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A phase diagram analysis on "The Environment and Directed Technical Change"

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Abstract

Analyzing a dynamic general equilibrium model that incorporates endogenous and directed technical change and environmental constraints, Acemoglu et al. (2012) present thought-provoking discussions on green growth and environmental disaster. For the clarity of argument, they place a restriction on the initial technology levels (Assumption 1). By means of the phase diagram, this note shows that without the assumption the same arguments can be extended. In other words, their results remain valid in the wide range of the initial technology levels. Also, it is shown that there exists a threshold of the relative technology level, which determines the future of the environment: disaster or restoration.

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1. Introduction

One of the biggest concerns in the global economy is the environment. World Bank (2012) states that although economic growth has lifted more than 660 million people out of poverty over the past 20 years, it has often come at the expense of the environment. In fact, human society has faced different types of threats and impediments to economic growth. Local environmental pollutions, which often cause severe diseases and destruction of nature, have been commonly observed in fast-growing economies. It now faces planet-wide environmental problems, such as global warming and extinction of various species. It is hoped that technological innovations will solve the problems.

Recently, using a two sector model of directed technical change (Acemoglu, 1998, 2002), the seminal paper by Acemoglu et al. (2012) provides thought-provoking discussions on the above issue.[1] In concrete, the final good is produced from "dirty" and "clean" inputs, which are in turn produced using labor and a continuum of sector specific machines. The production of dirty inputs damages the environment. To make the points clear, assuming that the initial technology ratio between the two sectors is in a certain range, they analyze the possibility of environmental disaster and green growth. Relaxing this assumption, this note examines the dynamics of the technology. As a result, it is shown that that their results remain robust in the wide range of the initial technology levels. In addition, there exists a threshold of the relative technology level, which determines the future of the environment: disaster or restoration. These findings have important implications not only in the dynamics of the economy but also in the environment because our study shows that environmental disaster can occurs in wide range of parameters.

2. The model

Since we will reexamine the dynamics of the endogenous and directed technical change in Acemoglu et al. (2012), only the production side of their model will be briefly presented.

The unique final good Y_t is produced competitively using "clean" and "dirty" inputs, Y_{ct} and Y_{dt}, according to the following aggregate production function

$$Y_t = \left(Y_{ct}^{(\varepsilon-1)/\varepsilon} + Y_{dt}^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}, \tag{1}$$

where $\varepsilon \in [0,+\infty]$ is the elasticity of substitution between Y_{ct} and Y_{dt}. These inputs are in turn produced using labor and a continuum of sector-specific machines according to the following production functions:

$$Y_{ct} = L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di \quad \text{and} \quad Y_{dt} = L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di \, . \tag{2}$$

Suppose that labor supply is normalized to 1. Then, labor market clearing requires the following:

$$L_{ct} + L_{dt} \le 1 \, . \tag{3}$$

Machines are supplied by monopolistic competitive firms and producing one unit of them requires ψ units of final goods. Therefore, market clearing for the final good implies

$$C_t + \psi\left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) = Y_t \, . \tag{4}$$

Assuming that the number of scientists is normalized to 1, market clearing for scientists takes the form

$$s_{ct} + s_{dt} \le 1 \, , \tag{5}$$

where s_{ct} is the number of scientists working in sector *c* and s_{dt} is that in sector *d*.

Suppose that A_{ct} is the average productivity in sector *c* and A_{dt} is that in sector *d*. Then,

$$A_{ct} = \int_0^1 A_{cit} di \qquad \text{and} \qquad A_{dt} = \int_0^1 A_{dit} di \, . \tag{6}$$

The innovation possibility frontiers, A_{ct} and A_{dt}, are assumed to evolve according to the following first-order deference equations:

$$A_{jt} = (1 + \gamma \eta_j s_{jt}) A_{jt-1} \, , \quad j = c, d \, , \tag{7}$$

where η_j is the innovation productivity in sector j. Also, the environmental quality S_t is assumed to change according to the following:

$$S_{t+1} = -\xi Y_{dt} + (1+\delta) S_t \, , \tag{8}$$

where $\xi > 0$ measures the rate of environment degradation resulting from the production of dirty input and $\delta > 0$ is the rate of "environmental regeneration."

Since the final good market is competitive, the relative price of the two inputs becomes

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[1] See also Goulder (2013), which provides new developments in the literature.

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{Y_{ct}}{Y_{dt}}\right)^{-1/\varepsilon}. \tag{9}$$

Normalizing the price of the final good to 1 at each point in time, the zero profit condition implies

$$\left[p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} = 1. \tag{10}$$

The equilibrium profits of machine producers whose technology is A_{jt}, π_{jit}, can be written as

$$\pi_{jit} = (1-\alpha)\alpha p_{jt}^{1/(1-\alpha)} L_{jt} A_{jit}, \quad j = c, d. \tag{11}$$

Using (7) and (8), the expected profits Π_{jt} can be calculated as

$$\Pi_{jt} = \int_0^1 \pi_{jit} di = \eta_j (1+\gamma)(1-\alpha)\alpha p_{jt}^{1/(1-\alpha)} L_{jt} A_{jt-1}, \quad j = c, d. \tag{12}$$

Taking (10) and (11) into account, from the above the relative benefit from research in sector *c* to in sector *d* can be decomposed as

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \times \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{1/(1-\alpha)}}_{price\ effect} \times \underbrace{\frac{L_{ct}}{L_{dt}}}_{market\ size\ effect} \times \underbrace{\frac{A_{ct}}{A_{dt}}}_{direct\ productivity\ effect}. \tag{13}$$

The above clearly indicates the importance of the *price effect* and *market size effect* in the directed technical change (Acemoglu, 1998, 2002). Taking the equilibrium into account, (13) is further rewritten as:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}. \tag{14}$$

where $\varphi \equiv (1-\alpha)/(1-\varepsilon)$. Similarly in Acemoglu et. al. (2012), assuming that the two inputs are substitute ($\varepsilon > 1$), $\varphi < 0$. Substituting the scientist resource constraint (5) into the above, we have

$$\frac{\Pi_{ct}}{\Pi_{dt}} = f(s_{ct}; a_{t-1}) = \frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d(1-s_{ct})}\right)^{-\varphi-1} a_{t-1}^{-\varphi}, \tag{15}$$

where $a_t \equiv A_{ct}/A_{dt}$ is the relative technology level in sector *c* compared to in sector *d*. Although Acemoglu et. al. (2012) analyze the possible three cases: $\varphi + 1 > 0$, $\varphi + 1 < 0$, and $\varphi + 1 = 0$, in detail (see Appendix B on page 161), we will assume $\varphi + 1 > 0$ or equivalently $\varepsilon < (2-\alpha)/(1-\alpha)$ to clarify our contribution.[2] In this case, $f(s_{ct};a_{t-1})$ is decreasing in s_{ct}. Therefore, given the previous technology ratio a_{t-1}, the allocation of scientists s_{ct} is determined so as to equalize the profits in the two sectors, or equivalently $\Pi_{ct}/\Pi_{dt} = 1$, as s_{ct}^* in Fig.1.

For the law of motion of a_t, we obtain it from (7) as below:

$$a_t = \frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d(1-s_{ct})} a_{t-1}. \tag{16}$$

Hence, once the allocation of scientists s_{ct} is determined by (15), the relative technology level a_t changes according to (16). The change in a_t in turn shifts the $f(\cdot)$ curve, which determines s_{ct+1} and hence a_{t+1}. In other words, utilizing equations (15) and (16), we can characterize the time-paths of s_{ct} and a_t recursively. However, this is not always the case. No interior solutions to $f(s_{ct};a_{t-1}) = 1$ exist, depending on a_{t-1}.

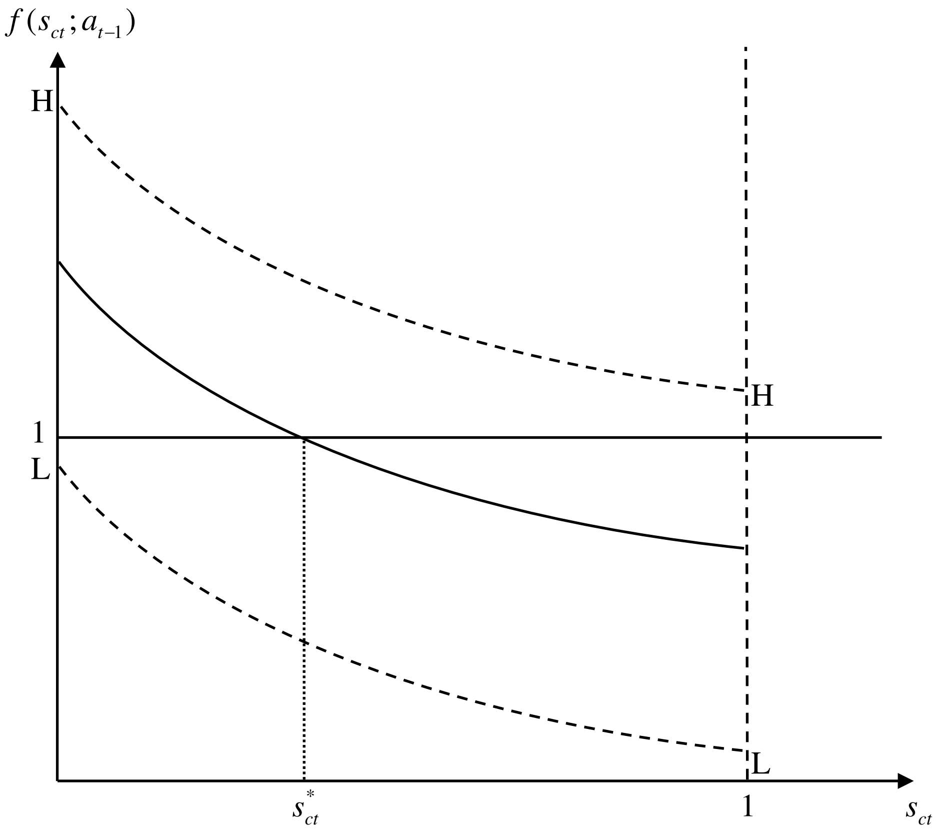

Fig.1 Determination of Scientist Allocation

2 The case in which $\varphi+1=0$ is trivial in the sense that s_{ct} always takes a corner solution of zero or one. Although the case in which $\varphi+1<0$ is very interesting in that multiple equilibria may arises and hence complex dynamics may emerge, the analysis becomes too complicated to clarify the main point of the paper.

Since $f(s_{ct};a_{t-1})$ is increasing in a_{t-1}, an increase in a_{t-1} shifts up the $f(s_{ct};a_{t-1})$ curve. If a_{t-1} is large enough, $f(s_{ct};a_{t-1})$ is always higher than 1 in the [0,1] interval, as the HH curve in Fig. 1. This emerges when $f(1;a_{t-1}) > 1$, i.e.,

$$(\eta_c/\eta_d)(1+\gamma\eta_c)^{-\varphi-1}a_{t-1}^{-\varphi} > 1 \quad \text{or} \quad a_{t-1} > \overline{a} \equiv (1+\gamma\eta_c)^{-(\varphi+1)/\varphi}(\eta_c/\eta_d)^{1/\varphi}. \tag{17}$$

If this is the case, then the innovations take place only in sector *c* or $s_{ct}^* = 1$ because the profitability of research in sector *c* is higher than sector *d* regardless of the allocation of scientists s_{ct}.

If, in contrast, the $f(s_{ct};a_{t-1})$ function is the LL curve in Fig.1, then $f(s_{ct};a_{t-1})$ is always small than 1 in the [0,1] interval. This takes place when a_{t-1} is small enough to satisfy $f(0;a_{t-1}) < 1$, i.e.,

$$(\eta_c/\eta_d)(1+\gamma\eta_d)^{\varphi+1}a_{t-1}^{-\varphi} < 1 \quad \text{or} \quad a_{t-1} < \underline{a} \equiv (1+\gamma\eta_d)^{(\varphi+1)/\varphi}(\eta_c/\eta_d)^{1/\varphi}. \tag{18}$$

If this is the case, then the innovations take place only in sector *d* or $s_{ct}^* = 0$ because the profitability of research in sector *d* is higher than sector *c* regardless of the allocation of scientists s_{ct}.

By definition,

$$\frac{\overline{a}}{\underline{a}} = \frac{(1+\gamma\eta_c)^{-(\varphi+1)/\varphi}}{(1+\gamma\eta_d)^{(\varphi+1)/\varphi}} = \frac{(1+\gamma\eta_c)^{-(\varphi+1)/\varphi}}{[(1+\gamma\eta_d)^{-1}]^{-(\varphi+1)/\varphi}} = \left[\frac{1+\gamma\eta_c}{(1+\gamma\eta_d)^{-1}}\right]^{-(\varphi+1)/\varphi}$$

$$= [(1+\gamma\eta_c)(1+\gamma\eta_d)]^{-(\varphi+1)/\varphi} > 1. \tag{19}$$

Therefore, $\overline{a} > \underline{a}$. These observations bring us to the following proposition.

Proposition 1

Suppose that the initial technology ratio a_0 *is sufficiently large to satisfy* $a_0 \geq \overline{a}$. *Then the innovation occurs only in the clean sector. In contrast, suppose that the initial technology ratio* a_0 *is sufficiently small to satisfy* $a_0 \leq \underline{a}$. *Then the innovation occurs only in the dirty sector.*

Here, it is worth mentioning about Assumption 1 in Acemoglu et al. (2012, p.139), although this does not affect the robustness of their analysis. Using the notations in this paper, it can be expressed that $a_0 < \min(\underline{a}, \overline{a})$. Hence, the innovations occur only in the dirty sector.

In the equilibrium, output of the two inputs and the final goods can be expressed by the technology levels only as follows:

$$Y_{ct} = A_t^{-(\alpha+\varphi)/\varphi} A_{ct} A_{dt}^{\alpha+\varphi}, \quad Y_{dt} = A_t^{-(\alpha+\varphi)/\varphi} A_{ct}^{\alpha+\varphi} A_{dt}, \text{ and } \quad Y_t = A_t^{-1/\varphi} A_{ct} A_{dt}, \tag{20}$$

where $A_t = \left(A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)$, which stands for the economy-wide technology level. Since technology levels are not downgraded and the production functions of the two inputs are Cobb-Douglass, both inputs are always produced in equilibrium.

3. The dynamics of relative technology level

The dynamic general equilibrium can be classified into three cases, depending on the initial relative technology level a_0. They are (i) $a_0 \le \underline{a}$, (ii) $\underline{a} < a_0 < \overline{a}$, and (iii) $\overline{a} < a_0$.

3-1. Environmental disaster: $a_0 \le \underline{a}$.

When the initial technology ratio a_0 is smaller than or equal to $\underline{a}$, then a_t is determined by the following equation because $s_{ct}^* = 0$:

$$a_t = (1 + \gamma\eta_d)^{-1} a_{t-1}. \tag{21}$$

The above implies

$$A_{ct} = A_{c0} \text{ and } A_{dt} = (1 + \gamma\eta_d) A_{dt-1}. \tag{22}$$

As (20) shows, although no technological advances occur in sector *c*, Y_{ct} increases over time if $\alpha + \varphi$ is positive. Quite naturally, however, Y_{dt} surely grows (faster than Y_{ct}). It is evident from (9) that the environmental quality S_t declines as Y_{dt} increases. In the end, S_t goes to zero, which means environmental disaster.

3-2. Environmental restoration: $\overline{a} \le a_0$.

When the initial technology ratio a_0 is larger than or equal to $\overline{a}$, then a_t is determined by the following equation because $s_{ct}^* = 1$:

$$a_t = (1 + \gamma\eta_c) a_{t-1}. \tag{23}$$

The above implies

$$A_{ct} = (1 + \gamma\eta_c) A_{ct-1} \text{ and } A_{dt} = A_{d0}. \tag{24}$$

Taking $A_t = \left(A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)$ into account, the second equation in (20) is rewritten as:

$$Y_{dt}/A_{dt} = \left(A_{ct}^{\varphi} + A_{dt}^{\varphi}\right)^{-\frac{\alpha+\varphi}{\varphi}} A_{ct}^{\alpha+\varphi} = \left\{\left[\left(A_{ct}^{\varphi}/A_{dt}^{\varphi}\right)+1\right]\left(A_{dt}^{\varphi}\right)\right\}^{-\frac{\alpha+\varphi}{\varphi}} A_{ct}^{\alpha+\varphi} = \left(1+a_t^{-\varphi}\right)^{-\frac{\alpha+\varphi}{\varphi}} . \tag{25}$$

Since $\lim_{t\to\infty} a_t = \infty$ and $\varphi < 0$, $\lim_{t\to\infty}\left(1+a_t^{-\varphi}\right) = \infty$. Realizing that $A_{dt} = A_{d0}$ for all t,

$\lim_{t\to\infty}\left(Y_{dt}/A_{d0}\right) = 0$ and hence $\lim_{t\to\infty} Y_{dt} = 0$ if $\alpha + \varphi < 0$,

$\lim_{t\to\infty}\left(Y_{dt}/A_{d0}\right) = 1$ and hence $\lim_{t\to\infty} Y_{dt} = A_{d0}$ if $\alpha + \varphi = 0$,

$\lim_{t\to\infty}\left(Y_{dt}/A_{d0}\right) = \infty$ and hence $\lim_{t\to\infty} Y_{dt} = \infty$ if $\alpha + \varphi > 0$.

As long as $\alpha + \varphi \le 0$, or Y_{dt} or equivalently $\varepsilon \ge 1/(1-\alpha)$, which implies that the two inputs are *strong substitutes* approaches to a finite value, either zero or A_{d0}, in the long run. This implies that the environmental quality S_t increase over time. As a result, S_t reaches at the maximum level $\overline{S}$, which means environmental restoration.

As, however, Acemoglu et. al. (2012, p.142) correctly point out and the above shows, when $\alpha + \varphi > 0$, or equivalently $\varepsilon < 1/(1-\alpha)$, which implies that the two inputs are *weak substitutes,* then Y_{dt} grows at a positive rate towards infinity even if the technological advance takes place only in the clean sector. Hence, a disaster is inevitable without some kind of intervention, such as industrial policies and environmental regulations.

3-3. Knife-edge dynamics: $\underline{a} < a_0 < \overline{a}$

When the initial technology ratio a_0 is in the interval of $(\underline{a}, \overline{a})$, the equilibrium value of s_{ct} is determined by $\Pi_{ct}/\Pi_{dt} = 1$, i.e., $f(s_{ct}; a_{t-1}) = 1$ from (15). Hence, we have

$$\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d(1-s_{ct})} = a_{t-1}^{-\varphi/(\varphi+1)}\left(\frac{\eta_c}{\eta_d}\right)^{1/(\varphi+1)} . \tag{26}$$

The above always holds in equilibrium when $\underline{a} < a_t < \overline{a}$. In other words, s_{ct} is adjusted in the labor market for scientists to satisfy (26). Substitution of (26) into (16) gives the following first-order difference equation of a_t that characterizes the equilibrium dynamics:

$$a_t = \left[(\eta_c/\eta_d)a_{t-1}\right]^{1/(\varphi+1)} \equiv h(a_{t-1}) . \tag{27}$$

Since $0 < \varphi + 1 < 1$, the function $h(a_{t-1})$ is an increasing and convex function as follows:

$h(0) = 0$,

$h'(a_{t-1}) = \left[1/(1+\varphi)\right]\left(\eta_c/\eta_d\right)^{1/(1+\varphi)} a_t^{-\varphi/(1+\varphi)} > 0$,

$h''(a_{t-1}) = -\left[\varphi/(1+\varphi)^2\right]\left(\eta_c/\eta_d\right)^{1/(1+\varphi)} a_t^{-(1+2\varphi)/(1+\varphi)} > 0$.

Suppose that the positive fixed point of $a_t = h(a_{t-1})$ is a^*. Then, $a^* = (\eta_c/\eta_d)^{1/\varphi}$, and hence,

$h'(a^*) = 1/(1+\varphi) > 1$.

The above implies that a^* is an unstable fixed point.

3-4. Complete characterization of dynamics

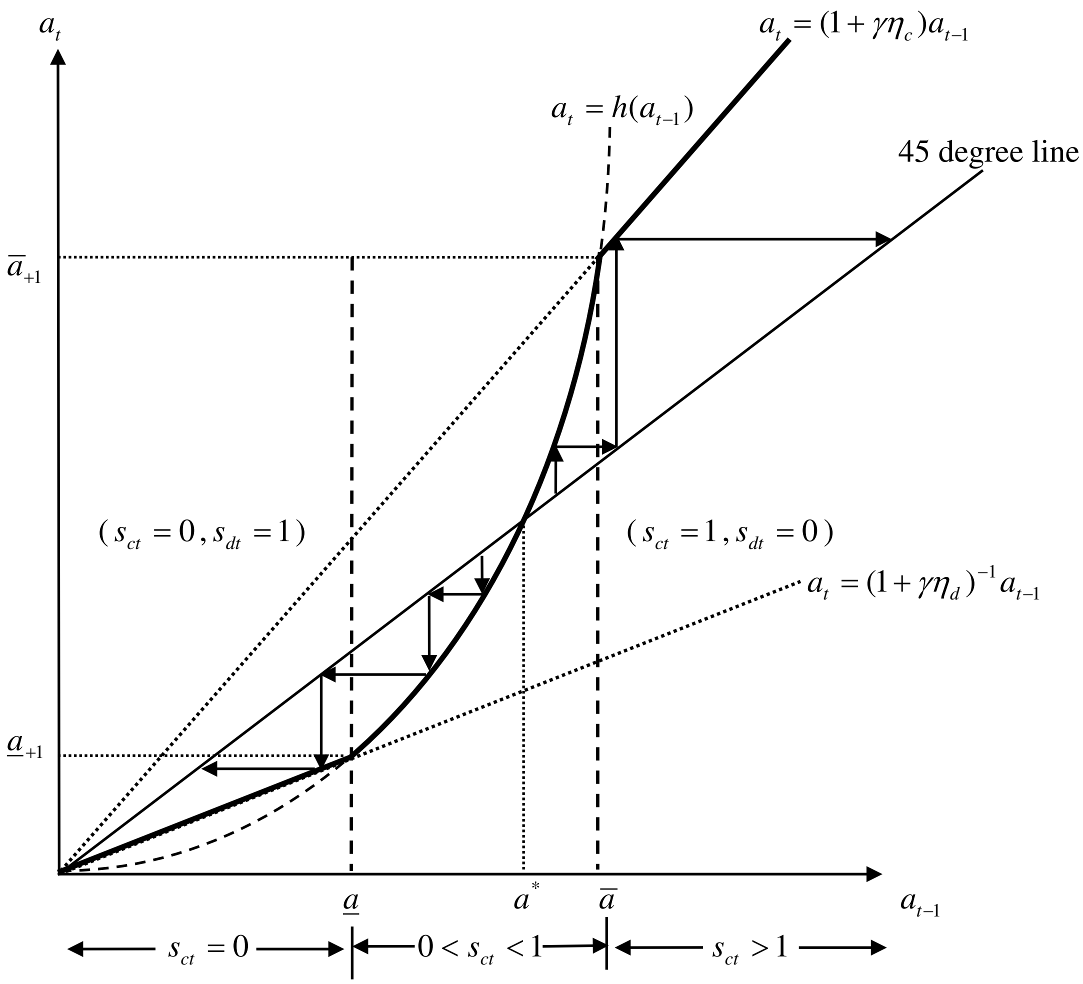

Fig.2 Equilibrium Dynamics of a_t

Consider the system of equations (21) and (27). Solving it for a_{t-1}, we have

$$a_{t-1} = (1+\gamma\eta_d)^{(\varphi+1)/\varphi}(\eta_c/\eta_d)^{1/\varphi},$$

which is equal to $\underline{a}$ in (18). This implies that the $a_t = h(a_{t-1})$ curve intersects the $a_t = (1+\gamma\eta_d)^{-1}a_{t-1}$ curve at $\underline{a}$, as is shown in Fig 2. Similarly, solving the system of equations that consists of (24) and (27) gives

$$a_{t-1} = (1+\gamma\eta_c)^{-(\varphi+1)/\varphi}(\eta_c/\eta_d)^{1/\varphi},$$

which is equal to $\bar{a}$ in (17). Hence, the $a_t = h(a_{t-1})$ curve intersects the $a_t = (1+\gamma\eta_c)a_{t-1}$ at $\bar{a}$, as is shown in Fig 2.

Noticing that $(1+\gamma\eta_d)^{-1} < 1 < (1+\gamma\eta_c)$, we can depict the phase diagram for a_t shown in Fig. 2. This leads us to the following main proposition of this note.

Proposition 2

There exists a threshold technology ratio a^*. *When the initial ratio is below it, then the economy will experience the environmental disaster soon or later. When the initial ratio is above it, then it will restore the environment in the end if and only if the two inputs are strong substitutes.*

The analysis implies that the environmental disaster can occurs in wider range of parameters than Acemoglu et. al. (2012) has assumed. Moreover, it is shown that the threshold technology ratio a^* is important in determining the dynamic path of the economy.

4. Concluding remarks

Directed technological change by Acemoglu (1998, 2002) is a crucial and useful addition not only to the growth literature but also to the environmental economics. Acemoglu et. al. (2012) clearly shows it and provides an interesting discussion on the relationship between growth and environment. It is no doubt that their model will be a canonical model in the field.

To make the point clear, they restrict the range of initial technology levels. This note examines the dynamics of the relative technology ratio without the restriction. As a result, it is shown that that their discussions remain valid in the wide range of the initial technology levels. The analysis developed in this paper is important not only in understanding their model in depth but also in examining policy implications in the relationship between growth and environment. It is hoped that the note will be a useful supplement to Acemoglu et. al. (2012) and stimulates future research.

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