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Learning-by-doing and the optimal Ramsey interest rate

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Abstract

We add learning-by-doing to firms technology in an imperfectly competitive Ramsey model and study optimal interest rate. Our main result is that the Ramsey allocation features an inverse relationship between optimal nominal interest rate and the degree of learning rate. We show that a sufficiently high degree of learning rate can reinstate the optimality of the Friedman rule.

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1. Introduction

This paper studies the role of learning-by-doing (LBD) in determining the optimal nominal interest rate. To this end we supplement a canonical imperfectly competitive Ramsey model by embedding LBD mechanism to firms technology. As in Cooper and Johri (2002), learning occurs at firms which accumulate a knowledge based intangible capital, commonly known as organizational capital (OC), as a by-product of production. These firms operate in a market with monopolistic competition which gives them the power to choose prices. In this setting, the firms endogenously control the amount of learning (which in turn influences future productivity) by varying the markup of prices over marginal costs in order to maximize the present value of lifetime profits. Our main finding is that the optimal nominal interest rate falls with the degree of learning rate - the higher the rate of learning the lower the optimal interest rate. In the limit, a sufficient degree of learning rate can reinstate the optimality of the Friedman rule of zero nominal interest rate.

Our finding is consistent with the results in Schmitt-Grohé and Uribe (2004); Chugh (2006) and other imperfectly competitive Ramsey models. As first explained by Schmitt-Grohé and Uribe (2004), in imperfectly competitive Ramsey economies the interest rate indirectly taxes profit income and as a result the Friedman rule ceases to be optimal. Profit income represents payments to a fixed factor, producers' monopoly power, which the Ramsey planner would like to tax as heavily as possible because it would be non-distortionary. With confiscation of profits ruled out, the nominal interest rate acquires the auxiliary role of indirectly taxing profits. They also show that the optimal nominal interest rate is increasing in the degree of market power. All these standard imperfectly competitive Ramsey models have one feature in common - firms face a static profit maximization problem the solution of which gives rise to a constant markup of prices over costs. The firms profit maximization implies that the markup (market power) is fixed across time and state and is governed by a single parameter, namely the price elasticity of demand. However, monopolistic firms in our model face a dynamic profit maximization problem as a current price (markup) change by them not only affects their current revenue and profits, it also affects their stock of organizational capital, productivity, costs, and hence profits in all future periods. Therefore, firms choose their prices and output such that current marginal cost equals to the current marginal revenue *plus* the present discounted value of all future benefits, in terms of profit, caused by a marginal change in output. Essentially, the presence of learning-by-doing gives rise to a theory of endogenous markup determination at the firm level. The higher the learning rate, the higher the marginal future benefit caused by a marginal reduction of the current price, and hence the lower the optimal markup set by the firms. And in line with Schmitt-Grohé and Uribe (2004), the lower the markup the lower the optimal nominal interest rate.

The remainder of the paper is organized as follows. The next section presents and describes the model while section 3 discusses about parameterizations and functional forms. Section 4 analyzes the results and section 5 concludes.

2. The model

2.1. Production: Final Goods and Intermediate Goods

Final goods producers are perfectly-competitive convert a continuum of differentiated intermediate goods into final goods using the following CES technology $y_t = [\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di]^{\frac{\eta}{\eta-1}}$. The typical final good producer assembles intermediate good quantities y_{it} to maximize profits, resulting in the usual downward-sloping demand schedule:

$$y_{it} = (p_{it}/p_t)^{-\eta} y_t, \tag{1}$$

where p_t denotes the nominal price of the final good and p_{it} denotes the nominal price of the intermediate good i.

There is a continuum of intermediate goods producers, indexed by the letter i, operate in a monopolistically competitive economy. Each such producer uses organizational capital (OC), h_{it}, and labor services, n_{it} to produce a differentiated intermediate good y_{it} using the following production technology:

$$y_{it} = z_t F[h_{it}, n_{it}] = z_t n_{it}^{\alpha} h_{it}^{\theta} \tag{2}$$

The technology differs from a standard neo-classical production function because the firm carries a stock of organizational capital which is an input in the production technology. Organizational capital refers to the information accumulated by the firm, through the process of past production, regarding how best to organize its production activities and deploy its inputs. As a result, the higher the level of organizational capital, the more productive the firm. Following Cooper and Johri (2002), who provide evidence on this specification, we assume that organizational capital is accumulated according to

$$h_{i,t+1} = h_{it}^{\gamma} y_{it}^{\varepsilon}, \tag{3}$$

An important implication of the presence of LBD is that the pricing problem at the firm level becomes dynamic. The decision problem of the representative firm i is to choose the plans for n_{it}, h_{it+1}, and p_{it} so as to maximize the present discounted value of life-time profits:

$$\sum_{t=0}^{\infty} Q_t p_t \{(p_{it}/p_t) y_{it} - w_t n_{it}\}$$

subject to the law of motion for the stock of organizational capital and the demand function for good i, given in Eqs. (3) and (1), respectively, and taking as given the aggregate demand y_t, the aggregate price level p_t, and the initial stock of OC, h_{t-1}. Here, Q_t denotes the consumer's stochastic discount factor, derived below, for risk-free assets.

Let $p_t\, mc_{it}$ and $p_t\, \psi_{it}$ be the lagrange multipliers associated with the constraints (1) and (3) respectively. Since all intermediate firms face the same wage rate, face the same downward sloping demand curves, and have access to the same production technology, we restrict our attention to a symmetric equilibrium and drop all the subscripts i. Then the first-order conditions of the firm's maximization problem with respect to labor, organizational capital, and price are, respectively,

$$w_t = mc_t \alpha \frac{y_t}{n_t} \tag{4}$$

$$\psi_t = E_t Q_{t+1} \pi_{t+1} \left\{ mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \psi_{t+1} \gamma h_{t+1}^{\gamma-1} y_{t+1}^{\varepsilon} \right\} \tag{5}$$

$$\begin{aligned} mc_t &= \left(1 - \frac{1}{\eta}\right) + \psi_t \varepsilon h_t^{\gamma} y_t^{\varepsilon-1} \\ &= \left(1 - \frac{1}{\eta}\right) + E_t Q_{t+1} \pi_{t+1} \left\{ mc_{t+1} \theta \frac{y_{t+1}}{h_{t+1}} + \psi_{t+1} \gamma h_{t+1}^{\gamma-1} y_{t+1}^{\varepsilon} \right\} \varepsilon h_t^{\gamma} y_t^{\varepsilon-1}, \end{aligned} \tag{6}$$

where, $\pi_t = \frac{p_t}{p_{t-1}}$. Equation (4) is standard. When $mc_t < 1$, labor price w_t is less than the corresponding social marginal product $\alpha \frac{y_t}{n_t}$. Note that the lagrange multiplier mc_t has the interpretation of marginal costs which can be seen more clearly if we rearrange (4) as, $mc_t = \frac{w_t}{z_t F_n(h_t, n_t)}$. Given all else the same, a larger stock of organizational capital, h_t, implies a lower marginal cost, mc_t.

Equation (5) determines the optimal use of organizational capital by the firm. One additional unit of organizational capital has a (marginal) value, in terms of profits, of ψ_t to the producer in the current period. The right hand side of (5) measures the value of having available an additional unit of organizational capital for use by the firm in the following period.

Finally, the optimal price setting condition (6) captures the nature of the dynamic trade-off that arises when intermediate goods producers face a downward sloping demand curve. The first term on the left side is the classical expression for marginal revenue in the static monopoly problem. In the absence of LBD (i,e., when $\theta = 0$), this standard measure of marginal revenue is equated to the marginal cost, mc_t, appearing on the left side. However, in the presence of LBD, this practice is not optimal. Pricing decision in the current period has consequences for future profits which is captured by the second term on the right side. This term can be interpreted as the expected present value of future marginal revenues stemming from a marginal sale today. The expression $\varepsilon h_t^{\gamma} y_t^{\varepsilon-1}$ ($= \frac{\partial h_{t+1}}{\partial y_t} \frac{\partial y_t}{\partial P_t}$) represents the marginal change in organizational capital in period $t+1$ due to a change in price in period t. The expression $Q_{t+1}[..]$ represents the present value, in terms of profits, of this period $t+1$ additional unit of organizational capital.

Also, Eq. (6) clearly shows that the markup of prices over marginal cost, which we denote by $\Omega_t = \frac{p_t}{mc_t} = \frac{1}{mc_t}$ is endogenous and time varying. In absence of any learning-by-doing effect, the markup, $\eta/(\eta - 1)$, is constant over time and state and completely governed by the parameter price elasticity of demand, η. However, in presence of LBD, firms essentially contro how much they wish to learn by varying the markup of prices over marginal costs in order to maximize their present discounted value of lifetime profits. The stronger the LBD effects (i.e. the stronger the dynamic link between current level of production and future level of OC stock) the larger the the second term in the right hand side of Eq. (6), and as a result the lower the markup.

2.2. Households

There is a measure-one continuum of identical, infinitely-lived households who maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t), \tag{7}$$

where, c_{1t} denotes consumption of cash goods, c_{2t} denotes consumption of credit goods, n_t denotes fraction of household's unit time endowment devoted to labor, and $\beta \in (0, 1)$ denotes the subjective discount factor. The household faces two sequences of constraints. The constraint on them in asset market (budget constraint) in period t is given by

$$\frac{M_t}{p_{t-1}} + \frac{B_t}{p_{t-1}} = (1 - \tau^n_{t-1})w_{t-1}n_{t-1} + R_{t-1}\frac{B_{t-1}}{p_{t-1}} + \frac{M_{t-1}}{p_{t-1}} - c_{1t-1} - c_{2t-1} + pr_{t-1}, \tag{8}$$

where M_t is the nominal money held at the end of securities-market trading in period t, B_t is the nominal, risk-free one-period bond held at the end of securities-market trading in period t, R_t is the gross nominal interest rate on these bonds, and p_t is the nominal price. w_t is the real wage rate and subject to a proportional tax rate τ^n_t. As the owner of the firms the household receives profit, pr_t, on a lump-sum basis with a one-period lag. We follow the same timing convention used in standard cash-credit goods environments, e.g. see Chugh (2007). The household also faces a second constraint as purchases of the cash good are subject to a cash-in-advance constraint

$$c_{1t} \leq \frac{M_t}{p_t}. \tag{9}$$

Let λ_t and ϕ_t denote the lagrange multipliers on the flow budget constraint and the cash-in-advance constraint respectively. Then the first-order conditions of the household's maximization problem are (8)-(9) holding with equality and

$$c_{1t}: \quad u_{1t} - \phi_t - \beta E_t\lambda_{t+1} = 0, \tag{10}$$

$$c_{2t}: \quad u_{2t} - \beta E_t\lambda_{t+1} = 0, \tag{11}$$

$$n_t: \quad -u_{3t} + \beta E_t\left[\lambda_{t+1}(1 - \tau^n_t)w_t\right] = 0, \tag{12}$$

$$M_t: \quad -\frac{\lambda_t}{p_{t-1}} + \frac{\phi_t}{p_t} + \beta E_t\frac{\lambda_{t+1}}{p_t} = 0, \tag{13}$$

$$B_t: \quad -\frac{\lambda_t}{p_{t-1}} + \beta E_t\frac{R_t\lambda_{t+1}}{p_t} = 0, \tag{14}$$

where u_{1t} denotes the value of marginal utility of cash good in period t (similarly for u_{2t}), and u_{3t} denotes the value of marginal utility of labor in period t. All these first order conditions have standard interpretations. Equation (14) gives rise to a standard Fisher equation which, after some manipulation, can be expressed in terms of marginal utilities as

$$1 = R_tE_t\left[\frac{\beta u_{1t+1}}{u_{1t}}\frac{1}{\pi_{t+1}}\right]. \tag{15}$$

This gives us the pricing formula for a one-period risk-free nominal bond as $Q_{t+1} = \left(\frac{\beta u_{1t+1}}{u_{1t}}\frac{1}{\pi_{t+1}}\right)$.

2.3. The Government

The government faces an exogenous, stochastic and unproductive stream of real expenditures denoted by g_t. These expenditures are financed through labor income taxation, money creation, and issuance of one-period, risk-free, nominal debt. That is the government faces the following budget constraint:

$$M_t + B_t + p_{t-1}\tau^n_{t-1}w_{t-1}n_{t-1} = M_{t-1} + R_{t-1}B_{t-1} + p_{t-1}g_{t-1}. \tag{16}$$

As discussed in Chugh (2007), aggregating the household and the government budget constraints yield the economy-wide resource frontier as

$$c_{1t-1} + c_{2t-1} + g_{t-1} = y_{t-1}. \tag{17}$$

2.4. Equilibrium

In our model a competitive monetary equilibrium is a set of endogenous plans $\{c_{1t}, c_{2t}, n_t, w_t, h_{t+1}, M_t, B_t, mc_t, \Psi_t, \pi_t\}$, such that the household maximizes utility taking as given prices and policies; the firms maximizes profit taking as given the wage rate, and the demand function; the labor market clears, the bond market clears, the money-market clears, the government budget constraint and the aggregate resource constraint are satisfied.

The Ramsey equilibrium is the unique competitive equilibrium that maximizes the household's expected lifetime utility.

Formally, we can define the Ramsey Equilibrium as a set of stationary processes $\{c_{1t}, c_{2t}, n_t, h_{t+1}, M_t, B_t, mc_t, \Psi_t, \pi_t, \tau_t^n, R_t\}$ that maximize: $E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, n_t)$, subject to the competitive equilibrium conditions (1)-(6), (15), (16)-(17) given exogenous process g_t, and z_t, values of all the variables dated $t < 0$, the values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$.

3. Parameterization and Functional Forms

The time unit in our model is one quarter. We follow Chugh (2007) in choosing the utility function and assume that the period utility function takes the specification $\ln c_t - \frac{\zeta}{1+\mu} n_t^{1+\mu}$, where $c_t = [(1-\sigma)c_{1t}^{\upsilon} + \sigma c_{2t}^{\upsilon}]^{\frac{1}{\upsilon}}$. As in Chugh (2007), we set $\beta = .9902$, $\mu = 1.7$, $\sigma = 0.62$ and $\upsilon = 0.79$. The preference parameter ζ was calibrated so that in the steady-state of the model without learning-by-doing the consumer spends about one-third of his time working. We hold the corresponding value of ζ (9.73) constant in all the environments considered in the paper. In our baseline case, we choose $\theta = 0.14$, $\gamma = 0.5$, and $\varepsilon = (1-\gamma) = 0.5$ in line with Cooper and Johri (2002). The exogenous processes for government spending, g_t, and productivity, z_t, are assumed to follow independent AR(1) in their logarithms, $\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g$ and $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z$ respectively, with $\epsilon_t^z \sim iidN(0, \sigma_z^2)$ and $\epsilon_t^g \sim iidN(0, \sigma_g^2)$. $\bar{g}$ is the steady-state level of government spending and we calibrate this value so that government spending constitutes 17 percent of steady-state output. We choose the first-order autocorrelation parameters $\rho_z = 0.95$ and $\rho_g = 0.97$, the standard deviation parameters $\sigma_z = 0.007$ and $\sigma_g = 0.02$ in line with Chugh (2007) and the RBC literature. We set the price elasticity of demand parameter $\eta = 5$, and the initial liabilities to government B_1/P_0 so that in the nonstochastic steady-state the government debt-to-GDP ratio is 44 percent per year.

4. Results

We characterize and solve the Ramsey equilibrium numerically using the methodology outlined in Schmitt-Grohé and Uribe (2012). Assuming that the initial state of the economy is the asymptotic Ramsey steady-state, we conduct 100 simulations of 500 periods each. For each simulation, we then compute a number of key moments and report the averages of these moments over the 100 simulations in Table 3. Each panel of the table corresponds to a different degree of learning rate[1], determined by the parameter θ.

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[1] As in Cooper and Johri (2002), learning rate is calculated as $2^{\theta} - 1$. Note that doubling the experience, h_t, increases output by a factor of 2^{θ}, where θ is the value of the elasticity of labor input with respect to experience in (2).

The top panel presents results for the model with no LBD ($\theta = 0$). The results resem-

Table 1: Dynamic properties of Ramsey allocation

Variable	Mean	Std. Dev.	Auto. corr.	Corr(x,y)
		No Learning ($\theta = 0$)		
τ^n	0.2718	0.0264	0.9565	0.2401
$\pi - 1$	-0.50316	7.8263	0.09592	-0.0759
$R - 1$	3.4767	0.0842	0.9097	-0.0811
y	0.2745	0.0036	0.7895	1.0000
n	0.3239	0.0012	0.8890	0.0439
c	0.2195	0.0042	0.8296	0.5930
		5% Learning ($\theta = 0.07$)		
τ^n	0.2532	0.0275	0.9479	0.1210
$\pi - 1$	-1.6204	7.5974	0.5964	-0.0525
$R - 1$	2.3148	0.0814	0.9140	-0.0567
y	0.2903	0.0035	0.8059	1.0000
n	0.3341	0.0012	0.8935	0.0423
c	0.2321	0.0041	0.8363	0.6186
		10% Learning ($\theta = 0.14$)		
τ^n	0.2345	0.0292	0.9294	0.0020
$\pi - 1$	-2.7644	7.3274	0.6682	-0.0275
$R - 1$	1.1250	0.0782	0.9178	-0.0313
y	0.3076	0.0035	0.8224	1.0000
n	0.3452	0.0012	0.8979	0.0401
c	0.2459	0.0040	0.8439	0.6449
		15% Learning ($\theta = 0.20$)		
τ^n	0.2185	0.0311	0.9123	-0.0970
$\pi - 1$	-3.7782	7.0493	0.7014	-0.0043
$R - 1$	0.0707	0.0750	0.9208	-0.0082
y	0.3238	0.0035	0.8364	1.0000
n	0.3555	0.0012	0.9016	0.0378
c	0.2589	0.0039	0.8510	0.6681

Note: The net inflation rate, $\pi - 1$, and the net nominal interest rate, $R - 1$, are expressed in percent per year.

ble the prescriptions of earlier flexible-price Ramsey models with imperfectly competitive product markets. Two especially notable findings are - a) the Friedman rule of a zero net nominal interest rate is not optimal, and b) inflation is very volatile over time. The reason for high inflation volatility is that inflation is used by the Ramsey government to make riskless nominal debt state-contingent in real terms. In explaining the non-optimality of the Friedman Rule, Schmitt-Grohé and Uribe (2004) first prove that in imperfectly competitive Ramsey models the nominal interest rate actually indirectly taxes profit income. And as profit income represents payments to a fixed factor, producers' monopoly power, the Ramsey planner would like to tax it as heavily as possible because it would be non-distortionary. With confiscation of profits ruled out, the nominal interest rate acquires the auxiliary role of indirectly taxing profits. Thus, the Friedman Rule of a zero net nominal interest rate ceases to be optimal once product markets exhibit monopoly power. They also show that optimal interest is increasing in the degree of market power.

However, in our model, as the learning rate increases from 0% to 15% the average nominal interest rate decreases from 3.48% to 0.07% per year. With a sufficiently high degree of learning rate our imperfectly competitive Ramsey model reinstates the optimality of the Friedman Rule[2]. Although not obvious at this point, our optimal interest rate result is in fact consistent with the finding in Schmitt-Grohé and Uribe (2004) that optimal interest rate increases with the degree of market power (markup). As equation (6) shows, the markup of prices over marginal cost is time varying in our model and there is an inverse relationship between the markup and the learning rate. We can see this more clearly by looking at the steady-state markup

$$\Omega^{ss} = \frac{\eta}{\eta - 1} \times \frac{1 - \beta\gamma - \beta\theta\varepsilon}{1 - \beta\gamma} < \frac{\eta}{\eta - 1}. \tag{18}$$

First, note that in presence of LBD, steady-state markup is no longer governed by a single parameter, η, it now also depends on the values of LBD parameters, θ and ε and on the discount factor, β. The inequality in equation (18) highlights the fact that under LBD the steady-state markup is smaller than in the standard static monopolistic case, in which the markup equals $\eta/(\eta - 1)$. In particular, as the learning rate increases (i.e. the value of θ increases), the markup of prices over cost falls[3]. As the learning rate increases from 0% to 15% the markup of prices over marginal cost decreases from 1.25 to 1.0048. The reason the markup is lower is that under LBD the pricing decision of the monopolistic firms becomes a dynamic one. The firms know that a current price/production change not only affects their current revenue, it also affects their stock of organizational capital, productivity, costs, and hence profits in all future periods. Therefore, the monopolistic firms no longer follow a static pricing rule of equating time t marginal cost, mc_t, and time t marginal revenue, $\eta/(\eta - 1)$. Instead, they now maximize lifetime profits and choose prices and output such that marginal cost equals to the marginal revenue *and* the present value of all future benefits, in terms of profit, generated by a marginal change in output and OC stock. Also note that optimal labor income tax rate and inflation rate both are falling with the degree of learning rate. And, although small, the volatility of inflation falls with higher learning rate. The reason for the falling labor income rate is that when the learning rate goes up the labor income tax base rises as both employment and wages increase as the economy becomes more productive and competitive. Inflation falls with the learning rate because inflation and nominal interest rate has a direct relationship through the Fisher relation(15). Finally, optimal inflation becomes more persistent with higher degree of learning rate[4].

5. Conclusion

This paper characterizes optimal Ramsey interest rate in presence of learning-by-doing ef-

[2]With our baseline parameter values, a learning rate of 15.5% reinstate the Friedman Rule of a zero net nominal interest rate

[3]To ensure the nonnegativity of the steady-state net markup $\Omega - 1$, we impose the parameter restriction that $\theta \leq \frac{2\eta - 2\beta\gamma\eta + \beta\gamma - 1}{\beta\eta(1-\gamma)}$.

[4]In Talukdar (2014), we employ a sticky-price Ramsey model and show that LBD can generate stable and persistent optimal inflation and a counter cyclical labor income tax.

fects in firms production technology. Our central finding is that optimal interest rate has an inverse relationship with the degree of learning rate. In presence of sufficient degree of learning, our model can reestablish the optimality of the Friedman rule. The key for our finding is an intertemporal link between the current level of production and the future level firms productivity. The presence of LBD endogenously lowers the markup of prices over marginal costs (market power) which calls for a reduction of the optimal nominal interest rate.

Aknowlagements

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