Do banks satisfy the Modigliani-Miller theorem?

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Abstract

The capital structure of banks has become the focus of an extended debate among policy-makers, regulators and academics. The seminal Modigliani-Miller (1958) theorem is seen as supportive of regulators' drive to require higher equity capital to banks. This raises the question on to what extent does Modigliani-Miller theorem hold for banks. This article brings a new insight of the Modigliani-Miller theorem by considering the implicit government guarantee offered to banks. Our theorem shows that a bank does not satisfy the Modigliani-Miller theorem. The main result indicates that banks will favor leverage instead of equity.

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1 Introduction

The Modigliani-Miller (1958) theorem is a cornerstone of corporate finance theory. It provides conditions under which changes in a firm's capital structure do not affect its fundamental value. This means that, if the theorem holds, banks' overall cost of financing is invariant to leverage. This theorem has become central to the debate on banking regulation since the regulator relies on it to justify his policy of requiring more equity. Yet, a recent controversial debate around the relevancy of the Modigliani-Miller theorem regarding the banking sector has been raised since the 2008 financial crisis. The reason comes from the stiff combat carried out by the banking sector to mitigate the Basel regulation consequences on the banks profitability. The pros supports the theory applicability on banks while the cons argue that banks cannot be considered as classical firms.

The research question, posed in this paper, is related to the application of the Modigliani-Miller theorem on the banking sector. But before further processing, let us discuss the contradictory arguments raised in the literature.

The supporters of this theorem believe that bank leverage is excessive, and should be curtailed by regulations requiring much larger equity issuance. They conclude that an increase in equity financing will not affect the cost of bank funding significantly aside from tax factors (Kashyap, Stein, and Hansen (2010)). Indeed, bank equity is not socially expensive and high leverage is not necessary for banks to perform all their socially valuable functions, including lending, deposit taking and issuing money-like securities. In addition, a higher level of debt results in more frequent and costly monitoring of the activities of the bank, which means that higher leverage may create higher costs in terms of governance and risk taking. The fact that banks choose high leverage does not imply that this is socially optimal, and, except for government subsidies and viewed from an ex ante perspective, high leverage may not even be privately optimal for banks (Admati, DeMarzo, Hellwig and Pfleiderer (2011)). Mehran and Thakor (2011) find that total bank value is positively correlated with bank capital. However, Brun, Fraisse and Thesmar (2013) suggest intuitively that Modigliani-Miller might hold only in the long run. In any cases, Admati and Hellwig (2013) recommend that banks should be restricted from paying profits to stockholders until their equity value is above 20 percent of assets.

The critics of this theorem believe that the main reason which breaks the Modigliani-Miller theorem is the government interventions (tax shield, deposit insurance, implicit government guarantee) that reward leverage. These policies that subsidize debt and indirectly penalize equity are distortive. Hence, the break of the theorem induces that capital structure can affect firm value. In that case, raising equity may be costly. Remember that Miller (1995) writes that ”The M&M propositions are ex ante propositions. They are concerned with having equity, not with raising equity.” Indeed, If the total firm value is affected, it must be due to how frictions are affected by the change, not simply because of a change in the funding mix of debt and equity (Pfleiderer (2010)).
A well-known first departure from the Modigliani-Miller conditions is the corporate taxation. Replacing bank’s debt by equity induce a loss on tax deductions given that firms could have passed this free money on to shareholders in the form of higher returns. However, bankruptcy costs induce deadweight losses (legal fees, loss of reputation etc.) when a firm goes bankrupt, which compensate up to a certain point the benefit of tax deductions. Nevertheless, a firm facing debt overhang problem (Myers (1977)) cannot issue new debt given that default becomes likely. Furthermore, there are also agency cost arguments supporting the debt financing by banks (Gertler, Kiyotaki and Queralto (2010)) since the debt provides a discipline on management, which reduces the marginal costs of debt relative to those on equity. Moreover, a context of asymmetric information may raise the cost of external equity and favor debt financing as well (Myers and Majluf (1984)). Note however that a strategic use of debt can benefit to the uninformed party (Chemla and Faure-Grimaud (2001)). In addition, Myerson (2013) notices that this effect of expensive external equity would not apply when a regulator requires the firm to sell new equity shares under a transparent regulatory requirement instead of a discretionary decision of its management. As a consequence, debt might be an optimal response to costly monitoring and information asymmetry (Villamil (2006)). But even all these arguments treat banks as firms that produce loans and ignore banks as producers of liquid financial claims. The Modigliani-Miller’s theorem is not applicable to banks because the debt-equity neutrality assigns a zero weight to the social value of liquidity while there is a market premium for the liquidity production (DeAngelo and Stulz (2013)).

A second departure is the safety net (deposit insurance and implicit government guarantee). First, the deposit liabilities of banks are close to riskless rate because of the free explicit deposit insurance made by the government, which may give banks an incentive to substitute equity with deposit. Second and most important to us, the implicit government guarantee, which is the major government subsidy to banks. Furthermore, Miller (1995) notices that "Much of the academic literature on banking has routinely treated the insurance program as a net subsidy, enabling banks to obtain funds at less than an appropriately risk-adjusted cost". In the same vein, Tsesmelidakis and Merton (2012) notice that banks changed their financing strategy in the course of the crisis by “deliberately taking advantage of their privileged status” of firms benefiting from implicit guarantees. They also shed light on the substantial wealth transfers to financial institutions that result from their "implied unofficial too-big-to-fail status”. Such a protection can be modeled as a free put option issued by the government on the bank’s debt and owned by the bank’s bondholders. This is the obvious reason why bank shareholders benefit from higher leverage. This is not a social benefit because it is a transfer from tax payers to shareholders. This private benefit is reinforced by moral hazard, which can impose substantial social costs of excessive leverage and risk-taking. Vickers (2012) recalls that prospective bail-outs cheapen the private but not social cost of debt relative to equity, which leads to a major reason why Modigliani-Miller theorem is not fully applicable to banks.
What can we learn from this on-going debate? It is that the costs and benefits of higher capital requirements for banks still remain uncertain when facing the arguments of the pros vs. the cons (see e.g Mehran and Thakor (2011) vs. DeAngelo and Stulz (2013)). Hence, the irrelevancy nature of the theorem applied to the banking sector is not trivial despite the common wisdom that it is an expected or even obvious result. It is true that the effect of the government guarantees to the bank’s value has been extensively discussed (Keeley (1990), Gueyie and Lai (2003), Nier and Baumann (2006) etc.); but it is worth emphasizing that surprisingly, in these important typical papers, few or even no references were made to Modigliani and Miller (1958). As a consequence, it may be premature to state that the question of government guarantees within the banking sector has been already investigated, letting no room for additional studies.

Therefore, the motivation of this paper is to address the research question on the relevancy of the Modigliani-Miller theorem to the banking sector. To our best knowledge, most of these papers are empirical studies and none of them has brought a formal proof likely to close the debate. This paper fills this gap despite the large number of published works on this issue. We argue that this research is relevant because it treats the relevancy of this theorem using a theoretical approach based on non-arbitrage reasoning inspired by the seminal article (Modigliani and Miller (1958)). Contrary to empirical studies subject to various biases (data, period, economic context, estimation method etc.), non-arbitrage reasoning encompasses all-at-once the possible trading strategies within the Modigliani-Miller paradigm where only arbitragers can modify the firm’s market value equilibrium.

Given our research agenda, we will introduce, within the Modigliani-Miller setting, the new hypothesis of the existence of governmental implicit guarantees offered to banks. Therefore, the contribution of this paper is to generalize the Modigliani-Miller theorem to include government’s implicit guarantees. The Modigliani-Miller theorem is reformulated for a market composed of classical firms and guaranteed firms such as banks. We offer new formal evidence that the bank’s value turns out to depend on capital structure. The consequence is that leverage is optimal for banks.

2 The model

2.1 Basic notations and definitions

In this section, we re-derive the Modigliani-Miller (1958) setting. Let us define a financial market as a set of firms. Let $S$ be the market value of the common shares of a company and let $D$ be the market value of its debt at time $t = 0$. The value $V$ of the firm is defined as $V := S + D$. We denote by $X$ the random income, for each period of the infinite horizon setting on the assets owned by the company before deduction of the debt interest. We assume that $X \geq 0$ and $X \neq 0$. Recall that $V = \mathbb{E}X/w$ where $w \geq r$ is the weighted average cost of capital as supposed in Modigliani-Miller (1958). Hence $V \leq \mathbb{E}X/r$, i.e. $\mathbb{E}X \geq rV$. The net income received by the shareholders is given by $Y := X - rD$ where $r$ is the interest rate of
the debt \( D \) fixed at time 0. Therefore, a firm is characterized at time \( t = 0 \) by the quadruplet \((S, D, X, r)\). We say that the portfolio structure of an agent’s investment is \((S, D)\) at time \( t = 0 \) if the initial endowment of the agent’s investment is \( S \) and \( D \) is the debt the agent faces. This means that the agent must pay the amount \( rD \) to the lender of his debt for the next period. To each capital structure \((S, D)\) is associated a net income we also denote by \( Y \). For example, when an investor owns the proportion \( \alpha \in [0, 1] \) of a company \((S, D, X, r)\), his capital structure is \((\alpha S, \alpha D)\) and the associated net income is \( Y = \alpha(X - rD) \). In the following, when the firms (respectively the investments) are designated by the names \( A, B, \) etc., the associated quadruplets are denoted by \((S_A, D_A, X_A, r_A)\), \((S_B, D_B, X_B, r_B)\) etc. (respectively the couples \((S_A, D_A)\), \((S_B, D_B)\) etc.) and the net incomes \( Y_A, Y_B \). Given a random variable \( x \), we denote by \( \mathbb{E}x \) the expectation of \( x \).

Recall that, in classical arbitrage theory, a market’s equilibrium is reached when there is no arbitrage opportunity. In the following, we recall the type of arbitrage opportunity which is considered by Modigliani and Miller (1958). Precisely, an agent realizes an arbitrage opportunity between two firms if they have the same profile in the sense that the expected incomes are the same, the risk is the same but they do not have the same value. To do so, the agent switches his investment for the overvalued firm to the cheapest one so that he obtains a positive net gain.

**Definition 2.1.** Let us consider a market composed of firms identified by their quadruplet \((S, D, X, r)\). We say that an agent realizes an arbitrage opportunity of the first kind if the following conditions holds:

1. There exists a firm \((S_A, D_A, X_A, r)\) such that \( V_A = S_A + D_A \) is overvalued, i.e., there exists another firm \((S_B, D_B, X_B, r)\) such that \( \mathbb{E}X_A = \mathbb{E}X_B \), the risk of \( X_B \) is the same as the risk of \( X_B \) but \( V_B = S_B + D_B < V_A \).

2. The agent may partially replicate the firm \((S_A, D_A, X_A, r)\) by investing in the firm \((S_B, D_B, X_B, r)\) and a bond of interest rate \( r \). He obtains a larger expected net income, i.e., he may construct an investment of associated quadruplet \((\alpha S_A, \alpha D_A, \tilde{X}_A, r)\), \( \alpha \in [0, 1] \), such that the net income \( \tilde{Y}_A \) satisfies \( \mathbb{E}\tilde{Y}_A > \alpha \mathbb{E}Y_A \).

The Modigliani-Miller (1958) model supposes the absence of arbitrage opportunity of the first kind. The concept of risk does not need to be clearly identified. We only suppose that an agent acting on the market can measure and compare the risk of two companies. In Modigliani-Miller (1958), the income \( X \) characterizes the risk. The seminal Modigliani-Miller theorem states that, under absence of arbitrage opportunity of the first kind, the value of a firm is independent of its capital structure.

### 2.2 The generalization of the Modigliani-Miller theorem to guaranteed firms

In this section, we generalize the Modigliani-Miller theorem by introducing the existence of guaranteed firms (firms \( GF \)) along with classical firms (firms \( CF \)). The main objective is to compare their values. We suppose that the \( GFs \) are guaranteed by a government.
the same notations, a GF is by definition a firm such that the net income of \((S, D, X, r)\) is given by

\[
Y := Z - rD, \quad Z = X + (rD - X)^+. \tag{2.1}
\]

Here, we use the notation \(x^+ = \max(x, 0)\) to characterize an option contract pay-off. We consider the free governmental implicit guarantee as equivalent to a free put option. Recall that Merton (1977) was the first academician to recognize that financial guarantee acts as a put option on the guaranteed underlying assets. In our setting, \(Z\) includes this put option. Contrary to the model for CFs proposed by Modigliani-Miller, it is more justified to suppose that the GFs benefit from the risk-free interest rate \(r\) as they are guaranteed by the government. In the following, we define \(Z := X\) for a CF. We assume without loss of generality that \((rD - X)^+ \neq 0\) for a GF. Applying the Modigliani-Miller theorem, we immediately deduce that the value \(V\) of a GF \((S, D, X, r)\) only depends on \((X, D, r)\) under absence of arbitrage opportunity of the first kind.

When \(D \neq 0\), it is natural to think that there cannot exist both a GF and a CF with the same characteristics \((S, D, X, r)\). In the contrary case, the value \(V = S + D\) would be the same. This contradicts the intuition that an agent should naturally prefer to invest in the GF \((S, D, X, r)\) rather than in the CF \((S, D, X, r)\). Recall that the GF offers positive income \(Y = (X - rD)^+\) to the shareholders while the CF offers the more risky net income \(Y = X - rD\). In its current form, the Modigliani-Miller framework permits this contradiction as agents are indifferent to a CF and a GF having the same value. We suggest to take into account the behaviors of the agents, acting on a market composed of CFs and GFs, by describing their preference between two firms having the same value. In the following, we introduce a new type of arbitrage opportunity, we call of second type. As we shall see, such an arbitrage opportunity may appear in presence of guaranteed firms as soon as they have the same value but distinct capital structures.

**Definition 2.2.** Suppose that the financial market is defined by the Modigliani-Miller setting. Let us consider a market composed of CFs and GFs identified by their quadruplet \((S, D, X, r)\). We say that an agent realizes an arbitrage opportunity of the second kind if the following conditions holds:

1. There exists two firms \((S_A, D_A, X_A, r_A)\) and \((S_B, D_B, X_B, r_B)\) such that \(\mathbb{E}X_A = \mathbb{E}X_B\), the risk of \(X_B\) is the same than the risk of \(X_A\), \(V_A = V_B\) and \(D_B \geq D_A\),

2. When investing the same amount \(\alpha S_A\) in \((S_A, D_A, X_A, r_A)\) and \((S_B, D_B, X_B, r_B)\), the agent prefers investing in firm \(B\) if the following relation is observed:

\[
\mathbb{E}\alpha(S_A/S_B)(Z_B - r_BD_B) > \mathbb{E}\alpha(Z_A - r_BD_A). \tag{2.2}
\]

The random variable \(\alpha(S_A/S_B)(Z_B - r_BD_B)\) is the net income the agent obtains by investing the capital \(\alpha S_A\) in firm \(B\). Observe that, if the Modigliani-Miller financial market
is only composed of CFs as in Modigliani-Miller (1958), the absence of arbitrage opportunity of the second kind also holds.

In the following, we make the natural assumption that a guaranteed firm benefits from a lower interest rate (e.g. the risk-free interest rate) than a classical firm having the same income distribution. Observe that a CF \((S_A, D_A, X_A, r_A)\) should not benefit from the risk-free interest rate \(r\) but \(r_A > r\). The discrepancy \((r_A - r)\) should reflect the option value resulting from insurance guarantees against default.

**Theorem 2.3.** Suppose that the financial market is defined by the Modigliani-Miller setting. Consider a classical firm \((\text{CF}) (S_A, D_A, X_A, r_A)\) and a guaranteed firm \((\text{GF}) (S_B, D_B, X_B, r_B)\) such that the random variables \(X_A\) and \(X_B\) admit the same distribution and \(r_A \geq r_B\).

1. Assume that the market does not admit any arbitrage opportunity of the first kind. Then, \(V_A \leq V_B\).
2. Assume that the market does not admit any arbitrage opportunity of the first and second kind. Then, \(V_A < V_B\).

The initial Modigliani-Miller framework does not allow to conclude that the values of a CF and a GF are distinct in general. However, in absence of arbitrage opportunity of the second kind, the value of a CF appears to be strictly inferior to the value of a GF having the same profile. **In conclusion, the guarantee increases the value of firms.**

**Theorem 2.4.** Suppose that the financial market is defined by the Modigliani-Miller setting. Assume that each GF of the market may borrow and lend money at the same risk-free interest rate \(r\). Consider two GFs \((S_A, D_A, X_A, r)\) and \((S_B, D_B, X_B, r)\) such that the random variables \(X_A\) and \(X_B\) admit the same distribution.

1. Assume that the market does not admit any arbitrage opportunity of the first kind. Then, \(D_A \leq D_B\) implies \(V_A \leq V_B\).
2. If the market does not admit any arbitrage opportunity of first and second kind, then \(D_A < D_B\) implies \(V_A < V_B\).

**Remark 2.5.** Considering a bank as a guaranteed firm, the result of the last theorem implies that leverage is optimal for bank.

### 3 Policy implications

The theoretical result of this research contradicts the shared belief on the relevance of the Modigliani-Miller theorem in the banking sector as suggested by the bulk of the academic literature. Policy implications can be derived from this paper. The first implication concerns the Government interventions. Indeed, there are abundant examples of interventions that create distortions to the Modigliani-Miller theorem such as tax shield, deposit insurances or implicit guarantees. For example, the Basel Committee asks for more equity to banks
whereas the Governments subsidize debt. Another example stems from the Governments willingness to reduce risk-taking in the banking sector, while offering an implicit guarantee to banks, which creates an incentive to excessive leverage. As shown in this extended version of the Modigliani-Miller theorem, a bank cannot be longer considered as a classical firm given the safety net offered by the governments. A comprehensive policy needs to reconcile the Governments interventions with the Basel regulator requirements given that the first creates various incentives to debt while the last asks for more equity arguing the Modigliani-Miller theorem.

Given that under this revised version of the Modigliani-Miller theorem, debt is optimal, a second policy implication would recommend the Irving Fisher (1936) 100% reserve proposal in the banking system to avoid a major financial crisis. Various banking reforms have been addressed in several countries. The U.S. favored the separation with some speculative activities while the U.K adopted ring-fencing principle. France implemented a light version of the Likkannen reform by choosing the filiarization mechanism. Surprisingly, no one considered the simplest reform that one could imagine. Indeed, effective policy would have asked for opening the debate on the 100% reserve proposal for which insured deposits should be invested exclusively in safe monetary instrument such as Treasury bills. Under this framework, no Government interventions to bailout banks would be expected since the deposits remain safe by nature.

4 Conclusion

This article derives a new model that generalizes the Modigliani-Miller (1958) theorem by including the implicit guarantee offered by governments to some firms. In the banking sector where this guarantee is generally free, the Modigliani-Miller theorem becomes irrelevant as banks are no longer sharing the firms’ characteristics. In that case, banks will prefer debt to equity. However, if this guarantee is sold at a normal actuarial rate, the Modigliani-Miller theorem holds as banks can no longer be considered as different from classical firms.

A future research may investigate the impact of the assumed market participants’ behavior on the Modigliani-Miller economic implications.
5 Appendix

Proof of Theorem 2.3. Suppose that $V_A > V_B$. An agent owning the fraction $\alpha$ of the common shares of the CF $A$ obtains at time $t = 1$ the income $Y_A = \alpha(X_A - r_AD_A)$. He could also sell his shares $\alpha S_A$, borrow the capital $\alpha D_A$ and invest the total amount in GF $B$. In this case, the associated portfolio structure is the same, i.e. $(\alpha S_A, \alpha D_A)$, but the net income is

$$\tilde{Y}_B = \frac{\alpha V_A}{S_B} (X_B + E(r_B D_B - X_B)^+ - r_B D_B) - r_B D_A \alpha. \quad (5.1)$$

As $r_A \geq r_B$, then $Y_A \leq \alpha(X_A - r_B D_A)$. Then, an easy calculation yields

$$E\tilde{Y}_B - Y_A \geq \frac{\alpha}{S_B} \left[ (V_A - S_B) E X_A + V_A E (r_B D_B - X_B)^+ - V_A D_B r_B \right]. \quad (5.2)$$

Since $V_A - S_B > V_B - S_B = D_B$ and $E X_A - r_B V_A \geq E X_A - r_A V_A \geq 0$ (as $A$ is a CF), we deduce that

$$E\tilde{Y}_B - Y_A \geq \frac{\alpha D_B}{S_B} \left[ E X_A - r_B V_A + V_A E (r_B D_B - X_B)^+ \right] \geq 0. \quad (5.3)$$

Therefore, there exists an arbitrage opportunity of the first kind, hence a contradiction. This shows that $V_A \leq V_B$.

Assume that there is no arbitrage opportunity of the second kind. Suppose for a moment that $V_A = V_B$. As $A$ is a CF, the Modigliani-Miller theorem asserts that $V_A$ does not depend on its capital structure. It is then sufficient to consider a CF $A'$ such that $V_{A'} = V_A$ and $A'$ has the same capital structure $(S_B, D_B)$ than $B$. The equality $V_{A'} = V_B$ leads to an arbitrage opportunity of the second kind between CF $A'$ and GF $B$, hence a contradiction. □

Proof of Theorem 2.4. Let us consider an agent owning the fraction $\alpha$ of the common shares of a GF $(S, D, X, r)$. This corresponds to the capital structure $(\alpha S, \alpha D)$. At time $t = 1$, he obtains the net income $Y = \alpha(X + (rD - X)^+ - rD)$. As there is no arbitrage opportunity, the following inequality holds:

$$E X + E(rD - X)^+ \geq rV. \quad (5.4)$$

Indeed, otherwise he could also sell his shares $\alpha S$, borrow the capital $\alpha D$ and invest the total amount in the risk-free bond of interest rate $r$. The corresponding structure should also be $(\alpha S, \alpha D)$. The generated net income should be $\tilde{Y} = \alpha Sr$ and $E \tilde{Y} > E Y$, i.e. a contradiction.

Let us consider two GFs $A$ and $B$ such that $X_A$ and $X_B$ admit the same distribution and $D_A \leq D_B$. Suppose that $V_A > V_B$. An agent owning the fraction $\alpha$ of the common shares of the GF $A$ obtains at time $t = 1$ the income $Y_A = \alpha(X_A + (rD_A - X_A)^+ - rD_A)$. He could also sell his shares $\alpha S_A$, borrow the capital $\alpha D_A$ and invest the total amount in GF $B$. In
this case, the associated capital structure is the same, i.e. \((\alpha S_A, \alpha D_A)\), but the net income is

\[
\bar{Y}_B = \frac{\alpha V_A}{S_B} (X_B + (rD_B - X_B)^+) - rD_A \alpha.
\]

(5.5)

An easy calculation yields

\[
\mathbb{E}\bar{Y}_B - \mathbb{E}Y_A = \frac{\alpha}{S_B} [ (V_A - S_B) \mathbb{E}X_A + V_A \mathbb{E}(rD_B - X_B)^+ - S_B \mathbb{E}(rD_A - X_A)^+ - V_A D_B r].
\]

(5.6)

With \(V_A := V_B + \epsilon, \epsilon > 0\), we get that

\[
\mathbb{E}\bar{Y}_B - \mathbb{E}Y_A = \frac{\alpha}{S_B} [D_B \mathbb{E}X_A + V_B \mathbb{E}(rD_B - X_B)^+ - S_B \mathbb{E}(rD_A - X_A)^+ - rD_B V_B]
\]

(5.7)

\[
+ \frac{\alpha \epsilon}{S_B} [ \mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rD_B ].
\]

(5.8)

Since \(D_A \leq D_B\), it follows that

\[
\mathbb{E}\bar{Y}_B - \mathbb{E}Y_A \geq \frac{\alpha D_B}{S_B} [ \mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rV_B ]
\]

(5.9)

\[
+ \frac{\alpha \epsilon}{S_B} [ \mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rD_B ].
\]

(5.10)

By (5.4), the second term above is greater than \(\alpha \epsilon S_B\) and the first term is positive i.e. a contradiction. We deduce that \(V_A \leq V_B\). Similarly, if \((V_B - S_A) \mathbb{E}X_A + V_B \mathbb{E}(rD_A - X_A)^+ - S_A \mathbb{E}(rD_B - X_B)^+ - V_B D_A r > 0\), then the condition \(V_B > V_A\) generates an arbitrage opportunity of the first kind.

Suppose that the market does not admit any arbitrage opportunity of second kind. We already know that \(D_A < D_B\) implies \(V_A \leq V_B\). Suppose that \(V_B = V_A\). Since \(D_A < D_B\), there exists any arbitrage opportunity of second kind. Indeed, an agent owning the fraction \(\alpha\) of the common shares of the GF \(A\) obtains at time \(t = 1\) the income \(Y = \alpha(X_A + (rD_A - X_A)^+ - rD_A)\). If he invests the same amount \(\alpha S_A\) in GF \(B\), he obtains \(\bar{Y} = \alpha(S_A/S_B)(X_B + (rD_B - X_B)^+ - rD_B)\). A straightforward calculation yields

\[
\mathbb{E}\bar{Y} - \mathbb{E}Y = \frac{D_B - D_A}{S_B} (\mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rV_B).
\]

(5.11)

Moreover, \(\mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rV_B > 0\). Indeed, otherwise (5.4) implies

\[
0 = \mathbb{E}X_B + \mathbb{E}(rD_B - X_B)^+ - rV_B > \mathbb{E}X_A + \mathbb{E}(rD_A - X_A)^+ - rV_A \geq 0
\]

(5.12)

i.e. a contradiction. The conclusion follows.
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