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Contributions of a noisy chaotic model to the stressed Value-at-Risk

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Abstract

The weaknesses of current Value-at-Risk (VaR) measure led the Basel Committee to revise the Basel II market risk framework. A stressed VaR measure is introduced to incorporate the violent behaviour of financial markets during crisis periods. This requirement allows the pro-cyclicality of the current VaR to be removed. However, this solution does not solve the problem related to the VaR estimation, including the choice of an appropriate model in a parametric approach. The forecasts of those models must comply with the assumptions of unconditional coverage and independence. In this paper, we evaluate the contribution of a noisy chaotic model for estimating the VaR measure in a crisis period. The simultaneous consideration of heteroskedastic and chaotic structures leads to a better forecast of the returns (Kyrtsou and Terraza (2010)). This clarification relative to the GARCH (1,1) model is used in this paper for predicting the stressed VaR of a portfolio built according to the mean-Gini criterion. The forecasting exercise, evaluated by backtesting tests, shows an outperformance of the Mackey-Glass-GARCH (1,1) model.

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1 Introduction

The underestimation of market risk is considered as one of the triggers of the 2007-2008 financial crisis. According to the Basel Committee's recommendations, the minimum capital requirement can be determined by standard methods (1993) or by using an internal model (1995) based on Value-at-Risk(VaR). The prudential standards, revised in the Basel II framework, led to the introduction of a stressed VaR. This allows banks to discard the pro-cyclicality in the determination of capital. The minimal amount of capital is given by a linear combination of these two VaR:

$$c = max\{VaR_{t-1}; m_c * VaR_{AVG}\} + max\{SVaR_{t-1}; m_s * SVaR_{AVG}\} \tag{1}$$

Where c is the minimum amount of required capital, VaR_{t-1} and VaR_{AVG} are, respectively, the current VaR in $t-1$ and the mean of the VaR_{t-1} over a period of 60 days; $SVaR_{t-1}$ is the stressed VaR in $t-1$ and $SVaR_{AVG}$ is the mean of the stressed VaR in $t-1$ over a period of 60 days. m_c et m_s are multiplicative factors between 3 and 4 determined by the supervisory authorities. In spite of this revision, the use of a stressed VaR does not solve the difficulties encountered in the underestimation of market risk.
The classical modelling of the VaR measure assumes the nullity of the mean of returns. However, chaotic structures in financial time series have been detected, e.g. Kyrtsou and Terraza (2003), Guégan and Mercier (2005), Guégan and Hoummiya (2005), Guégan (2009) among others. For instance, Kyrtsou and Terraza (2003) showed an improvement of forecasts when the heteroskedastic structures in the variance equation and chaotic structures in the mean equation are considered simultaneously.
Building on those approaches , the goal of this paper is to evaluate the ability of the Mackey-Glass-GARCH(1,1) model to forecast a stressed VaR. This VaR is computed for a portfolio containing only banking assets whose weights are determined by the mean-Gini criterion. To measure the contribution of the inclusion of chaotics structures, we compare results of the Mackey-Glass-GARCH (1,1) and GARCH (1,1) models. We apply two backtesting tests to confirm the outperformance of the noisy chaotic model.
Our paper is organized as follows: Section 2 introduces the stressed VaR and the Mackey-Glass-GARCH model. Section 3 presents our main results on a banking portfolio and some economic implications. Section 4 provides some concluding remarks.

2 Methodology for estimating stressed Value-at-Risk

The introduction of VaR as a measure of market risk was initiated by regulators such as the Basel Committee and the European Commission. The development of methods for estimating VaR has failed to solve the problems caused by this measure. The use of a stressed VaR is relevant if modelling takes into account chaotic structures of financial time series.

2.1 Stressed VaR: a response to the pro-cyclicality of classical Value-at-Risk

Einhorn(2008) implies 'the VaR is like an airbag that works all the time except when you have an accident'. This statement summarizes the uncertainties about the VaR measure. Brown and Tolikas(2006) and Danielsson(2009) mentioned its inaccuracies in estimating losses. Artzner and al.(1999) argue that it is not a coherent measure since it violates the subadditivity property. Damodaran(2007) pointed out the difficulty for the VaR measure to take into account non-market risks and its inaccuracies for long time periods.

Alternative measures are proposed but these are better than the VaR measure only in specific cases. Despite limitations of the VaR measure, it remains the principal instrument used by banks and regulatory authorities to estimate extreme risks.
VaR estimations are closely linked with economic cycles. Market euphoria following a favorable evolution in the real economy, frequently led to an underestimation of risks. This leads to an abusive granting of credit and an excessive consumption of households (BIS (2001)). The identification of self-fulfilling prophecies (Merton (1948)) in the behavior of financial institutions shows an underestimation of risk in periods of crisis [see Figure 1]. Market risks' assessment by the VaR tends to minimize the amount of potential losses during euphoria periods because of an increase of prices and a decrease of the volatility. Meanwhile, Basel 2 imposes the use of an historical of 250 days for VaR estimation, which leads to a bias due to the internal ratings of banks (BIS (2001)). The pro-cyclicality inherent in the risk measures based on self-fulfilling expectations of the agents does not allow a clear identification of the signs of an increased risk. The VaR measure is pro-cyclical since it is closely linked to the volatility. During periods of instability, the amount of the classical VaR can be multiplied by 2.6 compared with an estimated VaR in a period of euphoria: the movements in the financial markets are amplified (BCBS(2009)).
For all these reasons, the Basel Committee introduced the stressed VaR. Determined from a period of stress, it aims at reducing the pro-cyclicality of capital and thus leads to an increase of 110% of capital.

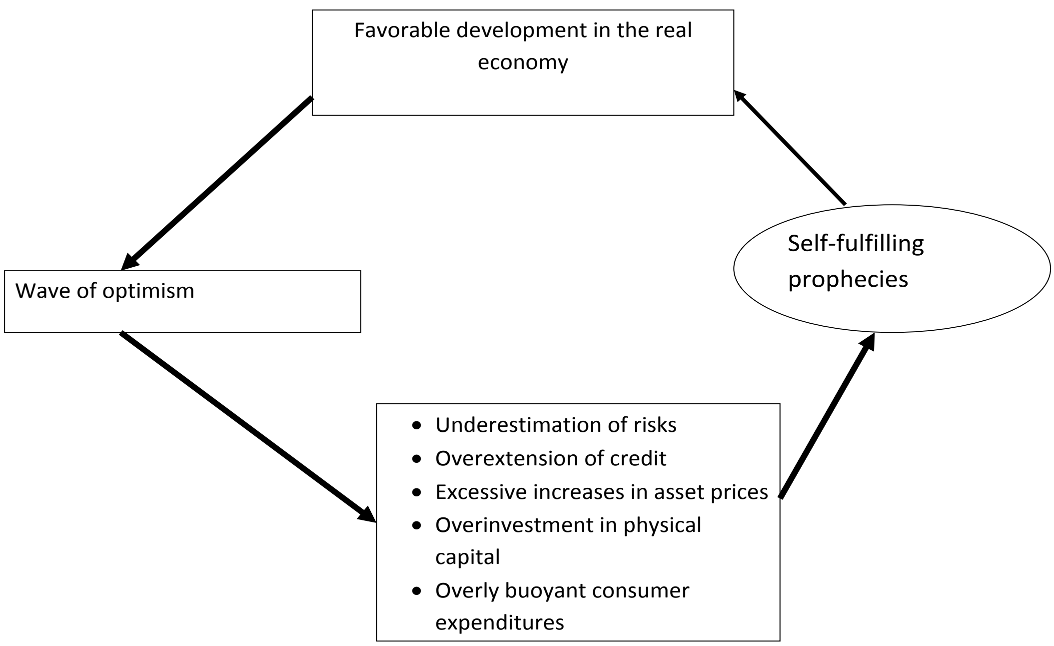

Figure 1: Pro-cyclicality in risk measures.

Factors of pro-cyclicality in risk measures according to the 71st report of BIS (2001)

2.2 How to estimate the stressed VaR?

Despite the introduction of the stressed VaR, its modelling is not well-established. Stochastic models, including GARCH-type models, are extensively used despite poor results. To overcome the shortcomings of stochastic models, we propose using a chaotic-stochastic approach. Many studies have shown the presence of chaotic structures in financial time series. This might be useful, in a market risk framework, to model financial time series using chaotic-stochastic models. For this, we choose the Mackey-Glass-GARCH model. Introduced by Mackey and Glass (1977), this takes into account the chaotic structures. A combination of this model with a GARCH type process was proposed by Kyrtsou and Terraza (2003,2004,2010). The Mackey-Glass-GARCH model takes into account the chaotic structures in the mean equation and the strong volatility of financial time series. The model is specified as follows:

$$X_t = a\frac{X_{t-1}}{1+X_{t-1}^2} - \delta X_{t-1} + \epsilon_t \tag{2}$$

with $\epsilon_t | I_{t-1} \sim N(0, h_t)$ $\epsilon_t = z_t\sqrt{h_t}$ and $h_t = \alpha_0 + \alpha_1 {\epsilon_{t-1}}^2 + \beta_1 h_{t-1}$.
Where X_t is the returns series, a and δ are parameters to be estimated. I_{t-1} is the information set available at time t and h_t is the variance of the residuals.

3 Empiricals findings

In order to take into account both heteroskedastic and chaotic structures in the estimation of a stressed VaR, we propose to build a portfolio composed only of banking assets coming from CAC 40 index over a period of crisis ranging from 01/07/2007 to 11/04/2011, that is 1260 observations. We propose to select three banking assets: Crédit Agricole, Société Générale and BNP Paribas[1]. From these assets, we can build a portfolio according to the mean-Gini criterion.

3.1 Descriptive analysis and preliminary tests

The mean-Gini approach is an alternative method to the mean-variance approach of Markowitz(1952). It overcomes the assumptions of normal distribution of the returns and quadratic utility functions of investors. Moreover, Yitzhaki (1982) has shown that Gini coefficient satisfies the criterion of second-degree stochastic dominance. This makes the measure compatible with the theory of expected utility maximization. Shalit and Yitzhaki(1984) assume that the cumulative distribution for each observation of rank t is t/T. More precisely, Dorfman(1979) and Shalit and Yitzhaki(1984) define the Gini measure as follows:

$$\Gamma = 2cov(R_p, F_p(R_p)) \tag{3}$$

where R_p is the returns series of the portfolio whose cumulative probability density is defined by $F_p(R_p)$. One of the advantages of this criterion compared to the mean-variance criterion is the effect of the variability of an asset on the variability of the portfolio. The

[1]The Financial Stability Board recommends to 29 credit institutions to strengthen their capital due to the risk that these institutions are likely to weigh on the international financial system in case of default (too big to fail). BNP Paribas, Société Générale and Crédit Agricole are part of these institutions.

Gini coefficient of the portfolio is defined by:

$$\Gamma_p = 2cov(R_p, F_p(R_p)) \tag{4}$$

$$R_p = \sum_{i=1}^{N} x_i R_i \tag{5}$$

$$\sum_{i=1}^{N} x_i = 1 \tag{6}$$

Where R_i is the return to asset i, N is the number of assets and x_i is the weight of asset i in the portfolio. The Gini coefficient is defined by:

$$\Gamma_p = 2\sum_{i=1}^{N} x_i cov(R_i, F_p(R_p)) \tag{7}$$

From the equation 7, the portfolio's risk can be decomposed as a weighted sum of the covariances between the variables R_i and the cumulative distribution of the portfolio. In the mean-variance analysis, the risk of the portfolio is represented by the variance of the portfolio, defined by:

$$V(R_p) = \sum_{i=1}^{N} x_i cov(R_i, R_p) \tag{8}$$

We note that in the case of the mean-Gini criterion, the portfolio is represented by the cumulative distribution of its returns F_p. In the mean-variance criterion, it is represented by its returns R_p. The mean-Gini analysis solves the following optimization problem:

$$\min \Gamma_p \tag{9}$$

Under the following constraints:

$$\mu_p = \overline{\mu} \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1 \tag{11}$$

$$x_i \geq 0, \forall i = 1, 2, , N \tag{12}$$

With $\overline{\mu}$ the expected average returns. The solution of this optimization program determines the relative weights to each asset. Table 1 gives the weight for each asset.

Weightings according to the mean-Gini criterion	
Asset	Weight
Crédit Agricole	0.406
BNP Paribas	0.515
Société Générale	0.08

Table 1: Weightings according to the mean-Gini criterion

The obtained weights reflect the climate in the marketplace and investors' interest in each asset[2]. The asset Société Générale, which has the lowest weight in the banking portfolio, is also the one that suffers the largest losses. It is subject to a high volatility following the Greek debt episode. In contrast, BNP Paribas is the asset which represents more than 50 % of the weight of the portfolio. It owes its attractiveness to its position of leader among banking institutions. This makes it the least risky during the crisis. The evolution of the portfolio returns series is similar to that of selected assets. The use of a diversification, however, leads to a sharp decrease of the skewness and kurtosis and therefore of the Jarque-Bera statistic. The portfolio construction based on detrended price series provides an integrated time series of order 0. This is confirmed by unit root tests[3].

Descriptive statistics of portfolio returns series		
Mean		-0.001124
Standard deviation		0.023931
Skewness		0.020109
Kurtosis		6.687953
Jarque-Bera		713.5707
Unit root tests of portfolio returns series		
Test	Estimation	Critical Value
ADF	-24.174	-3.41
KPSS	0.096416	0.146
ERS	0.119552	5.62

Table 2: Descriptives statistics and unit root tests

GPH test			
Observations	d	t-statistic	P-value
$T^{0.4} = 17$	0.0448	0.2221	0.8270
$T^{0.5} = 35$	0.03610	1.7110	0.3645
$T^{0.6} = 72$	-0.0367	-0.4387	0.6622
$T^{0.7} = 147$	-0.0528	-0.9344	0.3516
$T^{0.8} = 302$	-0.0226	-0.5778	0.5638

Table 3: GPH test

The generating process of the portfolio structure is non-identically and independently distributed, i.e. the null hypothesis of BDS test is rejected[4] [Figure 2]. Also, the application of Geweke and Porter-Hudak (GPH) test indicates that there are no long memory structures [Table 3].

To detect the presence of chaotic structures, we apply the algorithms of Wolf et al. (1985) and of Rosenstein et al. (1992). They can estimate the largest Lyapunov exponent

[2]The weights determined by the mean-variance criterion for the assets Crédit Agricole, BNP Paribas and Sociéte Générale are respectively equal to 0.46, 0.5, 0.03. They are substantially different from those obtained by the mean-Gini criterion, especially for the asset Société Générale.

[3]ADF: Augmented Dickey-Fuller(1981) test, KPSS: Kwiatkowski-Phillips-Schmidt-Shin(1992) test, ERS: Elliot-Rothenberg-Stock(1996) test

[4]BDS test is used to test the null hypothesis of an identically and independently distributed distribution against an unspecified alternative hypothesis

Figure 2: BDS test on returns series

Estimation of the largest Lyapunov exponent		
Parameters	m	τ
	2	2
Algorithm of Wolf and al.		
LLE		0.3803
Algorithm of Rosenstein and al.		
LLE		1.5866

Table 4: Estimation of the largest Lyapunov exponent

whose sign determines the nature of the generating process. Table 4 shows that the largest Lyapunov exponent of returns is positive, confirming the presence of chaotic structures in the returns series. The consideration of these structures should lead to a substantial improvement of forecasts.

3.2 Modelling of returns in a stressed period: comparing stochastic and chaotic-stochastic approaches

The analyses, carried out previously, revealed the presence of heteroskedastic and chaotic structures. To judge the relevance and usefulness of taking into account chaotic structures in the modelling of returns, we compare the Mackey-Glass-GARCH (1,1) model with the GARCH (1,1) model in terms of the quality of the residuals[5]. According to the results of the estimation [Table 5], the GARCH model seems appropriate for modelling the returns

[5]The choice of a $c + GARCH(1,1)$ model without ARMA process is due to the statistically insignificance of parameters of ARMA process.

of the portfolio. Apart from the constant, all parameters are significant.

GARCH(1,1) model		
Parameters	coefficients	Probability
c	-0.0003	0.4909
α_0	0.000012	<0.0001
α_1	0.200086	<0.0001
β_1	0.791429	<0.0001

Table 5: Estimation of the c+GARCH(1,1) model

Mackey-Glass-GARCH(1,1)model		
Parameters	coefficients	Probability
a	-14.2987	0.0169
τ	-14.6566	0.0145
α_0	7.469e-06	0.0004
α_1	0.157963	<0.0001
β_1	0.837255	<0.0001

Table 6: Estimation of the Mackey-Glass-GARCH(1,1) model

The significance of parameters of the Mackey-Glass-GARCH (1,1) model confirms its adequacy as model of the returns of the banking portfolio. The presence of chaotic structures is verified by the significance of parameters a and τ.

Given the relevance of the two models, we compare the residuals series. We retain, in this analysis, the standardized residual series. Tests of normality in the table 7 indicate a slight out-performance of the GARCH (1,1) model. It provides residuals less leptokurtic than the Mackey-Glass-GARCH(1,1) model. Table 8 shows that autocorrelation tests on residuals series reject the null hypothesis of non-autocorrelation. The two models are unable to take into account the phenomenon of volatility clustering in the series of portfolio returns.

Normality tests on standardized residuals			
Model	Kurtosis	Skewness	JB stat
GARCH(1,1)	3.7353	0.0527	28.9425
Mackey-Glass-GARCH(1,1)	3.8204	0.0472	35.749

Table 7: Normality tests on standardized residuals

Finally, the ARCH tests analyse the ability of different models to capture the phenomenon of intermittency [Table 9]. It appears that the GARCH (1,1) model is unable to take into account the high variability of the returns. This is unlike the Mackey-Glass-GARCH(1,1) model. Nonetheless, the choice of one of these models for estimating VaR requires the use of backtesting tests.

	Lags			
Model	2	5	10	15
GARCH(1,1)	0	0	0	0
Mackey-Glass-GARCH(1,1)	0	0	0,0002	0,0003

Table 8: P-values of autocorrelation tests on residuals series

	Lags			
Model	2	5	10	15
GARCH(1,1)	0.0015	0.0032	0.0268	0.0492
Mackey-Glass-GARCH(1,1)	0.1222	0.3949	0.3929	0.3815

Table 9: P-values of ARCH test on residuals series

3.3 Backtesting tests

We propose a forecasting exercise for evaluating the performance of the Mackey-Glass-GARCH (1,1) and GARCH (1,1) models. We retain long and short positions. We use the backtesting approach Kupiec(1995) and of Christoffersen (1998) that are most commonly used to evaluate the conditional and unconditional coverage provided by models. According to Christoffersen(1998), a VaR is valid if it satisfies 2 assumptions. The assumption of unconditional coverage is verified if the probability of an ex-post realization of a loss in excess relative to the ex-ante anticipated Value-at-Risk is equal to the coverage rate. Thus, the Value-at-Risk's forecasts for a coverage rate of $\alpha\%$ should not lead to over $\alpha\%$ of violations. Let:

$$I_t = \begin{cases} 1 & \text{if } r_t < VaR_t \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

with I_t an indicator function for comparing the observed returns and the estimated Value-at-Risk, while r_t corresponds to the observed returns at time t and VaR_t the Value-at-risk forecasted in t from the set of information available at time $t-1$. Kupiec(1995)'s test is built on the assumption H_0 as follows: $E(I_t) = \alpha$ where I_t is an indicator function of violations and α the coverage rate. Kupiec's statistic is given by:

$$LR_{uc} = -2ln[(1-\alpha)^{N-X}\alpha^X] + 2ln[(1-(X/N))^{N-X}(X/N)^X] \equiv \chi^2(1) \tag{14}$$

Where N is the number of violations and X the number of forecasts with $N < X$. The rate $\frac{N}{X}$ is the failure rate. If the calculated statistic LR_{uc} is less than the Chi square critical value for one degree of freedom, then the assumption H_0 is accepted. The assumption of independence considers the temporal realization of violations. It assumes that there are no clusters of violations. Violations of the Value-at-Risk at 2 different dates for the same coverage must be independently distributed. Christoffersen (1998) emphasizes the invalidity of a VaR's forecasts if they do not satisfy the assumptions

of unconditional coverage and independence. These two assumptions may be combined under the assumption of conditional coverage. This is satisfied when the conditional probability given all information available at $t-1$ of an exception in t is equal to the coverage rate α.
Christoffersen(1998)'s statistics are:

$$LR_{cc} = -2\{lnL[\Pi_{\alpha}, I_1(\alpha), ..., I_T(\alpha)] - lnL[\widehat{\Pi}, I_1(\alpha), ..., I_T(\alpha)]\} \equiv \chi^2(2) \tag{15}$$
$$LR_{ind} = -2\{lnL[\widehat{\Pi}_{\pi}, I_1(\alpha), ..., I_T(\alpha)] - lnL[\widehat{\Pi}, I_1(\alpha), ..., I_T(\alpha)]\} \equiv \chi^2(1) \tag{16}$$

where $\widehat{\pi}$ is the maximum likelihood estimator of the transition matrix under the alternative hypothesis and L(.), the log-likelihood of violations $I_t(\alpha)$. $\widehat{\Pi}_{\pi}$ and Π_{α} designate respectively the maximum likelihood estimator of the transition matrix under the assumption of independence and the maximum likelihood estimator of the transition matrix under the assumption of conditional coverage. In addition, we have the following result:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{17}$$

3.3.1 Within sample results

From table 10, we compare statistics LR_{uc} and LR_{ind} to a chi-square with one degree of freedom. It shows that the Mackey-Glass-GARCH (1,1) model provides acceptable predictions for the 0.95 quantile, unlike the GARCH (1,1). For 0.99 quantile, the two models provide acceptable predictions.

For the long position, the Mackey-Glass-GARCH (1,1) model seems sufficiently powerful to provide acceptable within sample predictions during a crisis period. The calculated statistics $[LR_{uc}, LR_{ind}]$ and LR_{cc} are respectively lower than the value of a chi-square at one and two degrees of freedom. The GARCH (1,1) model also validates the backtesting tests for the two positions, but the statistics calculated for this model are greater than those determined by the Mackey-Glass-GARCH (1,1) model. The Mackey-Glass-GARCH (1,1) model performs best in terms of the within sample predictions.

3.3.2 Out of sample predictions

Testing results for the out-of-sample forecasts [table 11] confirm the findings previously established. The Mackey-Glass-GARCH(1,1) model provides adequate conditional and unconditional coverage in short and long positions for the 0.95 and 0.05 quantile respectively. The statistics of the different tests clearly show the inadequacy of the GARCH (1,1) model to take into account the highly erratic market movements.

4 Conclusion and Perspectives

Proposals by the Basel Committee on Banking supervision to impose an estimation of a stressed VaR do not solve the matter of accuracy in modelling. The simultaneous integration of heteroskedastic and chaotic structures has already been the subject of a literature that has shown that their associations leads to an improvement of the forecasts. In this paper, we evaluate the relevance of this association in a context of severe crisis. We use the estimation of a stressed VaR on a portfolio composed only of banking assets over a highly disturbed period. We show that the backtesting tests confirm the outperformance

Backtesting tests			
Short position			
Quantile	LR_{uc}	LR_{ind}	LR_{cc}
MG GARCH(1,1) model			
0.95	0.637538	0.133283	0.786901
0.99	0.708546	.NaN	.NaN
GARCH(1,1) model			
0.95	3.0889	4.545	6.876
0.99	0.289	2.42	2.44
Long position			
Quantile	LR_{uc}	LR_{ind}	LR_{cc}
Mackey-Glass-GARCH(1,1) model			
0.05	0.0173	0.2126	0.2299
0.01	0.1597	0.6508	0.8105
GARCH(1,1) model			
0.05	0.9813	0.0968	1.0781
0.01	0.78146	0.1636	0.945

Table 10: Backtesting tests for in-the-sample predictions

of the Mackey-Glass-GARCH (1,1) model compared to the GARCH (1,1) model. The results we obtained open the way for further reflections on the robustness of the Mackey-Glass-GARCH(1,1) model: is it suitable only for crisis periods or not?

Backtesting tests			
Short position			
Quantile	LR_{uc}	LR_{ind}	LR_{cc}
MG GARCH(1,1) model			
0.95	1.3755	0.1715	1.5469
0.99	2.191	.NaN	.NaN
GARCH(1,1)model			
0.95	14.2204	7.4614	21.1569
0.99	27.965	.NaN	.NaN
Long position			
Quantile	LR_{uc}	LR_{ind}	LR_{cc}
Mackey-Glass-GARCH(1,1) model			
0.05	0.385656	0.494822	0.881211
0.01	0.1044	0.0324	0.1368
GARCH(1,1) model			
0.05	6.620363	0.852175	7.4921
0.01	0.1044	0.0487	0.1531

Table 11: Backtesting tests for out-the-sample predictions

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