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Job competition, employability and incentives for human capital formation

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Abstract

This note describes the effects on human capital formation of rank-order tournaments offering identical prizes to a given number of the ranked contestants. This compensation scheme is thought to resemble the selection processes in different areas of the public administration, particularly in Southern European countries. In the presence of contestants with identical ability, the incentives for educational effort are highest when the variance of final returns is maximized.

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1 Introduction

The labor market consequences of human capital formation have very often raised concerns about the efficiency and equity in the design of education systems. In some European countries, the share of state employees within the potentially active population today exceeds 20% (e.g. Denmark and Sweden; see Navarro and Tur (2012)). Considering the relevance of public employment, here we will focus on the educational role of the selection processes for state employees.

We will emphasize how, in the presence of a huge number of candidates for each position, many people's cognitive faculties are likely to be underemployed due to their discouraged, merely tentative preparation for tournaments. On the other extreme, there is another risk of low educational effort when jobs are too easy to achieve. This fact implies that there is some room for public optimization in terms of the employability after job competition; which may have an impact on the efficient allocation of physical and human resources in the administration. Moreover, since the degree of employability obviously affects the final income distribution, such optimization will also have a bearing on the equity of the resulting allocations.

In this paper we will present two models to illustrate such possibilities of optimization in terms of employability. The first one is inspired by Lazear and Rosen (1981)'s work on rank-order tournaments for homogeneous contenders in terms of ability. It turns out that our setting offers a neat employability criterion in terms of the variance of returns after the tournament. Finally, we extend the analysis to a model with heterogeneous participants, which offers a more precise intuition for the existing trade-offs.

2 Theory

2.1 A rank-order tournament with homogeneous contestants

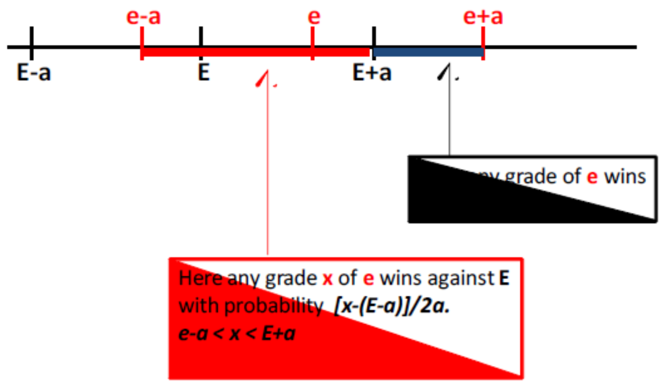
In this setting the payment differences depend on rank orders instead of performance distances. The prizes are set in advance and are independent of the output (grade) generated ex-post. More specifically, such prizes are identical and offered to a given number of the ranked contestants. The model consists of a single period, and therefore the prizes should be understood in terms of the present discounted value of future wage income that workers receive. The student-workers participating in the tournament are risk neutral, and their grade depends on their educational effort (e) and an idiosyncratic shock realization. Such randomness enters in an additive form, and the grade of any contestant $j = 1, ..., n$ (g_j) is equal to

$$g_j = e_j + \varepsilon_j$$

where ε_j stands for the idiosyncratic shock realization, uniformly distributed in $[-a, a]$, $a > 0$. As a result, the grade observed by the employer is uniformly distributed in $[e_j - a, e_j + a]$. Since the participants are homogeneous, here all choose the same education level E. Let an individual choose level e, while all others choose E.

This individual wins against a single other individual with a certain probability $p(e, E)$ (see diagram below).

How can we obtain the exact value of $p(e, E)$?. First of all, if $e \geqslant E$, for grades within the interval $[E + a, e + a]$ any score of our individual will beat his rival for sure. On the other hand, if $e < E$, for grades within $[e + a, E + a]$ it will be our player who will lose for sure. In any of the two previous cases, for grades between $e - a$ and $E + a$, any grade x of our individual wins against the rival with probability



$\frac{x-(E-a)}{2a}$. Therefore,

$$p \equiv p(e,E) = \frac{e-E}{2a} + \frac{1}{2a}\int_{e-a}^{E+a} \frac{x-(E-a)}{2a}dx = \frac{e-E}{2a} + \frac{1}{8a^2}\left[(x-(E-a))^2\right]_{e-a}^{E+a} =$$
$$= \frac{1}{2} + \frac{e-E}{2a} - \frac{(e-E)^2}{8a^2} \tag{1}$$

Then, the derivative of $p(e,E)$ with respect to e is

$$p' \equiv p'_e(e,E) = \frac{1}{2a} - \frac{(e-E)}{4a^2} \tag{2}$$

and if we consider a situation of symmetry where $e = E$, from (1) and (2) we can observe that

$$p(E,E) = \frac{1}{2};\ p'_e(E,E) = \frac{1}{2a} \tag{3}$$

Now, as we anticipated above, we assume a population of contenders of size n. Let us also assume that there are h jobs. The purpose of the public employer is thus finding the right value of h (h^*) such that the aggregate human capital of the population is maximized. To be selected, every competitor has to be in the top h, i.e. either he defeated *all* $(n-1)$; or he defeated exactly $(n-2)$ and was defeated by one; or he defeated exactly $(n-3)$ and two defeated him;...or he defeated exactly $(n-h)$ and $(h-1)$ defeated him. The probability of being in the top h is therefore:

$$\sum_{s=0}^{h-1} \binom{n-1}{s} p^{n-1-s}(1-p)^s \tag{4}$$

Using equation (4), the expected utility of our participant with education e when all others choose E is

$$w\left[\sum_{s=0}^{h-1}\binom{n-1}{s}p^{n-1-s}\left(1-p\right)^{s}\right]-c\left(e\right) \tag{5}$$

The derivative of (5) with respect to e taken at the point $e=E$ should be zero in a symmetric Nash equilibrium. Otherwise, when all choose E a single individual would choose a different education level.

$$wp'\sum_{s=0}^{h-1}\left(n-1-s\right)\binom{n-1}{s}p^{n-2-s}\left(1-p\right)^{s}-wp'\sum_{s=0}^{h-1}s\binom{n-1}{s}p^{n-1-s}\left(1-p\right)^{s-1}=c_{e}'\left(e\right) \tag{6}$$

Substituting in (6) the values of p and p'at $e=E$ according to (3),

$$\sum_{s=0}^{h-1}\left(n-1-2s\right)\binom{n-1}{s}=\frac{2^{n-1}a}{w}c_{e}'(E) \tag{7}$$

The left hand side first increases with h, but as $(h-1)$ becomes larger than $\frac{n-1}{2}$ it ends up decreasing with h, as more negative terms are added. This means that E first increases with the number of jobs and then starts falling, but that happens when the number of jobs is *roughly one half of the population*. It is straightforward to check that a compensation scheme such that $h=h^{*}\simeq\frac{n}{2}$ also maximizes the variance of the participants' returns ex-post. Furthermore, the resulting optimal Gini coefficient is exactly equal to 50.

Mejia and Saint Pierre (2009)'s general equilibrium model yields a similar result for a decentralized market allocation of workers to jobs: an intermediate value of the prize inequality induces the highest accumulation of human capital. However, such result is not analytical: it comes from a numerical calibration and simulation of their model.

2.2 Benchmarking with heterogeneous participants

Another usual practice in the selection of personnel is setting a benchmark (b) and awarding a job to all candidates with a grade above. In this case there is no strategic behavior, since each candidate's chances to pass do not depend on the performance of others. We include this alternative setting because it will be useful to understand the intuitive reasons why the profile of aggregate education could slope downwards (for very low values of b). The basic intuition present in the following subsections was already advanced by Asali et al. (2014).

2.2.1 Overview

Our candidates for the positions learn for their entry examinations. When they get a job their remuneration is w. Their choice of education level e will provide them, when tested, with a random grade uniformly distributed in $[e-1, e+1]$. There are various types of candidates, characterized by their costs of learning each education level. In particular, for type η it costs $c(e, \eta)$ to learn the level e. We assume that $c(e, \eta)$ increases with e but decreases with η: for the highest types it costs less to learn.

Now the employer sets a benchmark b, such that all those who achieve a grade higher than b will get a job. Finally, there will be a precise correspondence between the benchmark and the number of positions available (the former will determine the latter).

According to their chances to succeed, the candidates are divided into 3 groups:

- Those whose preferred level of education e is high: $e > b+1$, so that they will obtain a job with probability 1. Since lower education levels cost less, they will reduce their education level to the lowest for which they will be accepted for sure, i.e.

$e = b + 1$.

- Those whose preferred level of education provides them with a position with a positive probability less than 1. For these individuals $b - 1 < e < b + 1$. When studying level e, their probability for being selected is $\frac{e+1-b}{2}$.

- Those who will not study for the examination and will never get a job, since for these individuals $e < b - 1$.

How do the candidates choose their level of education? If type η chooses a level of education e such that $b - 1 \leq e \leq b + 1$, his probability of getting a job is $\frac{e+1-b}{2}$ and his expected payoff:

$$w\frac{e+1-b}{2} - c(e, \eta) \tag{8}$$

This is maximized when

$$\frac{w}{2} = c'_e(e, \eta) \tag{9}$$

In order to have a maximum we assume convexity of $c(.,.)$. Furthermore, given a level of education e we can find the type $\eta = \eta(e)$ that solves the above equation. Our assumptions imply that the type $\eta(e)$ increases with e.

When the employer lowers the benchmark b, there are 4 basic effects: those aspirants with a preferred effort above the old $(b + 1)$ will lower their education level. Others used to work with an effort given by the condition (9), but now will lower their education level to the new $(b + 1)$. Some of those below the old $(b + 1)$ - who were previously accepted with a certain probability - will keep their effort level but improve their chances to get a job. Additionally, some new candidates, who so far did not study, will enter the competition by acquiring some education.

Whether the overall level of education has increased or decreased depends on the changes in the education levels and the number of competitors changing their levels. For instance, it is clear that when the benchmark is very low, the number of competitors with preferred education level above b is high, and they will all lower their effort when b falls. This will probably cause a net negative effect on the aggregate education level, which is likely to happen when the initial benchmark is low, when the number of jobs is large. On the other hand, when the benchmark is high, there are few individuals above the benchmark. They will lower their education when the benchmark falls, but their effect will be small (due to their small numbers) compared to those who newly joined the competition, hence the total effect is likely to be an increase in education.

We propose two measures of the aggregate education level of the population. The first measure is the total education induced by the competition (henceforth TE). This considers the education of *all* those who studied, including those who did not get a position because their grade was low. To simplify, we take the level they studied to, not their grade. The second measure is the education level of the employed (henceforth TEW). This includes only those whose grade was above the benchmark. We take the average of the population assuming that the unemployed have level zero.

Now we will present a discrete, illustrative example. Let us note that we could also present a general example with a continuum of types, which would convey the same basic intuition.

2.2.2 A discrete example with 2 types

Particular setting Let there be two types in the existing population of aspirants: $\eta_1 > \eta_2$. Let us also denote by $e_i^* \equiv (c_e')^{-1}\left(\frac{w}{2};\eta_i\right)$ the optimal level of education for type η_i resulting from the first order condition (9). There is a measure f_i of type η_i, with $f_1 + f_2 = 1$.

First of all, let us define $\delta \equiv e_1^* - e_2^* > 0$. A higher value of δ, the distance between both preferred efforts, means that either the heterogeneity is very strong and/or the skill premium is very high. In that case, the optimal effort by the best type will be much higher than the effort level of the worst type. The employer will hire a larger staff the lower is the benchmark b in place. We know that, for the benchmark to be really selective, necessarily $b > e_2^* - 1$; and for the benchmark to be accessible to some candidates, necessarily $b < e_1^* + 1$. We will divide now the range of possible values of b into 3 intervals/regions, starting from the highest values of b and gradually lowering the benchmark:

- **Region a):**

$$e_1^* + 1 > b \geq e_2^* + 1 \tag{10}$$

This is the interval with highest possible values of the benchmark and lowest admission chances for the aspirants. Since only the top-type individuals will have some chances to be hired, though only a share of them will succeed, we can conclude that here

$$TE(b) = e_1^* f_1 \text{ and } TEW(b) = e_1^* f_1 \frac{e_1^* + 1 - b}{2}$$

Given the benchmark b, if we denote by N the total number of workers finally hired, it is straightforward to see that $N = f_1 \frac{e_1^* + 1 - b}{2}$. Finally, if we apply the inequality restriction (10) to the previous expression, we can infer that, within region a),

$$0 \leq N \leq \frac{f_1 \delta}{2}$$

-Region b):

$$e_1^* - 1 \leq b \leq e_2^* + 1 \tag{11}$$

This region will exist only if $\delta < 2$. Otherwise we should only consider the regions a) and c).

Within this interval, aspirants of both types will be able to succeed or fail. This means that the profile of $TE(b)$ will be flat and will show the highest possible value for the aggregate educational effort. That is,

$$TE(b) = e_1^* f_1 + e_2^* (1 - f_1) \text{ and } TEW(b) = e_1^* f_1 \frac{e_1^* + 1 - b}{2} + e_2^* (1 - f_1) \frac{e_2^* + 1 - b}{2}$$

Moreover, it is clear that now $N = f_1 \frac{e_1^* + 1 - b}{2} + (1 - f_1) \frac{e_2^* + 1 - b}{2}$. Finally, by applying both restrictions in (11) to the last expression, we obtain that the hiring limits within region b) are

$$\frac{f_1 \delta}{2} \leq N \leq 1 - \frac{(1 - f_1)\,\delta}{2}$$

- **Region c):**

$$e_1^* - 1 \geq b \geq e_2^* - 1 \tag{12}$$

Along this interval, the top-type individuals will start to relax once their success is guaranteed. Therefore, here

$$TE(b) = (b+1)f_1 + (1-f_1)e_2^* \text{ and } TEW(b) = (b+1)f_1 + (1-f_1)e_2^* \frac{1 + e_2^* - b}{2}$$

Furthermore, $N = f_1 + (1 - f_1)\frac{1+e_2^*-b}{2}$. It is clear that $TE(b)$ in this interval exhibits a decreasing and always lower level of aggregate education than it shows in Region b), since the top-types are studying just for the passing grade. By differentiating $TEW(b)$ with respect to b within this region, we can derive that

$$\frac{dTEW(b)}{db} = f_1 - \frac{(1-f_1)e_2^*}{2} > (<)\ 0 \text{ iff } f_1 > (<)\ \frac{e_2^*}{2 + e_2^*}$$

That is, as we continue lowering b and increasing the admission probabilities, the aggregate stock of employed human capital may decrease if there are many top-type individuals who reduce their effort and the newcomers are sufficiently "awkward". This clarifies that considerably high job opportunities result in a lower aggregate schooling; and sometimes (if the most talented are many, and sufficiently better than the least talented) the aggregate schooling *of the labor force* could even fall. That is, lots of good jobs reduce the aggregate stock of human capital employed by the economy, since the employability of the worst types often involves some relaxation on the part of the best types.

Summary of all the previous information The aggregate education of the labor force ($TE(N)$) has in all cases one or more interior, absolute maxima. These maxima are located at $N = f_1$ (when $\delta > 2$) and at all points within the interval $f_1 \frac{\delta}{2} \leq N \leq 1 - (1 - f_1)\frac{\delta}{2}$ (when $\delta < 2$).

The aggregate education of the hired workers ($TEW(N)$) has an interior, absolute maximum at $N = 1 - (1 - f_1)\frac{\delta}{2}$ (when $\delta < 2$) and at $N = f_1$ (when $\delta > 2$), provided in both cases that $f_1 > \frac{e_2^*}{2+e_2^*}$.

3 Conclusions

This paper presents a framework for the evaluation of different levels of employability in the public administration. The criterion is their contribution to the formation of human capital: a crucial variable for the long run prospects of accumulation and growth. The suitability of this objective has been defended by authors like Docquier and Rapoport (2012).

We hope this might be useful to build different quantitative applications in the future, aiming to an assessment of administrative reforms in any particular economy. Another interesting extension may be studying the implications of the business cycle volatility on employability and, subsequently, on the educational effort of Ph.D. students, professors, research fellows, etc.

4 References

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