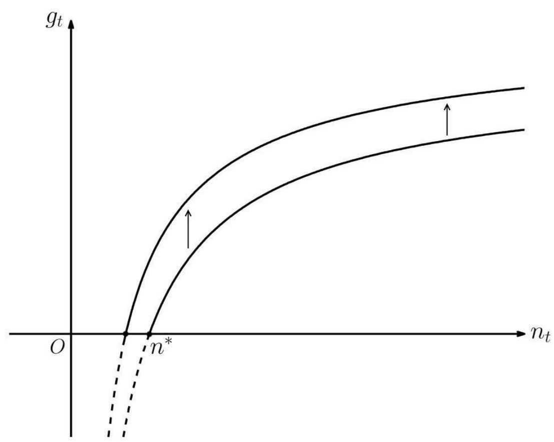

Figure 2-2

an increase in the commodity tax rate in our model can reduce the short-run growth rate. An intuitive explanation is as follows. $\frac{1+\tau}{a}L > (\beta-1)\rho$ and $\frac{1+\tau}{a}L > (\sigma+1)(\beta-1)\rho$ imply that σ is sufficiently small; that is, the elasticity of labor supply with respect to commodity taxation is sufficiently high.[5] An increase in the commodity tax rate reduces household demand for consumption goods and increases leisure time. Because the latter effect outweighs the former, the short-run growth rate declines. As discussed above, when the number of varieties increases, households decrease leisure time. Thus, the effect of labor supply becomes negligibly small. A rise in the commodity tax rate depresses demand for consumption goods, reallocating labor from production to R&D. Consequently, this leads to a higher long-run growth rate. On the other hand, $\frac{1+\tau}{a}L > (\beta-1)\rho$ and $\frac{1+\tau}{a}L < (\sigma+1)(\beta-1)\rho$ imply that σ is sufficiently large; that is, the elasticity of labor supply with respect to commodity taxation is sufficiently low. In this case, the effect of labor supply becomes negligible even in the short run. Thus, the mechanism is the same as in Futagami and Doi (2004).

In closing, we note two limitations of this study and thus directions for future work. First, this paper's main model employed a linear utility function for C_t, whereas Futagami and Doi (2004) use a logarithmic utility function with respect to C_t. Using the logarithmic function, we show that the economy initially jumps to the steady state and an increase in the commodity tax rate leads to an increase in the growth rate.[6] That is, Futagami and Doi's (2004) results do not change as a result of incorporating the labor–leisure choice: only if the utility functional form is changed and the labor–leisure choice included can the short-run result differ. As both linear and logarithmic utility functions are special functional forms, future research should examine the growth effects of commodity taxation under a CRRA utility function for C_t. Second, a clear limitation is that under this paper's approach the economy could not converge to a balanced growth path because of the additively separable utility function. To ensure the existence of a balanced growth path, we could modify the utility function as follows: $U_0 = \int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\theta}}{1-\theta}f(l_t)\right]dt$.[7] Future work could consider the potentially interesting changes that would result from such a

[5]From (9) and (13), a lower σ implies a larger effect of changes in τ on l_t. Therefore, the inverse of σ is related to the elasticity of labor supply with respect to commodity taxation.

[6]The proof of this is included in Appendix C.

[7]King et al. (1988) show that this functional form is consistent with the existence of a balanced growth path.

modification.

Appendix A. Conditions of $g_t > 0$, $L - l_t > 0$, and $E_t > 0$.

The number of varieties when $g_t = 0$ in (17) is defined as n^*. From Figure 1, the condition for positive growth is given by

$$n_0 > n^* \equiv \left(\frac{\mu}{\zeta}\right)^{(\beta-1)\sigma}.$$

The actual labor supply $L - l_t$ must also be positive. From (13) and $\dot{n}_t \geq 0$, leisure is maximized at time 0. Hence, $L - l_t > 0$ if the initial number of varieties satisfies the following condition:

$$n_0 > \hat{n} \equiv \left\{\frac{a(\beta+\tau-1)}{(1+\tau)L}\mu\right\}^{(\beta-1)\sigma}.$$

Next, we consider the condition $E_t > 0$. By using (16) and (17), we obtain

$$E_t = \frac{a\beta}{(\beta-1)}\left\{-\frac{1+\tau}{\beta+\tau-1}\eta + (2-\beta)\mu n_t^{-\frac{1}{(\beta-1)\sigma}}\right\},$$

where $\eta \equiv \frac{2-\beta}{a}L - (\beta-1)\rho$. To ensure $\eta < 0$, we assume for simplicity that $\beta > 2$.[8] As with the condition $L - l_t > 0$, the initial number of varieties satisfies the following condition so that $E_t > 0$:

$$n_0 > \tilde{n} \equiv \left\{\frac{(2-\beta)(\beta+\tau-1)\mu}{(1+\tau)\eta}\right\}^{(\beta-1)\sigma}.$$

We then consider the relationship between n^*, $\hat{n}$, and $\tilde{n}$. We calculate the following difference:

$$(n^*)^{\frac{1}{(\beta-1)\sigma}} - (\hat{n})^{\frac{1}{(\beta-1)\sigma}} = (\beta+\tau-1)\mu\left\{\frac{1}{\frac{1+\tau}{a}L - (\beta-1)\rho} - \frac{1}{\frac{1+\tau}{a}L}\right\} > 0,$$

$$(\hat{n})^{\frac{1}{(\beta-1)\sigma}} - (\tilde{n})^{\frac{1}{(\beta-1)\sigma}} = (\beta+\tau-1)\mu\left\{\frac{1}{\frac{1+\tau}{a}L} - \frac{1}{\frac{1+\tau}{a}L - \frac{(1+\tau)(\beta-1)\rho}{2-\beta}}\right\} > 0.$$

Thus, $n^* > \hat{n} > \tilde{n}$ holds. As a result, the condition that the growth rate, labor supply, and consumption expenditures all take positive values can be summarized as follows: $n_0 > n^*$.

Appendix B. Conditions under which the discounted utility remains finite.

To obtain the conditions under which the discounted utility remains finite, we first solve the differential equation (17). Let us define $z_t \equiv n_t^{\frac{1}{(\beta-1)\sigma}}$. Eq. (17) can be rewritten as follows:

$$\dot{z}_t = \frac{\zeta}{(\beta-1)\sigma}z_t - \frac{\mu}{(\beta-1)\sigma}.$$

[8] This assumption falls in the range of empirically plausible parameters. A detailed discussion is provided in Haruyama (2009).

Multiplying both sides by $e^{-\frac{\zeta}{(\beta-1)\sigma}t}$, we obtain

$$\frac{d}{dt}\left(z_t e^{-\frac{\zeta}{(\beta-1)\sigma}t}\right) = -\frac{\mu}{(\beta-1)\sigma}e^{-\frac{\zeta}{(\beta-1)\sigma}t}.$$

Integrating both sides from time 0 to time t yields

$$z_t = e^{\frac{\zeta}{(\beta-1)\sigma}t}\left(u_0 - \frac{\mu}{\zeta}\right) + \frac{\mu}{\zeta}.$$

From the definition of z_t, we obtain

$$n_t = \left\{e^{\frac{\zeta}{(\beta-1)\sigma}t}\left(n_0{}^{\frac{1}{(\beta-1)\sigma}} - \frac{\mu}{\zeta}\right) + \frac{\mu}{\zeta}\right\}^{(\beta-1)\sigma}. \tag{B.1}$$

Next, we express utility as a function of time t. Eqs. (16) and (17) yield

$$E_t = \frac{a\beta}{\beta-1}\left\{-\frac{(1+\tau)\eta}{\beta+\tau-1} + (2-\beta)\mu n_t^{-\frac{1}{(\beta-1)\sigma}}\right\}. \tag{B.2}$$

By using (4), (9), and (13), households' lifetime utility (1) is rewritten as follows:

$$U_0 = \int_0^\infty e^{-\rho t}\left(\frac{E_t}{q}n_t^{\frac{1}{\beta-1}} + \frac{q^{\frac{1-\sigma}{\sigma}}n_t^{-\frac{1-\sigma}{(\beta-1)\sigma}} - 1}{1-\sigma}\right)dt. \tag{B.3}$$

Substituting (B.1) and (B.2) into (B.3), we obtain

$$\begin{aligned}\frac{E_t}{q}n_t^{\frac{1}{\beta-1}} = \frac{a\beta}{(\beta-1)q}\Bigg[&-\frac{(1+\tau)\eta}{\beta+\tau-1}\left\{e^{\left(\frac{\zeta}{(\beta-1)\sigma}-\frac{\rho}{\sigma}\right)t}\left(n_0{}^{\frac{1}{(\beta-1)\sigma}} - \frac{\mu}{\zeta}\right) + e^{-\frac{\rho}{\sigma}t}\frac{\mu}{\zeta}\right\}^{\sigma} \\ &+ (2-\beta)\mu\left\{e^{\left(\frac{\zeta}{(\beta-1)\sigma}-\frac{\rho}{\sigma-1}\right)t}\left(n_0{}^{\frac{1}{(\beta-1)\sigma}} - \frac{\mu}{\zeta}\right) + e^{-\frac{\rho}{\sigma-1}t}\frac{\mu}{\zeta}\right\}^{\sigma-1}\Bigg],\end{aligned}$$

and

$$q^{\frac{1-\sigma}{\sigma}}n_t^{-\frac{1-\sigma}{(\beta-1)\sigma}} = q^{\frac{1-\sigma}{\sigma}}\left\{e^{\left(\frac{\zeta}{(\beta-1)\sigma}-\frac{\rho}{\sigma-1}\right)t}\left(n_0{}^{\frac{1}{(\beta-1)\sigma}} - \frac{\mu}{\zeta}\right) + e^{-\frac{\rho}{\sigma-1}t}\frac{\mu}{\zeta}\right\}^{\sigma-1}.$$

Thus, the condition under which the discounted utility remains finite is $\frac{\zeta}{(\beta-1)\sigma} - \frac{\rho}{\sigma-1} < \frac{\zeta}{(\beta-1)\sigma} - \frac{\rho}{\sigma} < 0$ when $\sigma > 1$ and $\frac{\zeta}{(\beta-1)\sigma} - \frac{\rho}{\sigma} < 0$ when $\sigma < 1$. As a result, we summarize with the following condition: $\frac{\zeta}{(\beta-1)\sigma} - \frac{\rho}{\sigma} < 0$. From the definition of ζ, this is rewritten as follows:

$$\frac{1+\tau}{a}L < (\beta+\tau)(\beta-1)\rho.$$

Appendix C. Logarithmic utility function.

We modify the utility function as follows:

$$U_0 = \int_0^\infty e^{-\rho t} \left(\log C_t + \frac{l_t^{1-\sigma} - 1}{1-\sigma} \right) dt.$$

Under this modification, the optimal conditions of (5) and (7) become

$$\lambda_t = \frac{1}{E_t}, \tag{C.1}$$

$$r_t - \frac{\dot{E}_t}{E_t} = \rho. \tag{C.2}$$

From (4), (6), (9), and (C.1), we obtain

$$l_t = E_t^{\frac{1}{\sigma}}, \tag{C.3}$$

$$Q_t = \frac{\beta}{\beta - 1}(1+\tau) n_t^{-\frac{1}{\beta-1}}. \tag{C.4}$$

Next, (11), (15), and (C.2) yield

$$\frac{\dot{E}_t}{E_t} = \frac{E_t}{\beta a} - g_t - \rho. \tag{C.5}$$

By using (12), (C.3), and (C.5), we obtain

$$\frac{\dot{E}_t}{E_t} = \frac{\beta + \tau}{\beta a(1+\tau)} E_t + \frac{1}{a} E_t^{\frac{1}{\sigma}} - \frac{L}{a} - \rho. \tag{C.6}$$

Eq. (C.6) formulates an autonomous dynamic system with respect to E_t. Next, (C.6) implies that the steady state is unstable; thus, the economy initially jumps to the steady state. The steady-state value E^s is determined by the following equation:

$$\frac{\beta + \tau}{\beta a(1+\tau)} E^s + \frac{1}{a}(E^s)^{\frac{1}{\sigma}} = \frac{L}{a} + \rho. \tag{C.7}$$

Taking the total differentials of (C.7) yields $\partial E^s / \partial \tau > 0$. We then investigate the effect of commodity taxation on economic growth. From (C.5), the steady-state growth rate, g^s, is as follows:

$$g^s = \frac{E^s}{\beta a} - \rho. \tag{C.8}$$

By using (C.8) and $\partial E^s / \partial \tau > 0$, we show that $\partial g^s / \partial \tau > 0$ holds. That is, an increase in the commodity tax rate raises the growth rate.

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