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### International Migration, Skill Acquisition and Matching Frictions

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#### Abstract

This paper investigates the impacts of the prospect of work abroad on a sending country's labor market outcomes in a search and matching environment with endogenous skill acquisition. Our model allows firms to adjust their demand for labor to changes in the prospect of work abroad. We show that an increase in the probability of work abroad for skilled (unskilled) workers discourages job entry in the skilled (unskilled) labor market, and makes it more difficult for skilled (unskilled) workers to find employment. Therefore, such change may not always result in an increase in the fraction of skilled workers, depending on the value of work abroad.

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# 1 Introduction

It has been widely accepted that the leave of skilled workers in developing countries would create a brain drain problem. However, some studies argue that such openness to international migration raises the return on human capital in a developing country, which, in turn, may increase the average level of human capital in the home country (among others, see Mountford, 1997; Stark, Helmenstein and Prskawetz, 1998; Stark and Wang, 2002). Recently, based on a migration model with job search, Fan and Stark (2007a) show that the prospect of work abroad would induce a brain gain and "educated unemployment" because workers who fail to get jobs abroad will choose to remain unemployed to engage in a repeated attempt to find jobs in foreign countries instead of taking jobs in the home country. With the assumption of exogenous wage rate in the home country, their analysis focuses only on the supply-side effect of labor market as individuals adjust to changes in the probability of work abroad by changing their skill acquisition and ignores the fact that these changes will also affect firms' demand of skilled workers.<sup>1</sup>

This paper complements the existing literature on brain drain by examining the effects of the probability of work abroad on the acquisition of skill, wage rate and unemployment in a general-equilibrium model. We introduce Mortensen (1982)-Pissaridis (1984) labor search-matching framework into a migration model. When considering determinants of the brain drain, previous literature only looks at wages. With labor market frictions, our model looks not only at wages but also at unemployment rates. Because workers with different levels of skill face different probabilities of work abroad, we consider two types of workers: skilled versus unskilled workers. Before entering the labor market and given the probabilities of work abroad for skilled (unskilled) workers, agents make decisions on skill acquisition.<sup>2</sup> Depending on agents' innate ability, they may choose to remain unskilled or to spend a fixed cost for skill training and become skilled workers. Skilled and unskilled workers face separate labor markets and different probabilities of work abroad. Both skilled and unskilled workers can look for jobs in the home country or abroad.<sup>3</sup> Unfilled vacancies and job seekers are brought together by a matching technology that exhibits constant returns. Wages are the outcome of Nash bargaining between each individual firm and worker.

We show that there exists a unique equilibrium. For skilled (unskilled) workers, the probability of work abroad is negatively correlated with the wage rate and the labor market tightness and positively correlated with the unemployment rate. An increase in the probability of work abroad may cause an increase or a decrease in the fraction of skilled workers. For skilled workers, if the value of work abroad is sufficiently high (low), then an increase in the probability of work abroad will result in an increase (a decrease) in the fraction of skilled workers. However, for unskilled workers, an increase in the probability of work abroad will

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<sup>1</sup>Fan and Stark (2007b) adopt a framework that is similar to that in Fan and Stark (2007a) to examine the effects of international migration distinguishing between the short run and the long run. They show that in the short run international migration can result in "educated unemployment" and overeducation, while a relaxed migration policy can prompt "take-off" of the economy in the long run.

<sup>2</sup>A labor searching-matching model with education choice is developed by Kolm and Tonin (2004) to argue that one of the factors behind the success of the Nordic countries is that welfare benefits are often tied to employment.

<sup>3</sup>Stark and Byra (2012) also consider the impact of migration of *unskilled* workers on skill formation in the home country. They show that the home country may suffer reduced aggregate skill formation.

cause the reversed effect on skill acquisition.

## 2 The Model

Consider a developing economy inhabited by a unit mass of workers that are risk neutral and discount the future at a constant rate  $r > 0$ . Before entering the labor market, each worker decides whether to invest in education and become skilled or remain unskilled. Therefore, workers are either skilled ( $H$ ) or unskilled ( $L$ ). We use the index  $i$  to distinguish their skill level,  $i = H, L$ . The share of skilled workers in the population is represented by  $b \in (0, 1)$ ; thus,  $1 - b$  is the fraction of workers that are unskilled.

There is also a large continuum of firms. Each firm can have at most one job, which is suited either for a skilled ( $H$ ) or for an unskilled ( $L$ ) worker. The index  $i$  is also used to distinguish between the two types of jobs,  $i = H, L$ . Firms must decide before searching for a worker whether they will open a skilled or an unskilled job. Skilled jobs are filled by a skilled worker. The flow of output produced by such a pair is  $y_H$ . By contrast, an unskilled job is filled by unskilled workers whose flow of productivity in these jobs is  $y_L$ .

### 2.1 Search and Matching

Each firm posts either a skilled or an unskilled vacancy and incurs a flow cost  $c_i$ ,  $i = H, L$ . Free entry determines endogenously the number of firms in each labor market. On the other hand, unemployed workers search for employment. In particular, skilled workers direct their search towards the skilled labor market, whereas unskilled workers search for unskilled jobs. Skilled and unskilled workers face a prospect of working in a foreign developed country. The migration probabilities for skilled and unskilled workers are respectively denoted by  $\delta_H$  and  $\delta_L$ , with  $\delta_i \in [0, 1]$ ,  $i = H, L$ . Employed and unemployed skilled (unskilled) workers face the same migration probability  $\delta_H$  ( $\delta_L$ ). To simplify our analysis, we assume that the sending country's skill pool remains unchanged because any migrating worker is replaced by an identical worker.

Job seekers and vacant jobs are matched randomly in a pair-wise fashion. The mass of successful job matches in each labor market is determined by the matching function  $M(u_i, v_i)$ , where  $u_i$  and  $v_i$  denote respectively the number of unemployed workers and vacancies of skill type  $i$ ,  $i = H, L$ . We define the labor market tightness in labor market  $i$  as  $\theta_i = v_i/u_i$ . The rates at which unskilled and skilled vacancies are filled are  $q(\theta_i) = M_i/v_i$ , where  $q'(\theta_i) < 0$ . The rate at which unemployed type  $i$  workers find jobs is  $m(\theta_i) = \theta_i q(\theta_i)$ , where  $m'(\theta_i) > 0$ . We also assume that matches dissolve at a rate  $s_i$ , which is specific to their type.

### 2.2 Asset Value Functions

Let  $\Pi$  and  $V$  be the values associated with a filled and unfilled vacancy, and  $E$ , and  $U$  the values associated with an employed and an unemployed worker, respectively. More specifically, let  $\Pi_i$  be the present discounted value associated with a firm of type  $i$  which is matched with a worker of skill  $i$  and  $V_i$  the expected income streams accrued to unfilled vacancy of type  $i$ . Then in steady state:

$$r\Pi_i = y_i - w_i - \gamma_i[\Pi_i - V_i], \quad \text{if } i = H, L, \quad (1)$$

$$rV_i = -c_i + q(\theta_i)[\Pi_i - V_i], \quad (2)$$

where  $w_i$  is the wage rate of a worker who has skill  $i = H, L$ . Because workers may find employment in a foreign country, given that  $\lambda_i$  is the exogenous job separation rate, then for unskilled workers,  $\gamma_L = \lambda_L + \delta_L$ , whereas for skilled workers,  $\gamma_H = \lambda_H + \delta_H$ .

In order for skilled workers to maintain skills, agents need to pay certain cost. We use  $\phi$  to denote this cost. The expected income streams accrued to employed workers are:

$$rE_H = w_H - \lambda_H(E_H - U_H) + \delta_H(E_H^f - E_H) - \phi, \quad (3)$$

$$rE_L = w_L - \lambda_L(E_L - U_L) + \delta_L(E_L^f - E_L). \quad (4)$$

In particular, equation (3) gives the flow income accrued to a skilled worker in a skilled position. The third term on the right-hand side (RHS) gives the change in this value because of the work abroad option, where  $E_H^f$  represents the value associated with an employed skilled worker in the foreign labor market. Similarly, the last term on the RHS of (4) gives the change in this value because of the work abroad option, where  $E_L^f$  represents the value associated with an employed unskilled worker in the foreign labor market. We assume that  $E_i^f > E_i$  so that workers will be willing to migrate to the foreign country if they have the chance to migrate.

The values associated with unemployed workers are:

$$rU_H = m(\theta_H)(E_H - U_H) + \delta_H(E_H^f - U_H) - \phi, \quad (5)$$

$$rU_L = m(\theta_L)(E_L - U_L) + \delta_L(E_L^f - U_L). \quad (6)$$

Free entry implies that, in equilibrium, the expected payoff of posting a vacancy is equal to zero, that is,

$$V_i = 0, \quad i = H, L. \quad (7)$$

### 2.3 Wage Determination

Once a worker meets a firm, they bargain over the wage rate. They solve a generalized Nash bargaining problem given by

$$\underset{w_i}{Max} (E_i - U_i)^\beta (\Pi_i - V_i)^{(1-\beta)},$$

where  $\beta \in (0, 1)$  represents the worker's bargaining strength. The solution to this problem gives

$$(1 - \beta)(E_i - U_i) = \beta(\Pi_i - V_i), \quad (8)$$

where  $E_i - U_i$  and  $\Pi_i - V_i$  are the worker's and the firm's surplus from the match, respectively.

Substituting for  $E_i - U_i$  and  $\Pi_i$ , using equations (1) - (6), in equation (8) and noting that  $V_i = 0$  (equation 7), we find

$$w_H = \frac{r + \lambda_H + \delta_H + m(\theta_H)}{r + \lambda_H + \delta_H + \beta m(\theta_H)} \beta y_H, \quad (9)$$

$$w_L = \frac{r + \lambda_L + \delta_L + m(\theta_L)}{r + \lambda_L + \delta_L + \beta m(\theta_L)} \beta y_L. \quad (10)$$

## 2.4 Endogenous Skill Acquisition

Before entering the labor market each individual decides whether to invest in education and become skilled or remain unskilled. Agents differ with respect to their cost of acquiring education. We denote the cost of acquiring training by  $z$  and assume that it is distributed uniformly over the closed interval  $[0, \bar{z}]$ . As agents enter the labor market in the state of unemployment, they compare the values of unemployment for skilled and unskilled workers when making their decision on skill acquisition. An agent will invest in education if the benefit from this decision exceeds the cost, that is, a worker will invest in education if

$$z \leq U_H - U_L.$$

Thus, all agents with a cost of education lower than some value  $z^*$  will invest in education, where  $z^*$  is given by

$$z^* = U_H - U_L.$$

In this case  $b^*$ , the fraction of workers that are skilled, is endogenous and is given by

$$b^* = \frac{z^*}{\bar{z}}. \quad (11)$$

## 3 Steady-State Equilibrium

Using the free-entry conditions (equation 7), we derive the following system

$$\frac{c_H}{q(\theta_H)} = \frac{1 - \beta}{r + \lambda_H + \delta_H + \beta m(\theta_H)} y_H, \quad (12)$$

$$\frac{c_L}{q(\theta_L)} = \frac{1 - \beta}{r + \lambda_L + \delta_L + \beta m(\theta_L)} y_L. \quad (13)$$

An equilibrium is a vector  $(\theta_H^*, \theta_L^*)$  that solves (12) and (13).

**Proposition 1 (Existence and Uniqueness)** *A steady-state equilibrium exists and is unique.*

**Proof.** The proof of Proposition 1 is presented in the Appendix. ■

At the steady-state equilibrium, the flow into unemployment equals the flow out of unemployment for each type of workers. The steady-state unemployment rates ( $u_i$ ) for skilled and unskilled workers are

$$u_H = \frac{\lambda_H + \delta_H}{\lambda_H + \delta_H + m(\theta_H)}, \quad (14)$$

$$u_L = \frac{\lambda_L + \delta_L}{\lambda_L + \delta_L + m(\theta_L)}. \quad (15)$$

### 3.1 Migration Probability

We first examine the effect of the probability of work abroad on the wage rates, labor market tightness and unemployment.

**Proposition 2** *If the probability of work abroad becomes higher for the type- $i$  workers, then  $\frac{d\theta_i^*}{d\delta_i} < 0$ ,  $\frac{dw_i}{d\delta_i} < 0$  and  $\frac{du_i}{d\delta_i} > 0$ ,  $i = H, L$ .*

**Proof.** The proof of Proposition 2 is presented in the Appendix. ■

For skilled (unskilled) workers, an increase in the probability of work abroad results in a higher separation rate between skilled (unskilled) workers and skilled (unskilled) jobs, which reduces firm's expected profit from the creation of a skilled (unskilled) position. Therefore, an increase in the prospect of work abroad for skilled (unskilled) workers discourages entry of skilled (unskilled) jobs and reduces the tightness of labor market for skilled (unskilled) workers. With the reduction in the tightness of labor market for skilled (unskilled) workers, skilled (unskilled) worker's bargaining power decreases and their wage rate becomes lower.<sup>4</sup> Moreover, the finding rate of skilled (unskilled) jobs for these workers goes down and thus their unemployment rate goes up.

We then examine the effects of the probability of work abroad on skill acquisition. Combining equations (5)-(6) and equation (8) and substituting away  $U_i$ ,  $i = H, L$ , we obtain the following equation

$$U_H - U_L = \frac{\beta}{1 - \beta} \left( \frac{c_H \theta_H}{r + \delta_H} - \frac{c_L \theta_L}{r + \delta_L} \right) + \frac{\delta_H}{r + \delta_H} (E_H^f - \phi) - \frac{\delta_L}{r + \delta_L} E_L^f. \quad (16)$$

Substituting (16) into (11) gives

$$b^* = \frac{1}{\bar{z}} \left[ \frac{\beta}{1 - \beta} \left( \frac{c_H \theta_H}{r + \delta_H} - \frac{c_L \theta_L}{r + \delta_L} \right) + \frac{\delta_H}{r + \delta_H} (E_H^f - \phi) - \frac{\delta_L}{r + \delta_L} E_L^f \right].$$

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<sup>4</sup>The result of migration on wages critically depends on the assumption that both employed and unemployed workers migrate with the same probability. If only unemployed workers were to get a chance to leave, the increase in the migration probability (for unskilled workers) would raise the wage for skilled workers because the firm would have to compensate employed workers for giving up the chance to migrate. We thank the referee for pointing this out.

Differentiating  $b^*$  with respect to  $\phi$  indicates that

$$\frac{db^*}{d\phi} = -\frac{\delta_H}{\bar{z}(r + \delta_H)} < 0.$$

An increase in  $\phi$  means that the cost of maintaining skills increases, lowering the incentive to acquire skills and reducing  $b^*$ .

Differentiating  $b^*$  with respect to  $\delta_H$  and  $\delta_L$  and evaluating it at the steady state, we have

$$\begin{aligned} \frac{db^*}{d\delta_H} &= \frac{1}{\bar{z}(r + \delta_H)^2} \left\{ -\frac{\beta c_H}{1 - \beta} \left[ \theta_H^* - (r + \delta_H) \frac{d\theta_H^*}{d\delta_H} \right] + r \left( E_H^f - \phi \right) \right\}, \\ \frac{db^*}{d\delta_L} &= \frac{1}{\bar{z}(r + \delta_L)^2} \left\{ \frac{\beta c_L}{1 - \beta} \left[ \theta_L^* - (r + \delta_L) \frac{d\theta_L^*}{d\delta_L} \right] + r E_L^f \right\}. \end{aligned}$$

Then  $\frac{db^*}{d\delta_H} > 0$  if  $E_H^f > \frac{\beta c_H}{r(1-\beta)} \left[ \theta_H^* - (r + \delta_H) \frac{d\theta_H^*}{d\delta_H} \right] + \phi$  and vice versa. Thus, an increase in the probability of work abroad for skilled workers will cause an increase (a decrease) in the fraction of skilled workers if the value of work abroad is sufficiently high (low). On the other hand, we have  $\frac{db^*}{d\delta_L} < 0$  if  $E_L^f > \frac{\beta c_L}{r(1-\beta)} \left[ \theta_L^* - (r + \delta_L) \frac{d\theta_L^*}{d\delta_L} \right]$  and vice versa. This indicates that an increase in the probability of work abroad for unskilled workers will cause a decrease (an increase) in the fraction of skilled workers if the value of work abroad is sufficiently high (low). We then have the following Proposition.

**Proposition 3** *An increase in the probability of work abroad for skilled (unskilled) workers can lead to an increase (decrease) in the fraction of skilled workers if the positive contribution of the work abroad option outweighs the emigration-induced adverse effect on skill acquisition incentives for agents, or a decrease (an increase) in the fraction of skilled workers if the latter effect dominates.*

There are two forces that work in opposite directions to affect agents' skill acquisition decisions. On the one hand, an increase in the probability of work abroad for skilled workers raises the return on human capital, which encourages skill acquisition. On the other hand, an increase in the probability of work abroad for skilled workers also discourages firms' entry in the skilled labor market, which decreases the bargaining position of skilled workers and lowers their wage and employment. This effect discourages skill acquisition. Therefore, these two forces work together to determine whether a prospect of work abroad can lead to an increase (or a decrease) in the fraction of skilled workers. However, an increase in the probability of work abroad for unskilled workers will cause the opposite effects on the skill acquisition.

## 4 Conclusions

This paper examines the effect of the probability of work abroad on the labor-market outcomes based on a migration model with job search and matching. We show that an increase in the probability of work abroad for skilled (unskilled) workers raises the unemployment

rate while reducing the labor market tightness and wage rate for skilled (unskilled) workers. Besides, such change in the probability of work abroad may cause an increase or a decrease in the fraction of skilled workers, depending on the value of working abroad.

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## 6 Appendix

This appendix provides the mathematical proofs of Propositions 1 and 2.

**Proof of Proposition 1.** From (12) and (13), we have

$$\frac{q(\theta_i)}{r + \lambda_i + \delta_i + \beta m(\theta_i)} - \frac{c_i}{(1-\beta)y_i} = 0, \quad i = H, L. \quad (17)$$

Define  $h_i(\theta_i) = \frac{q(\theta_i)}{r + \lambda_i + \delta_i + \beta m(\theta_i)} - \frac{c_i}{(1-\beta)y_i}$ . Note that

$$\frac{dh_i(\theta_i)}{d\theta_i} = \frac{[r + \lambda_i + \delta_i + \beta m(\theta_i)] \left[ \frac{dq(\theta_i)}{d\theta_i} \right] - \beta q(\theta_i) \left[ \frac{dm(\theta_i)}{d\theta_i} \right]}{[r + \lambda_i + \delta_i + \beta m(\theta_i)]^2} < 0.$$

Furthermore,  $\lim_{\theta_i \rightarrow 0} h_i(\theta_i) = \infty$  and  $\lim_{\theta_i \rightarrow \infty} h_i(\theta_i) = -\frac{c_i}{(1-\beta)y_i} < 0$ . Therefore, there exists a unique solution of  $\theta_i$  for (17).

Q.E.D.

**Proof of Proposition 2.** Totally differentiating (12) and (13) with respect to  $\theta_i$  and  $\delta_i$  yields:

$$\frac{d\theta_i^*}{d\delta_i} = \frac{q(\theta_i^*)}{[r + \lambda_i + \delta_i + \beta m(\theta_i^*)] \left( \frac{dq(\theta_i^*)}{d\theta_i} \right) - \beta q(\theta_i^*) \left( \frac{dm(\theta_i^*)}{d\theta_i} \right)} < 0.$$



Using (9) and (10) to differentiate  $w_i$  with respect to  $\delta_i$  gives:

$$\frac{dw_i}{d\delta_i} = \frac{\beta y_i (1 - \beta) (r + \lambda_i + \delta_i) \left( \frac{dm(\theta_i^*)}{d\theta_i} \right)}{[r + \lambda_i + \delta_i + \beta m(\theta_i^*)]^2} \left( \frac{d\theta_i^*}{d\delta_i} \right) < 0.$$

Using (14) and (15) to differentiate  $u_i$  with respect to  $\delta_i$  gives:

$$\frac{du_i}{d\delta_i} = \frac{m(\theta_i^*) - (\lambda_i + \delta_i) \left( \frac{dm(\theta_i^*)}{d\theta_i} \right) \left( \frac{d\theta_i^*}{d\delta_i} \right)}{[\lambda_i + \delta_i + m(\theta_i^*)]^2} > 0.$$

Q.E.D.