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A general factorial decomposition of the second Theil index of inequality with applications in environmental economics

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Abstract

In this paper, we propose a general factorial decomposition of the second Theil index of inequality at macroeconomic level. The factorial decomposition of the second Theil index of inequality in k multiplicative factors is shown. Such decomposition is detailed, on the one hand, considering the partial contribution of each factor and, on the other hand, taking into account the interactions between factors as a whole. The previous decomposition is extended to analyze the between- and within-group inequality components. Finally, the study of the determinants of global inequality in per-capita CO2 emissions is provided as an example of application.

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1. Introduction

In recent years, the study of inequality has caused a considerable interest in different fields of research. Although distributional aspects have been traditionally considered in income analysis[1], their application is spreading to other areas.

Among the different inequality measures that are additively decomposed, we propose the second Theil index of inequality (Theil, 1967) because it is the only index that is decomposed by population groups, is differentiable, symmetric, scale invariant and satisfies the Pigou-Dalton criterion (Sen, 1973; Bourguignon, 1979 and Cowell, 1998). The lower limit of this index is zero which is indicative of an equitable situation while its upper limit depends on the size of the sample. Similarly, the Theil index is characterized by being more sensitive to transfers at the bottom of the distribution (Shorrocks, 1980; Jenkins and Van Kerm, 2009), a very interesting feature for income and wealth inequality research (Bourguignon, 1979).

In environmental research, the study of inequality gains importance with the celebration of the United Nations Framework Convention on Climate Change in 1992 and with the entry into force of the Kyoto Protocol in 2005. In this vein, a number of authors have studied the factors causing inequality in CO_2 emissions. Thus, while Heil and Wodon (1997) used the decomposition of the Gini index proposed by Yitzhaki and Lerman (1991), Hedenus and Azar (2005) considered the use of the Atkinson index (Atkinson, 1970). Meanwhile, Padilla and Serrano (2006), Duro and Padilla (2006) and Remuzgo and Sarabia (2013) used the Theil index. Groot (2010) also measured such inequality with the Lorenz concentration curve whereas Mahony (2013) applied the log mean Divisia index.

The aim of this paper is to propose the second Theil index of inequality to study the role that each multiplicative factor plays in the explanation of inequality. As this index can be decomposed into the between- and within-group inequality components, the factorial decomposition can be extended to study driving forces behind these inequality components. To the best of our knowledge, this is the first attempt to provide the decomposition of the second Theil index of inequality in k factors, which allows to take a greater number of factors than the traditional decomposition approaches.

Given the special importance of inequality in environmental research, an application of this methodology to the global inequality in CO_2 emissions is shown. In particular, we decompose inequality in per-capita CO_2 emissions inequality taking the Kaya factors as reference. The analysis covers the regions considered by the International Energy Agency (IEA) in 1990 and 2012.

The structure of this paper is the following. In Section 2, the factorial decomposition of the second Theil index of inequality in k multiplicative factors is exhibit. Such decomposition is detailed, on the one hand, considering the partial contribution of each factor and, on the other hand, taking into account the interactions between factors as a whole. Next, a second decomposition by multiplying factors for analyzing the between- and within-group inequality components is described. In Section 4, an application of this methodology to the global inequality in CO_2 emissions is presented. Finally, some concluding remarks are presented.

[1] See Duro and Esteban (1998), Goerlich (2001), Sala-i-Martin (2002; 2006), Milanovic, (2005) and Bourgignon and Morrison (2002).

2. Factorial decomposition of the second Theil index of inequality

2.1 Considering the partial contribution of each factor

Let z_i be the variable of country i which is desirable to decompose by multiplying factors and defined, in turn, by the ratio of two variables: $z_i = x_i / y_i$.

The Theil index associated with z_i can be defined as

$$T(\underline{z},w) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{z_i}\right), \tag{1}$$

where $\underline{z} = (z_1,\ldots,z_n)$; w_i represents the share of country i in the world value of the variable Y; $\overline{Z}$ stands for the world average of the variable Z and, finally, log is the natural logarithm.

Let $F^{(1)}, F^{(2)}, \ldots, F^{(k)}$ be the k multiplying factors in which the variable Z is decomposed and let these k factors be defined by the ratio of two variables as follows:

$$Z = \frac{X}{Y} = F^{(1)} \times F^{(2)} \times \ldots \times F^{(k)} = \frac{X}{A^{(1)}} \times \frac{A^{(1)}}{A^{(2)}} \times \ldots \times \frac{A^{(k-1)}}{Y}. \tag{2}$$

Once the factorial decomposition is specified, the next step is to measure the contribution of each factor to the global inequality index. Then, we shall define k hypothetical vectors for the variable Z for each country by letting that, in each vector, only the value of one factor diverges from the global average value:

$$z_i^{F^{(1)}} = f_i^{(1)} \times \overline{F}^{(2)} \times \ldots \times \overline{F}^{(k)}, \tag{3}$$

$$z_i^{F^{(2)}} = \overline{F}^{(1)} \times f_i^{(2)} \times \ldots \times \overline{F}^{(k)}, \tag{4}$$

$$\vdots$$

$$z_i^{F^{(k)}} = \overline{F}^{(1)} \times \overline{F}^{(2)} \times \ldots \times f_i^{(k)}, \tag{5}$$

where $\overline{F}^{(1)}, \overline{F}^{(2)}, \ldots, \overline{F}^{(k)}$, represent the world averages of each factor, respectively.

Let us now calculate the degree of inequality related to each factor using the definition of the Theil index:

$$I^{F^{(r)}}(\underline{z}^{F^{(r)}}, w) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}^{F^{(r)}}}{z_i^{F^{(r)}}}\right), \quad r = 1,\ldots,k, \tag{6}$$

where $\overline{Z}^{F^{(r)}} = \sum_{i=1}^{n} w_i \cdot z_i^{F^{(r)}}$, so that, each index measures the partial contribution of each factor to global inequality.

Now, if we add the appropriate expression to the previous inequality indices, we shall obtain for the k factors their corresponding Theil index, using the global average of Z as a reference:

$$I^{F^{(r)}}(\underline{z}^{F^{(r)}}, w) + \log\left(\frac{\overline{Z}}{\overline{Z}^{F^{(r)}}}\right) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{z_i^{F^{(r)}}}\right) = T^{F^{(r)}}(\underline{z}^{F^{(r)}}, w), \quad r = 1,\ldots,k. \tag{7}$$

In consequence we can write:

$$\sum_{r=1}^{k} T(\underline{z}^{F^{(r)}}, w) = \sum_{r=1}^{k}\sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{z_i^{F^{(r)}}}\right) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{z_i}\right) = T(\underline{z}, w), \tag{8}$$

where $T(\underline{z}, w)$ is the Theil index of the variable *Z*.

Thus, for the *k* factors, the factorial decomposition can be expressed as follows,

$$T(\underline{z}, w) = \sum_{r=1}^{k} I^{F^{(r)}}(\underline{z}^{F^{(r)}}, w) + \sum_{r=1}^{k} \log\left(\frac{\overline{Z}}{\overline{Z}^{F^{(r)}}}\right). \tag{9}$$

The second term of the previous sum measures the differences between the global average of the variable *Z* and the average of the variable *Z* associated to the *k* hypothetical vectors. These adjusting components collect the correlations between the different factors considered in the analysis and can be expressed as follows:

$$\sum_{r=1}^{k} \log\left(\frac{\overline{Z}}{\overline{Z}^{F^{(r)}}}\right) = \log\left(\frac{\sigma_{F^{(1)}, F^{(2)} \ldots F^{(k)}}}{\overline{Z}^{F^{(1)}}} + 1\right) + \sum_{r=2}^{k} \log\left(\frac{\sigma_{F^{(r)}, F^{(r+1)} \ldots F^{(k)}} \prod_{j=1}^{r-1} \overline{F}^{(j)}}{\overline{Z}^{F^{(r)}}} + 1\right), \tag{10}$$

where $\sigma_{F^{(r)}, F^{(r+1)} \ldots F^{(k)}} = Cov(F^{(r)}, F^{(r+1)} \cdots F^{(k)})$ represents the covariance between $F^{(r)}$ and the product $F^{(r+1)} \cdots F^{(k)}$.

2.2 Considering the interactions between factors as a whole

Sometimes, mostly when you work with few factors, it is more appealing to know the effect of the interactions between factors jointly. In this case, we have to work as follows.

Let z_i be the variable of country *i* which is desirable to decompose by multiplying factors and defined, in turn, by the ratio of two variables: $z_i = x_i / y_i$. Let $F^{(1)}, F^{(2)}, \ldots, F^{(k)}$ be the *k* multiplying factors in which the variable *Z* is decomposed and let these *k* factors be defined by the ratio of two variables *X* and *Y*, as in equation (2).

As we are working with world averages, the Theil index can be expressed as:

$$T(\underline{z}, w) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{z_i}\right) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{f_i^{(1)} \times f_i^{(2)} \times \ldots \times f_i^{(k)}}\right). \tag{11}$$

Define $\mu^{F^{(r)}} = \sum_{i=1}^{n} w_i \cdot f_i^{(r)}$, if we multiply and divide the inside of the logarithm of the Theil index by $\prod_{r=1}^{k} \mu^{F^{(r)}}$, we have:

$$T(\underline{z}, w) = \sum_{i=1}^{n} w_i \cdot \log\left(\frac{\overline{Z}}{f_i^{(1)} \times f_i^{(2)} \times \ldots \times f_i^{(k)}} \times \frac{\prod_{r=1}^{k} \mu^{F^{(r)}}}{\prod_{r=1}^{k} \mu^{F^{(r)}}}\right). \tag{12}$$

Then,

$$T(\underline{z},w)=\sum_{r=1}^{k}\sum_{i=1}^{n}w_i\cdot\log\left(\frac{\mu^{F^{(r)}}}{f_i^{(r)}}\right)+\log\left(\frac{\overline{Z}}{\prod_{r=1}^{k}\mu^{F^{(r)}}}\right)=\sum_{r=1}^{k}I^{F^{(r)}}(\underline{z}^{F^{(r)}},w)+\log\left(\frac{\overline{Z}}{\prod_{r=1}^{k}\mu^{F^{(r)}}}\right), \quad (13)$$

where

$$\log\left(\frac{\overline{Z}}{\prod_{r=1}^{k}\mu^{F^{(r)}}}\right)=\sum_{r=1}^{k-1}\log\left(\frac{\sigma_{F^{(1)}\cdots F^{(r)},F^{(r+1)}}}{\mu^{F^{(1\cdots r)}}\times\mu^{F^{(r+1)}}}+1\right), \quad (14)$$

where $\mu^{F^{(1\cdots r)}}=E(F^{(1)}\cdots F^{(r)})$ represents the mean of the product $F^{(1)}\cdots F^{(k)}$.

Note that there are different interaction terms as possible combinations between the factors considered in the decomposition. For example, if k = 3, we have three possible combinations:

$$\log\left(\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(1)}}\cdot\mu^{F^{(2)}}\cdot\mu^{F^{(3)}}}\right)=\log\left(\frac{\mu^{F^{(12)}}}{\mu^{F^{(1)}}\cdot\mu^{F^{(2)}}}\times\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(12)}}\cdot\mu^{F^{(3)}}}\right)=\log\left(\frac{\sigma_{F^{(12)},F^{(3)}}}{\mu^{F^{(12)}}\times\mu^{F^{(3)}}}+1\right),$$

$$\log\left(\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(1)}}\cdot\mu^{F^{(2)}}\cdot\mu^{F^{(3)}}}\right)=\log\left(\frac{\mu^{F^{(13)}}}{\mu^{F^{(1)}}\cdot\mu^{F^{(3)}}}\times\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(13)}}\cdot\mu^{F^{(2)}}}\right)=\log\left(\frac{\sigma_{F^{(13)},F^{(2)}}}{\mu^{F^{(13)}}\times\mu^{F^{(2)}}}+1\right),$$

$$\log\left(\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(1)}}\cdot\mu^{F^{(2)}}\cdot\mu^{F^{(3)}}}\right)=\log\left(\frac{\mu^{F^{(23)}}}{\mu^{F^{(2)}}\cdot\mu^{F^{(3)}}}\times\frac{\mu^{F^{(1\cdots 3)}}}{\mu^{F^{(23)}}\cdot\mu^{F^{(1)}}}\right)=\log\left(\frac{\sigma_{F^{(23)},F^{(1)}}}{\mu^{F^{(23)}}\times\mu^{F^{(1)}}}+1\right).$$

3. Factorial decomposition of the between- and within-group inequality components

The inequality analysis by population groups allows to know what inequality percentage can be attributed to differences between population groups and what to differences within each group considered. The first component shows the inequality when we only consider the differences between the average inequalities of each region, while the second component is calculated as the weighted sum of the inequality values of each region (Theil, 1967; Shorrocks, 1980).

Specifically, the decomposition of the total inequality in the between- and within-group components is given by the following expression,

$$T(\underline{z},w)=T_B(\underline{z},w)+T_W(\underline{z},w)=\sum_{g=1}^{G}w_g\cdot\log\left(\frac{\overline{Z}}{\overline{Z}_g}\right)+\sum_{g=1}^{G}w_g\cdot T_g(\underline{z},w), \quad (15)$$

where $T_B(\underline{z},w)$ is the between-group inequality component, $T_W(\underline{z},w)$ is the within-group inequality component, w_g represents the share of the region g in the world value of the variable Y, $\overline{Z}_g$ denotes the average of the variable Z in the region g, $T_g(\underline{z}, w)$ is the inequality in the region g and, finally, G is the number of regions.

The expressions of the two inequality components show that both can be decomposed by multiplying factors. In the case of the within-group inequality component, we can see that it is a weighted average of regional Theil indices. Therefore, we have that,

$$T_W(\underline{z}, w) = \sum_{g=1}^{G} w_g \cdot T_g(\underline{z}, w)$$

$$= \sum_{r=1}^{k}\sum_{g=1}^{G} w_g \cdot I_g^{F^{(r)}}(\underline{z}, w) + \sum_{r=1}^{k}\sum_{g=1}^{G} w_g \cdot \log\left(\frac{\overline{Z}_g}{\overline{Z}_g^{F^{(r)}}}\right), \tag{16}$$

where $\overline{Z}_g^{F^{(r)}} = \sum_{i \in g} w_i \cdot z_i^{F^{(r)}}$.

In relation to the between-group inequality component, note that it is a population-weighted Theil index where the units of study are the regions.

4. Application to global distribution of per-capita CO_2 emissions

In this section, we apply the factorial decomposition of the second Theil index of inequality to study the determinants of global inequality in per-capita CO_2 emissions from fuel combustion, based on the Kaya identity (Kaya, 1989; Yamaji et al., 1991). In particular, per-capita CO_2 emissions (CO_2/POP) are expressed, for a given time period, as the product of carbon intensity of the energy mix (CO_2/E), energy intensity of the economy (E/GDP) and per-capita economic output (GDP/POP)[2]. All the variables are studied across the regions considered by the International Energy Agency (IEA): OECD Americas, OECD Asia Oceania, OECD Europe, Non-OECD Europe and Eurasia, Africa, Asia, China, Non-OECD Americas and Middle East in the years 1990 and 2012.

Table I shows the results obtained in the factorial decomposition of global inequality in per-capita CO_2 emissions. T^1 represents per-capita CO_2 emissions inequality depending on carbon intensity of the energy mix, T^2 stands for per-capita CO_2 emissions inequality determined by energy intensity of GDP and T^3 is per-capita CO_2 emissions inequality based on per-capita GDP. *Interaction*^{1} is the factorial correlation between carbon intensity of the energy mix and per-capita primary energy consumption; *interaction*^{2} is the factorial correlation between energy intensity of GDP and per-capita GDP and the term *interaction* comes from considering the correlations between factors as a whole.

Table I. Decomposition of inequality in per-capita CO_2 emissions by Kaya factors

Year	$T(\underline{c}, p)$	T^1	T^2	T^3		$interaction^1$	$interaction^2$		$interaction$
1990	0.8375	0.1171	0.2061	0.7166		0.1521	-0.3543		-0.2023
2012	0.5681	0.1112	0.1161	0.3979		0.0756	-0.1328		-0.0572

Source: Authors using IEA data.
Note: $T = T^1 + T^2 + T^3 + interaction$.

[2] All the variables have been taken from the International Energy Agency (IEA, 2014).

The results reveal that global inequality in per-capita CO_2 emissions had declined by 32 percent between 1990 and 2012. However, the global level of inequality was still significant in 2012. During all the period, the inequality level is mainly explained by disparities in the per-capita GDP level (T^3). The second major component was the energy intensity of economic output (T^2) whereas the carbon intensity of the energy mix (T^1) explained the least degree of inequality. Regarding the two interaction components, the covariance between the carbon intensity of energy mix and the per-capita energy use ($interaction^1$) has a positive sign, whereas the covariance between the energy intensity of GDP and the per-capita GDP ($interaction^2$) is negative which determines the sign of the *interaction* term.

Table II shows the findings obtained in the decomposition of global inequality in per-capita CO_2 emissions in the between- and within-group inequality components. Along with the values of both components it is also shown the relative importance of both components in total inequality.

Table II. Decomposition of inequality in per-capita CO_2 emissions in the between- and within-group inequality components

Year	$T(\underline{c}, p)$	$T_B(\underline{c}, p)$	$T_W(\underline{c}, p)$	$T_B(\underline{c}, p)$ (%)	$T_W(\underline{c}, p)$ (%)
1990	0.8375	0.5761	0.2614	68.79	31.21
2012	0.5681	0.3226	0.2455	56.78	43.22

Source: Authors using IEA data.

According to Table II, both components contributed to the explanation of overall inequality. Inequality associated with both inequality components had declined from 1990 to 2012. However, while the between-group inequality component had reduced its concentration by 44 percent, the inequality associated with the within-group inequality component had been slightly reduced passing from a value of 0.2614 in 1990 to that of 0.2455 in 2012.

Tables III and IV show the results of the factorial decomposition of the between- and within-group inequality components in per-capita CO_2 emissions, respectively.

Table III. Decomposition of the between-group inequality component in per-capita CO_2 emissions by Kaya factors

Year	$T_B(\underline{c}, p)$	T^1	T^2	T^3	$interaction^1$	$interaction^2$	$interaction$
1990	0,5761	2,7073	1,0660	0,5549	-3,6606	-0,0915	-3,7520
2012	0,3226	0,3226	0,1883	0,2561	0,0000	-0,4444	-0,4444

Source: Authors using IEA data.
Note: $T_B = T^1 + T^2 + T^3 + interaction$.

Table IV. Decomposition of the within-group inequality component in per-capita CO_2 emissions by Kaya factors

Year	$T_W(\underline{c}, p)$	T^1	T^2	T^3	$interaction^1$	$interaction^2$	$interaction$
1990	0,2614	0,1045	0,0907	0,1617	0,0723	-0,1677	-0,0954
2012	0,2455	0,2455	0,1525	0,1418	0,0000	-0,2943	-0,2943

Source: Authors using IEA data.
Note: $T_W = T^1 + T^2 + T^3 + interaction$.

Disparities in the carbon intensity of the energy mix (T^1) were the main contributor to inequality between regions in both years. The energy intensity of GDP (T^2) changes from being the second major factor in the explanation of total inequality in 1990 to becoming the factor with the least importance in 2012 while, the disparities in the per-capita GDP level (T^3) experiences the opposite direction between from 1990 to 2012.

It should be noted the large levels of inequality associated with T^1 and T^2 in 1990, which are counteracted by the interaction term. The high level of inequality in the carbon intensity of the energy mix (T^1) may come from differences in the use of energy forms, that is, some regions use high-carbon fossil fuels whereas others use renewable energy sources. Meanwhile, the elevated disparities in the energy intensity of GDP (T^2) may be due to the diversity in the structure of GDP by economic sector across the regions.

In 1990, inequality within regions was principally explained by disparities in the per-capita GDP level (T^3). However, differences in the carbon intensity of the energy mix (T^1) were the factor with the highest level of inequality in 2012.

In this case, given the groupings of countries in the Appendix, countries within the same region had approximately the same per-capita GDP while regions differ mainly by per-capita GDP. In this sense, a highest value for T^3 may be expected for the between-group inequality in Table III, compared to the within-group inequality component (Table IV).

Table V shows the decomposition of internal inequality in per-capita CO_2 emissions within each region considered by the IEA in 1990 and 2012.

Table V. Decomposition of regional inequality in per-capita CO_2 emissions by Kaya factors

Year	T_g ($\underline{c}$, p)	T^1	T^2	T^3		*interaction*[1]	*interaction*[2]		*interaction*
OECD Americas									
1990	0,2434	0,2434	0,1797	0,1224		0,0000	-0,3021		-0,3021
2012	0,1618	0,1618	0,1586	0,1148		0,0000	-0,2734		-0,2734
OECD Asia Oceania									
1990	0,0398	0,0398	0,0762	0,0570		0,0000	-0,1333		-0,1333
2012	0,0190	0,0190	0,0036	0,0038		0,0000	-0,0074		-0,0074
OECD Europe									
1990	0,1042	0,1042	0,1112	0,0806		0,0000	-0,1918		-0,1918
2012	0,0429	0,0429	0,0585	0,0489		0,0000	-0,1074		-0,1074
Non-OECD Europe and Eurasia									
1990	0,0843	0,0043	0,0529	0,0950		-0,0117	-0,0562		-0,0679
2012	0,1941	0,1941	0,1846	0,1180		0,0000	-0,3026		-0,3026
Africa									
1990	1,0592	0,5216	0,2997	0,3875		0,3466	-0,4963		-0,1497
2012	0,9875	0,9875	0,5513	0,4235		0,0000	-0,9748		-0,9748
Asia									
1990	0,3115	0,0624	0,0703	0,2219		0,1080	-0,1511		-0,0431
2012	0,2041	0,2041	0,0986	0,1484		0,0000	-0,2470		-0,2470

Source: Authors using IEA data.
Note: $T_g = T^1 + T^2 + T^3 + interaction$.

Table V. Decomposition of regional inequality in per-capita CO_2 emissions by Kaya factors (Continuation)

Year	$T_g(\underline{c}, p)$	T^1	T^2	T^3	*interaction*^{1}	*interaction*^{2}	*interaction*
China							
1990	0,0042	0,0004	0,0066	0,0676	0,0023	-0,0728	-0,0705
2012	0,0000	0,0000	0,0040	0,0110	0,0000	-0,0150	-0,0150
Non-OECD Americas							
1990	0,2283	0,0458	0,0676	0,0658	0,0691	-0,0200	0,0491
2012	0,1830	0,1830	0,1534	0,0753	0,0000	-0,2287	-0,2287
Middle East							
1990	0,3089	0,0007	0,1001	0,3731	-0,0054	-0,1597	-0,1650
2012	0,3942	0,3942	0,3746	0,3851	0,0000	-0,7597	-0,7597

Source: Authors using IEA data.
Note: $T_g = T^1 + T^2 + T^3 + interaction$.

In both years, Africa had been the region with the highest level of internal inequality, followed by Middle East and Asia. On the other hand, China had been the territory with the lowest regional inequality. Inequality had decreased in all the regions excluding Non-OECD Europe and Eurasia and Middle East.

As for the contribution of the different factors to the inequality within each geographical zone, it should be noted that, in 2012, disparities in the carbon intensity of the energy mix (T^1) were the main factor in explaining inequality in all the regions but OECD Europe and China.

5. Concluding remarks

We introduced the factorial decomposition of the second Theil index of inequality in k multiplicative factors. In particular, we describe two different decompositions. Firstly, we specify the conventional decomposition where the factorial correlations represent the importance of each factor. Secondly, we indicate the decomposition when is more appealing to consider all factors as a whole. In addition, the previous decomposition is extended to analyze the between- and within-group inequality components.

Based on the the Kaya identity, we apply the previous inequality tool to study the determinants of global inequality in per-capita CO_2 emissions. The results reveal that global inequality in per-capita CO_2 emissions had declined by 32 percent between 1990 and 2012, being the inequality mainly explained by disparities in the per-capita GDP level (T^3) in both years. The between-group inequality component had reduced its concentration by 44 percent whereas the inequality associated with the within-group inequality component had been slightly reduced. Finally, it should be note that, in both years, Africa had been the region with the highest level of internal inequality, followed by Middle East and Asia.

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Appendix

Classification of countries based on the regions covered by the IEA

OECD Americas: Canada, Chile, Mexico, United States.

OECD Asia Oceania: Australia, Israel, Japan, Korea, New Zealand.

OECD Europe: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom.

Non-OECD Europe and Eurasia: Albania, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Cyprus, Macedonia, Georgia, Gibraltar, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Malta, Moldova, Romania, Russia, Serbia, Tajikistan, Turkmenistan, Ukraine, Uzbekistan.

Africa: Algeria, Angola, Benin, Botswana, Cameroon, Congo, Democratic Republic of the Congo, Côte d'Ivoire, Egypt, Eritrea, Ethiopia, Gabon, Ghana, Kenya, Libya, Mauritius, Morocco, Mozambique, Nigeria, Senegal, South Africa, Sudan, Tanzania, Togo, Tunisia, Zambia, Zimbabwe.

Asia: Bangladesh, Brunei Darussalam, India, Indonesia, People's Republic of Korea, Malaysia, Mongolia, Myanmar, Nepal, Pakistan, Philippines, Singapore, Sri Lanka, Chinese Taipei, Thailand, Viet Nam.

China: People's Republic of China, Hong Kong (China).

Non-OECD Americas: Argentina, Bolivia, Brazil, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Jamaica, Netherlands Antilles, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay, Venezuela.

Middle East: Bahrain, Iran, Iraq, Jordan, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, Yemen.